Oddziaływania efektywne niezależne od gęstości

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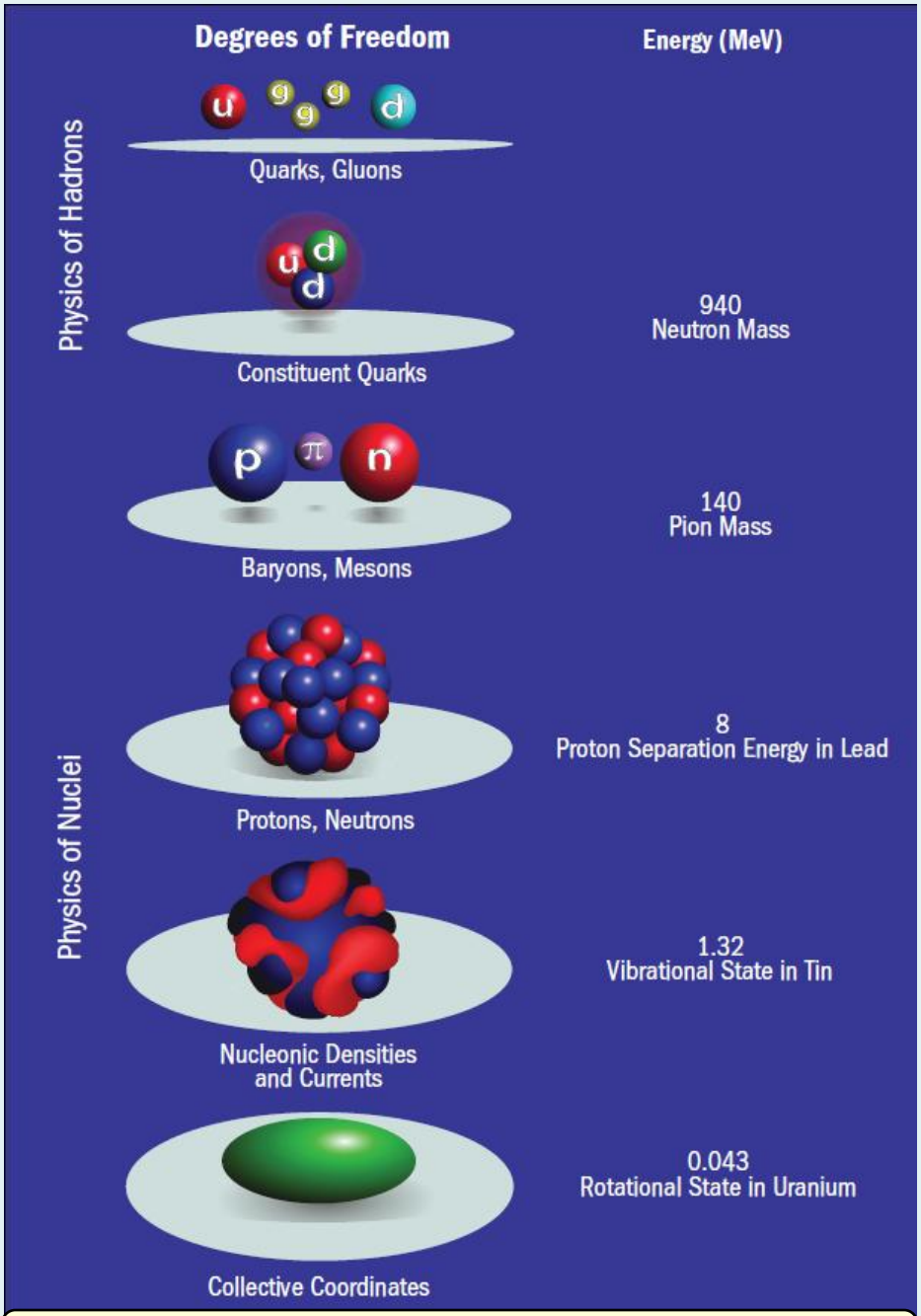
Outline

1. Introduction - effective theories in physics
2. Energy density functionals in nuclear physics
3. Effective interactions vs. functional generators
4. Regularized pseudopotentials
5. Three-body forces vs. density-dependent terms
6. New functionals without density dependence
7. Conclusions

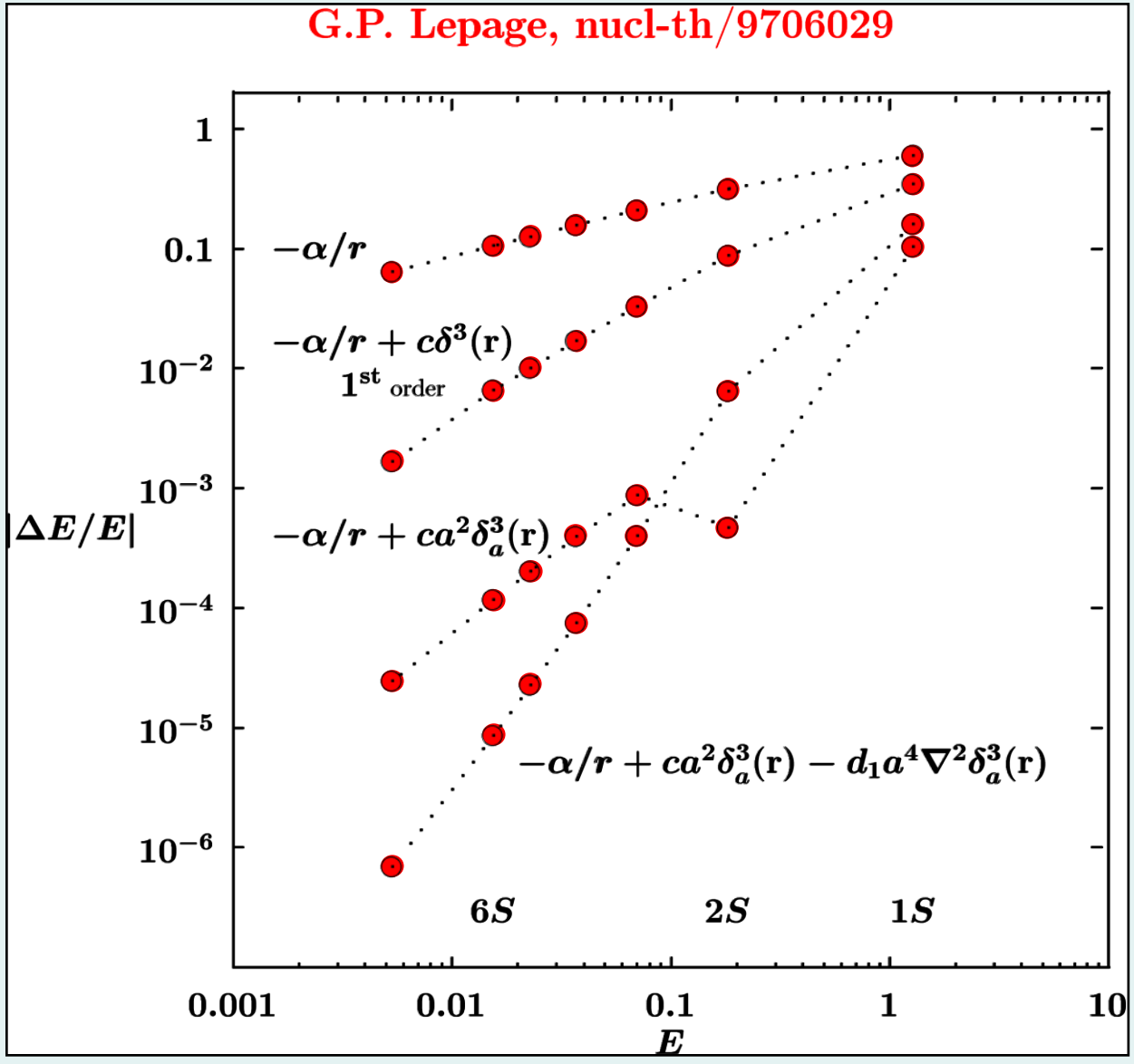


G.F. Bertsch, *et al.*, Scidac Review 6, 42 (2007)

- An effective theory (ET) is a theory which "effectively" captures what is physically relevant in a given domain.
- The most appropriate description of particle interactions in the language of quantum field theory (QFT) depends on the energy at which the interactions are studied.
- Objective reductionism (Weinberg): the convergence of arrows of scientific explanation.
- Emergence (Anderson): at each new level of complexity entirely new properties appear and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other.

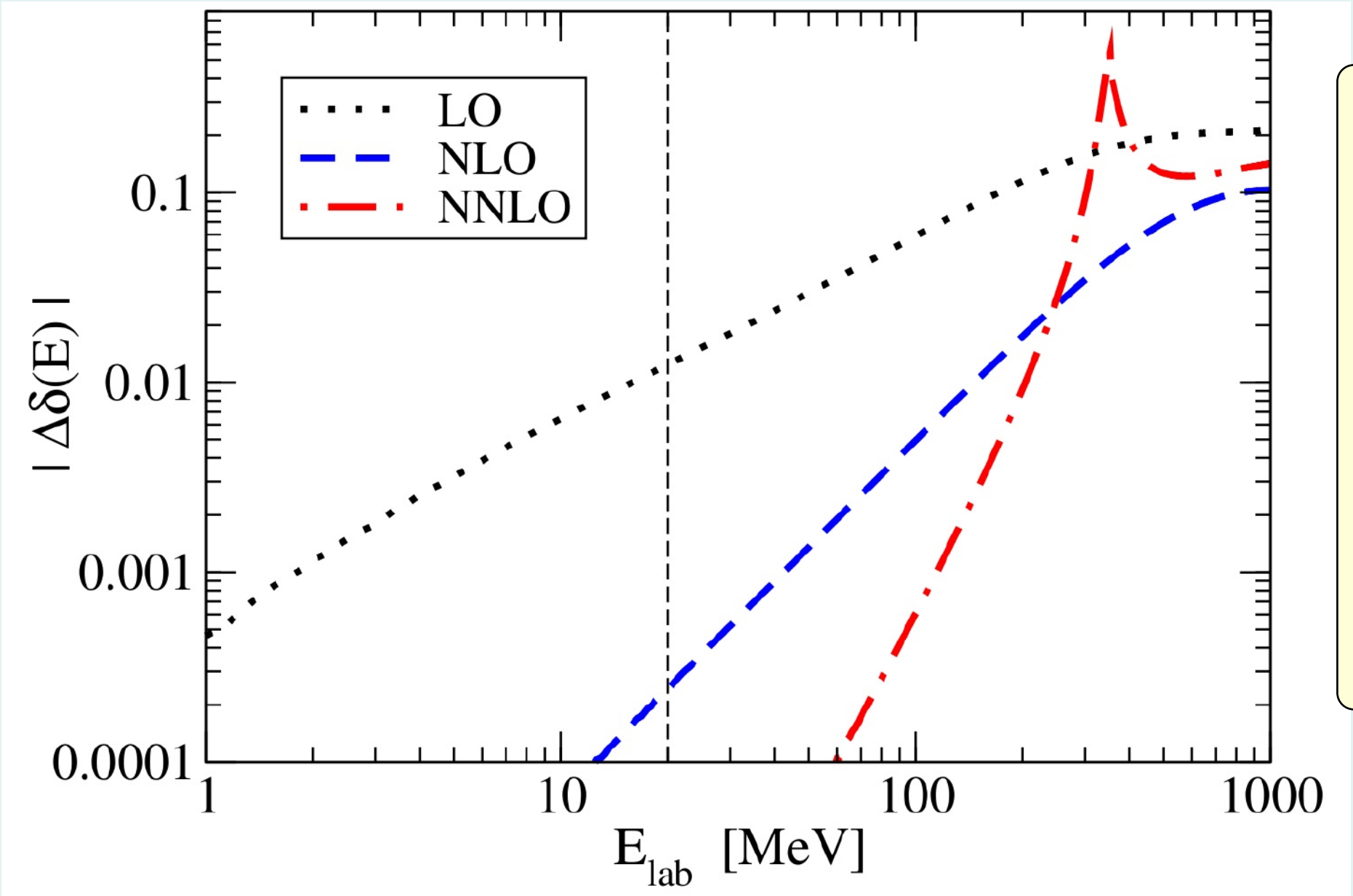
Elena Castellani, physics/0101039

Hydrogen atom perturbed near the center



Relative errors in the S-wave binding energies are plotted versus:
(i) the binding energy for the Coulomb theory
(ii) the Coulomb theory augmented with a delta function in first-order perturbation theory
(iii) the non-perturbative effective theory through a^2, and
(iv) the effective theory through a^4.

Phase shifts in the low-momentum expansion



Evgeny Epelbaum, arXiv:1001.3229

How the nuclear EDF is built?

$$E[\rho(\vec{r}_1,\vec{r}_2)] = \int\int \mathrm{d}\vec{r}_1 \mathrm{d}\vec{r}_2\, \mathcal{H}(\rho(\vec{r}_1,\vec{r}_2))$$

Energy Density Functional (EDF)

Energy Density

$$\mathcal{H}(\rho(\vec{r}_1,\vec{r}_2)) = V(\vec{r}_1-\vec{r}_2)\Big[\rho(\vec{r}_1)\rho(\vec{r}_2) - \rho(\vec{r}_1,\vec{r}_2)\rho(\vec{r}_2,\vec{r}_1)\Big]$$

Direct

Exchange

Phenomenological functional generators

- Gogny*

$$V(\vec{r}_1\vec{r}_2;\vec{r}\,'_1\vec{r}\,'_2)=\delta(\vec{r}_1-\vec{r}\,'_1)\delta(\vec{r}_2-\vec{r}\,'_2)V(\vec{r}_1-\vec{r}_2),$$

where,

$$V(\vec{r}_1-\vec{r}_2)=\sum\nolimits_{i=1,2}e^{-(\vec{r}_1-\vec{r}_2)^2/\mu_i^2}\times(W_i+B_iP_\sigma-H_iP_\tau-M_iP_\sigma P_\tau)$$

$$+t_3(1+P_\sigma)\delta(\vec{r}_1-\vec{r}_2)\rho^{1/3}\left[\tfrac{1}{2}(\vec{r}_1+\vec{r}_2)\right].$$

$P_\sigma=\frac{1}{2}(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)$ and $P_\tau=\frac{1}{2}(1+\vec{\tau}_1\cdot\vec{\tau}_2)$ are, respectively, the spin and isospin exchange operators of particles 1 and 2, $\rho(\vec{r})$ is the total density of the system at point $\vec{r}$, and μ_i=0.7 and 1.2 fm, W_i, B_i, H_i, M_i, and t_3 are parameters.

- Skyrme*

$$V(\vec{r}_1\vec{r}_2;\vec{r}\,'_1\vec{r}\,'_2)=\Big\{t_0(1+x_0P^\sigma)+\tfrac{1}{6}t_3(1+x_3P^\sigma)\rho^\alpha\left(\tfrac{1}{2}(\vec{r}_1+\vec{r}_2)\right)$$

$$+\tfrac{1}{2}t_1(1+x_1P^\sigma)[\vec{k}'^{*2}+\vec{k}^2]+t_2(1+x_2P^\sigma)\vec{k}'^{*}\cdot\vec{k}\Big\}\,\delta(\vec{r}_1-\vec{r}\,'_1)\delta(\vec{r}_2-\vec{r}\,'_2)\delta(\vec{r}_1-\vec{r}_2),$$

where the relative-momentum operators read $\hat{\vec{k}}=\frac{1}{2i}\left(\vec{\nabla}_1-\vec{\nabla}_2\right)$, $\hat{\vec{k}}'=\frac{1}{2i}\left(\vec{\nabla}\,'_1-\vec{\nabla}\,'_2\right)$.

*We omit the spin-orbit and tensor terms for simplicity.

Regularized pseudopotentials

We regularize delta interaction by using the Gaussian function,

$$\delta(\vec{r}) = \lim_{a\to 0} g_a(\vec{r}) = \lim_{a\to 0} \frac{e^{-\frac{\vec{r}^2}{a^2}}}{\left(a\sqrt{\pi}\right)^3}.$$

Then, the resulting central two-body regularized pseudopotential reads,

$$V(\vec{r}_1\vec{r}_2;\vec{r}\,'_1\vec{r}\,'_2) = \sum_{i=1}^{4} \hat{P}_i\hat{O}_i(\vec{k},\vec{k}\,')\delta(\vec{r}_1-\vec{r}\,'_1)\delta(\vec{r}_2-\vec{r}\,'_2)g_a(\vec{r}_1-\vec{r}_2),$$

where $\vec{k} = \frac{1}{2i}(\vec{\nabla}_1 - \vec{\nabla}_2)$ and $\vec{k}' = \frac{1}{2i}(\vec{\nabla}'_1 - \vec{\nabla}'_2)$ are the standard relative-momentum operators, and the Wigner, Bartlett, Heisenberg, and Majorana terms are given by the standard spin and isospin exchange operators, $\hat{P}_1 \equiv 1$, $\hat{P}_2 \equiv \hat{P}_\sigma$, $\hat{P}_3 \equiv -\hat{P}_\tau$, $\hat{P}_4 \equiv -\hat{P}_\sigma\hat{P}_\tau$.

To give a specific example, up to the second-order, that is, up to the next-to-leading-order (NLO) expansion, operators $\hat{O}_i(\vec{k},\vec{k}')$ read

$$\hat{O}_i(\vec{k},\vec{k}') = T_0^{(i)} + \frac{1}{2}T_1^{(i)}\left(\vec{k}'^{*2} + \vec{k}^2\right) + T_2^{(i)}\vec{k}'^{*}\cdot\vec{k},$$

where $T_k^{(i)}$ are the channel-dependent coupling constants.

J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Regularized pseudopotentials

Let us first assume that the differential operators $\hat{O}_i(\vec{k},\vec{k}')$ depend only on the sum of relative momenta, that is,

$$\hat{O}_i(\vec{k},\vec{k}') = \hat{O}_i\left(\vec{k}+\vec{k}'\right) = \hat{O}_i\left(\vec{k}-\vec{k}'^{*}\right), \quad \text{which requires that } T_2^{(i)} = -T_1^{(i)}.$$

Such particular differential operators commute with the locality deltas $\delta(\vec{r}_1' - \vec{r}_1)\delta(\vec{r}_2' - \vec{r}_2)$, and thus can be applied directly onto the regularized delta $g_a(\vec{r}_1 - \vec{r}_2)$. In such a case, the pseudopotential reduces to a simple local potential

$$V(\vec{r}) = \sum_{i=1}^{4} \hat{P}_i V_i(\vec{r}), = \sum_{i=1}^{4} \hat{P}_i \hat{O}_i(\vec{k}) g_a(\vec{r}),$$

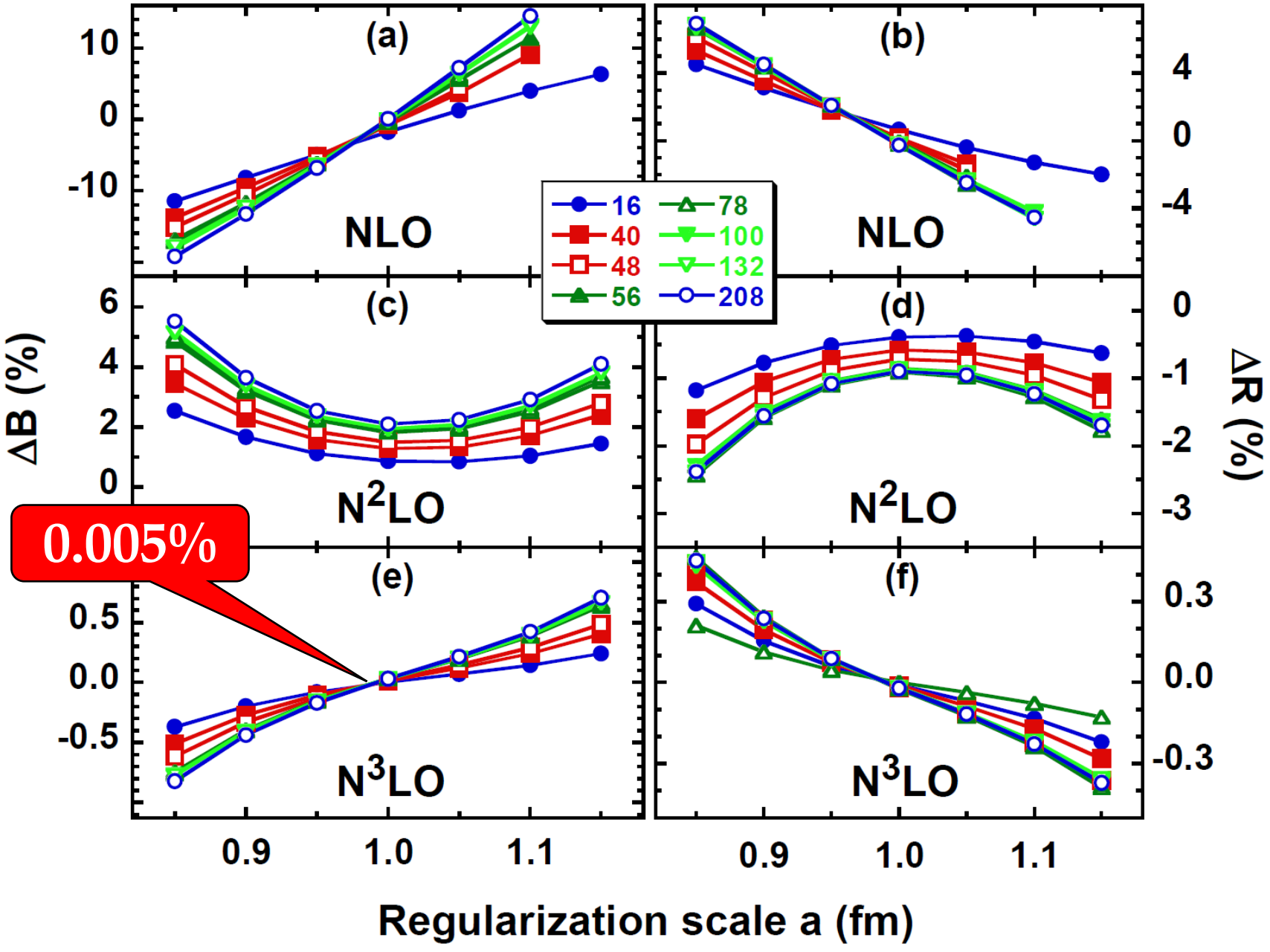
Moreover, since $\hat{O}_i(\vec{k})$ are scalar differential operators, the potentials must have forms of power series of Laplacians Δ in $\vec{r}$, that is,

$$V_i(\vec{r}) = \sum_{n=0}^{n_{max}} V_{2n}^{(i)} \Delta^n g_a(\vec{r}),$$

where $V_{2n}^{(i)}$ are the coupling constants at order $2n$.

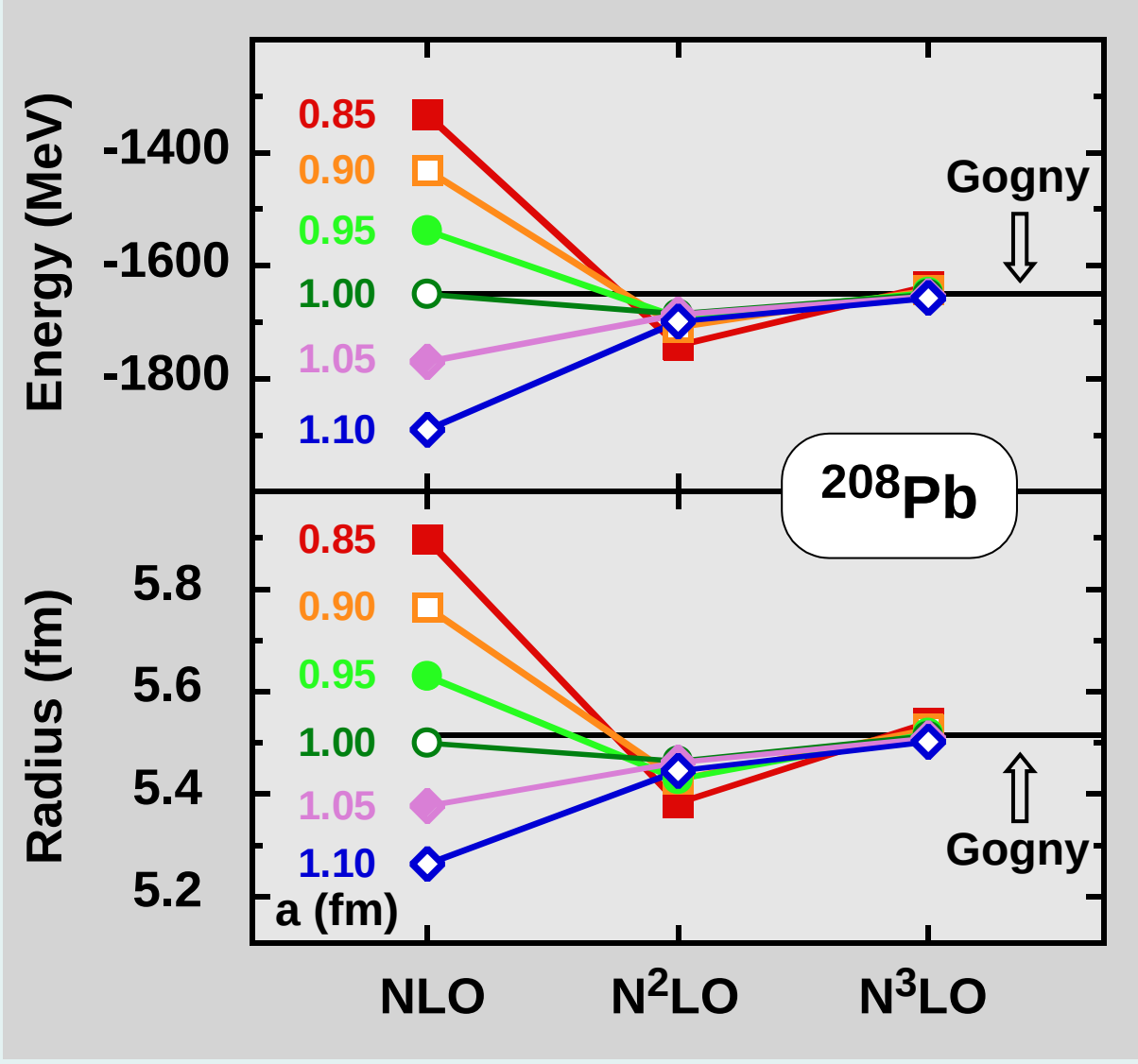
J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Regularized pseudopotentials vs. Gogny



J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

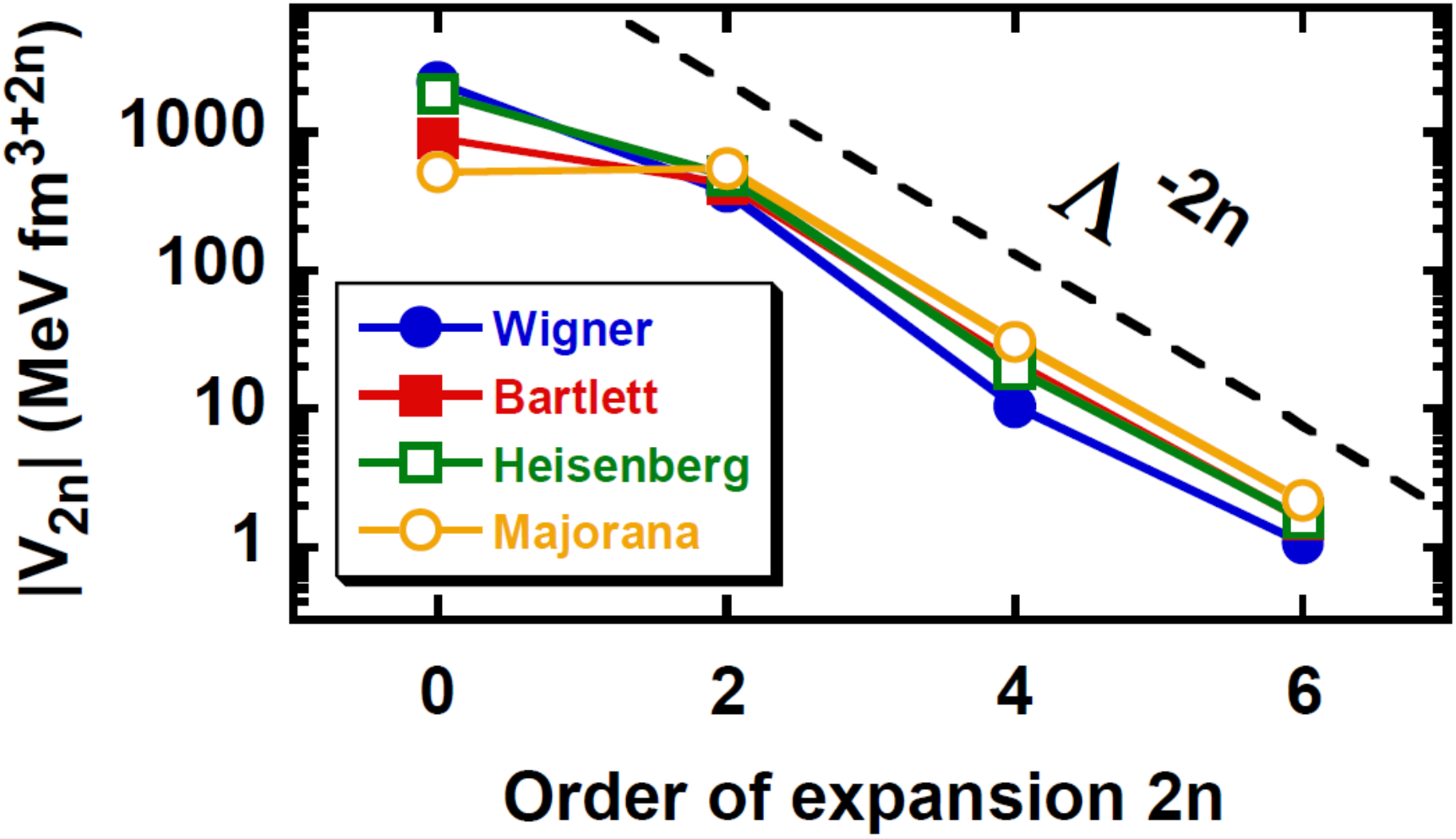
Regularized pseudopotentials vs. Gogny



J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Coupling constants of the regularized pseudopotentials

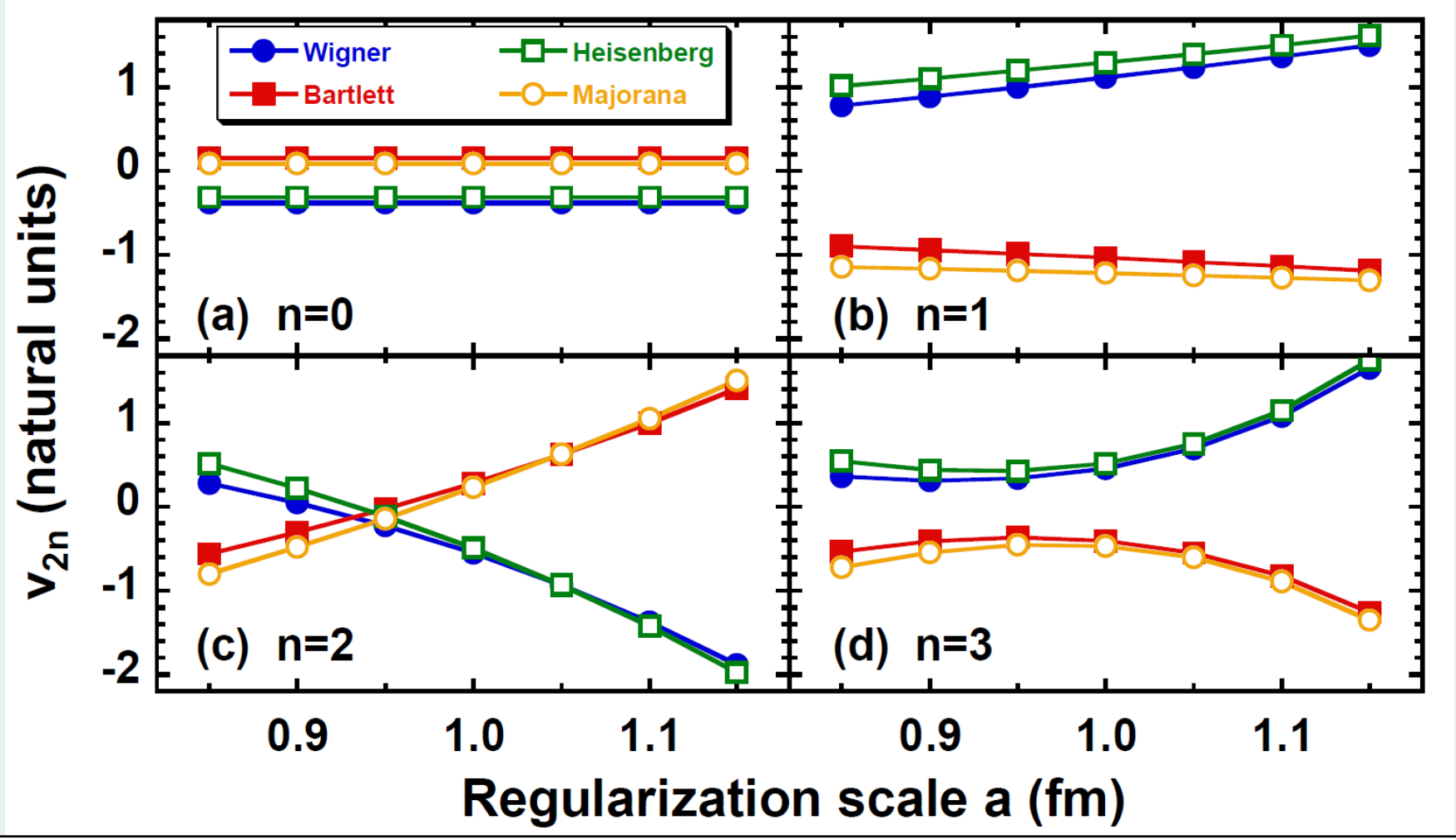
$\Lambda \approx 700$ MeV/$\hbar c \approx 3.8$ fm^{-1}



J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Coupling constants of the regularized pseudopotentials in natural units

$$v_{2n} = f^2 \Lambda^{2n} V_{2n} \text{ for } f = 35\ \text{MeV}/(\hbar c)^{3/2}$$



J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Skyrme's three-body interaction

1.C:
1.D.1

Nuclear Physics **9** (1959) 615—634; © *North-Holland Publishing Co., Amsterdam*

THE EFFECTIVE NUCLEAR POTENTIAL

T. H. R. SKYRME
Atomic Energy Research Establishment, Harwell, Didcot, Berks.

Received 18 October 1958

The effective interaction potential will *not* be the same as that defined in the self-consistent many-body theory (to which the variational principle is not applicable). The potential used in our analysis must contain three-body, and generally many-body, terms which describe the way in which interaction between two particles is influenced by the presence of others; the two-body terms alone should be related closely to the scattering between free nucleons.

These considerations have led to the following ansatz for the form of the effective potential:

$$T = \sum_{i<j}\sum t_{ij} + \sum_{i<j<k}\sum\sum t_{ijk} \qquad (2)$$

in which the many-body effects have been simulated by three-body terms alone, for the sake of simplicity of calculation.

The density dependence is born

PHYSICAL REVIEW C VOLUME 5, NUMBER 3 MARCH 1972

Hartree-Fock Calculations with Skyrme's Interaction. I. Spherical Nuclei*

D. Vautherin and D. M. Brink

For the three-body force Skyrme also assumed a zero-range force

$$v_{123}^{(3)} = t_3 \delta(\vec{r}_1 - \vec{r}_2)\delta(\vec{r}_2 - \vec{r}_3)\,. \qquad (7)$$

In the following we will show that for Hartree-Fock calculations of even-even nuclei, this force is equivalent to a two-body density-dependent interaction:

$$v_{12} = \tfrac{1}{6} t_3 (1 + P_\sigma)\delta(\vec{r}_1 - \vec{r}_2)\rho\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right). \qquad (8)$$

Such a term provides a simple phenomenological representation of many-body effects, and describes the way in which the interaction between two nucleons is influenced by the presence of others.

Volume 56B, number 3 PHYSICS LETTERS 28 April 1975

SPIN SATURATION AND THE SKYRME INTERACTION ☆

B.D. CHANG*

Received 18 March 1975

Of existing variants of the Skyrme interaction, those with strong three-body terms – in particular the variant SIII that is in best accord with experiment – overbind odd-mass and odd-odd nuclei and produce unstable spin-saturated Hartree-Fock ground states in nuclear matter and in even-even light nuclei. This difficulty can be removed either by imposition of an additional stability condition or by abandoning the three-body term in favor of the two-body density-dependent interaction equivalent to it in spin-saturated HF states.

The density dependence is exploited

1.E.2

Nuclear Physics **A238** (1975) 29—69; © *North-Holland Publishing Co., Amsterdam*

NUCLEAR GROUND-STATE PROPERTIES AND SELF-CONSISTENT CALCULATIONS WITH THE SKYRME INTERACTION

(I). Spherical description

M. BEINER, H. FLOCARD and NGUYEN VAN GIAI

TABLE 5

Binding energy per particle E/A, Fermi momentum k_F, incompressibility coefficient K, effective mass ratio m^*/m and symmetry coefficients ε_1 and ε_2 [see eq. (17)] in nuclear matter calculated with the interactions SII to SVI

	t_3 (MeV fm^6)	E/A (MeV)	k_F (fm^{-1})	K (MeV)	m^*/m	ε_1 (MeV)	ε_2 (MeV)
SVI	17000	−15.77	1.29	364	0.95	26.89	0.67
SIII	14000	−15.87	1.29	356	0.76	28.16	0.83
SII	9331	−16.00	1.30	342	0.58	34.2	1.10
SIV	5000	−15.98	1.31	325	0.47	31.22	1.37
SV	0	−16.06	1.32	306	0.38	32.72	1.70

The interactions have been ordered according to the decreasing values of the parameter t_3.

The density dependence causes havoc

Particle-number projection impossible

Strong self interaction present

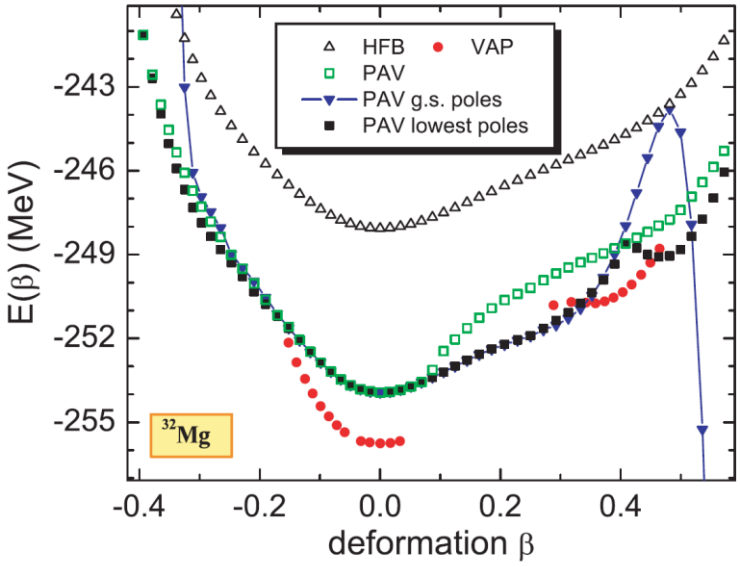
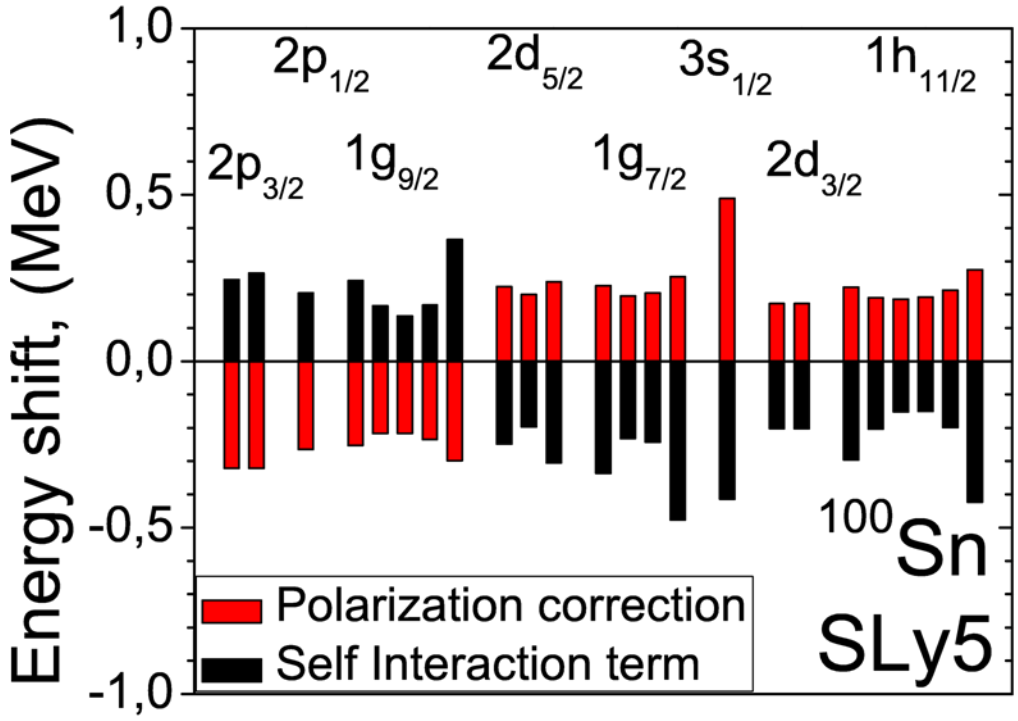


FIG. 11. (Color online) Deformation energy $E(\beta)$ as a function of quadrupole deformation β calculated for ^{32}Mg with the SIII force and volume pairing interaction. Results of the PAV HFB+LN calculations (squares and triangles) are compared with the VAP PNP results (dots). The standard HFB result is shown by open triangles.



J.D. *et al.*, Phys. Rev. C 76, 054315 (2007)

D. Tarpanov. *et al.*, to be published

First density-independent finite-range functional generator at a=0.8fm

Results with SV:

$\rho_{\rm sat}$	E/A	K_∞	J	L	$K_{\rm sym}$
0.1551 fm^{-3}	−16.05 MeV	305.7 MeV	32.82 MeV	96.09 MeV	24.17 MeV

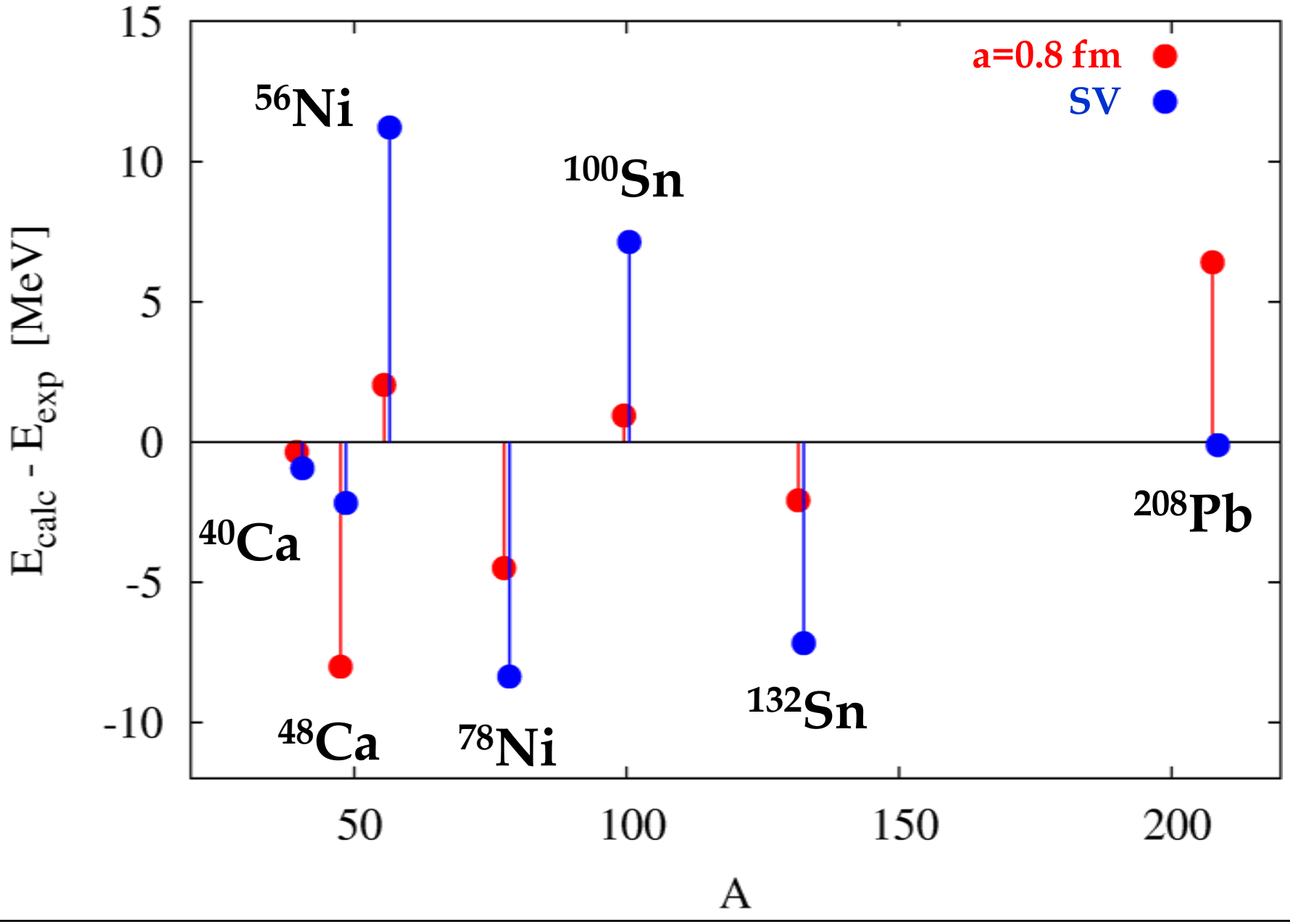
Nuc.	^{40}Ca	^{48}Ca	^{56}Ni	^{78}Ni	^{100}Sn	^{132}Sn	^{208}Pb
$E^{\rm exp}$	-342.050	-415.998	-483.991	-641.780	-824.921	-1102.827	-1636.350
$E^{\rm calc}$	-342.985	-418.177	-472.783	-650.148	-817.793	-1109.998	-1636.463
diff.	-0.935	-2.179	11.208	-8.368	7.128	-7.171	-0.113

Results with $a = 0.8$ fm:

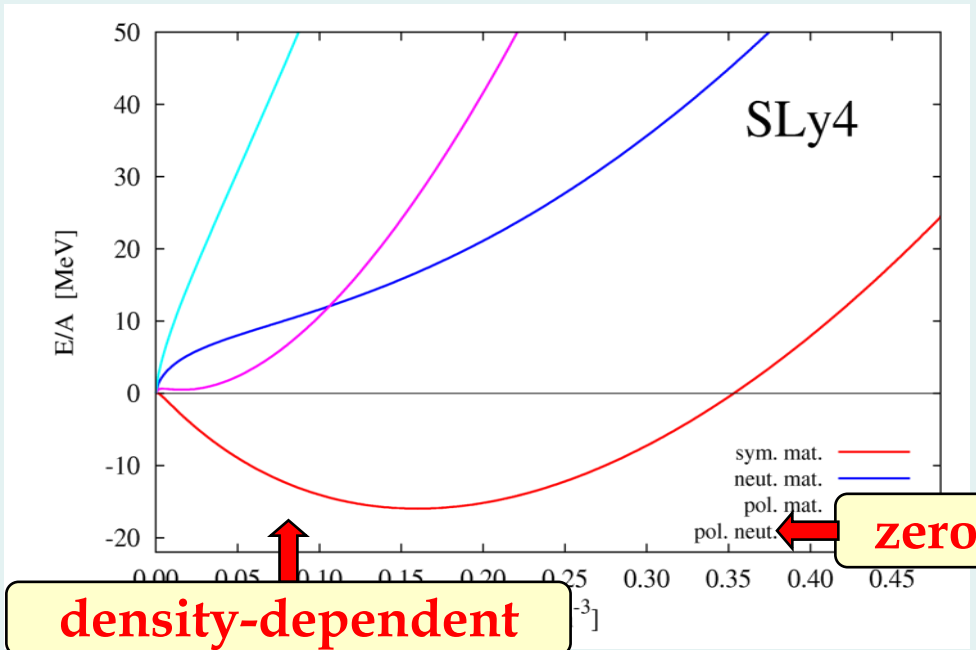
$\rho_{\rm sat}$	E/A	K_∞	J	L	$K_{\rm sym}$
0.1600 fm^{-3}	−16.00 MeV	230.0 MeV	32.00 MeV	100.2 MeV	83.26 MeV

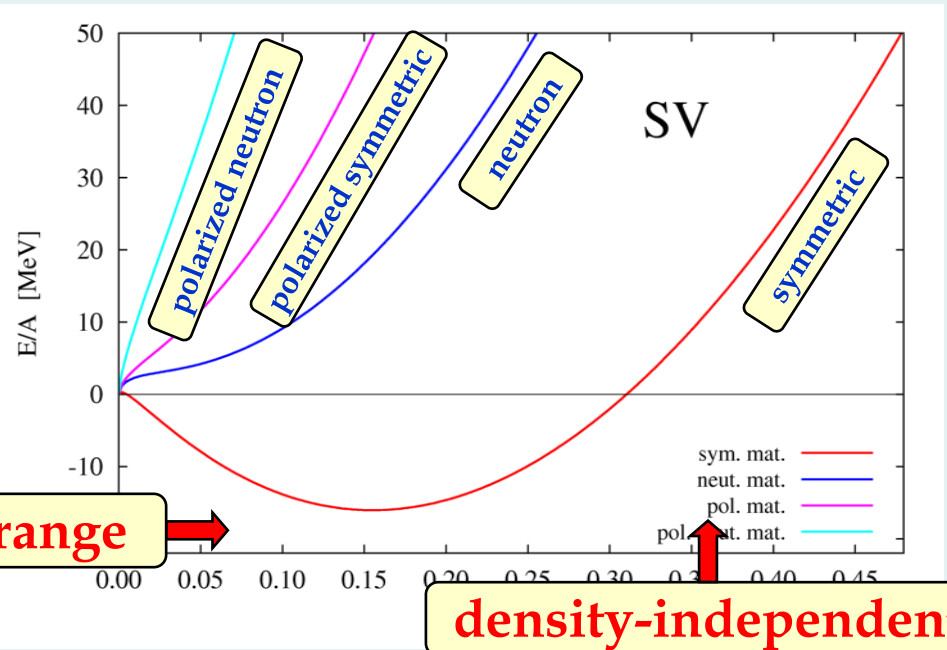
Nuc.	^{40}Ca	^{48}Ca	^{56}Ni	^{78}Ni	^{100}Sn	^{132}Sn	^{208}Pb
$E^{\rm exp}$	-342.050	-415.998	-483.991	-641.780	-824.921	-1102.827	-1636.350
$E^{\rm calc}_{10}$	-341.876	-422.960	-480.475	-644.712	-821.079	-1101.619	-1622.441
diff.	-0.355	-8.017	2.031	-4.495	0.941	-2.080	6.404

Masses of doubly magic nuclei



Equations of state

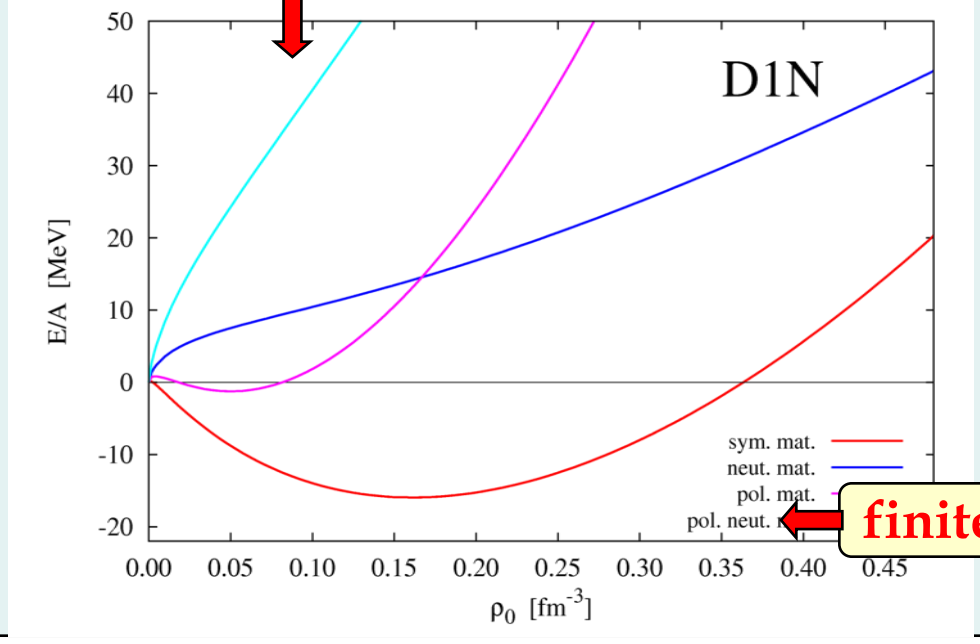


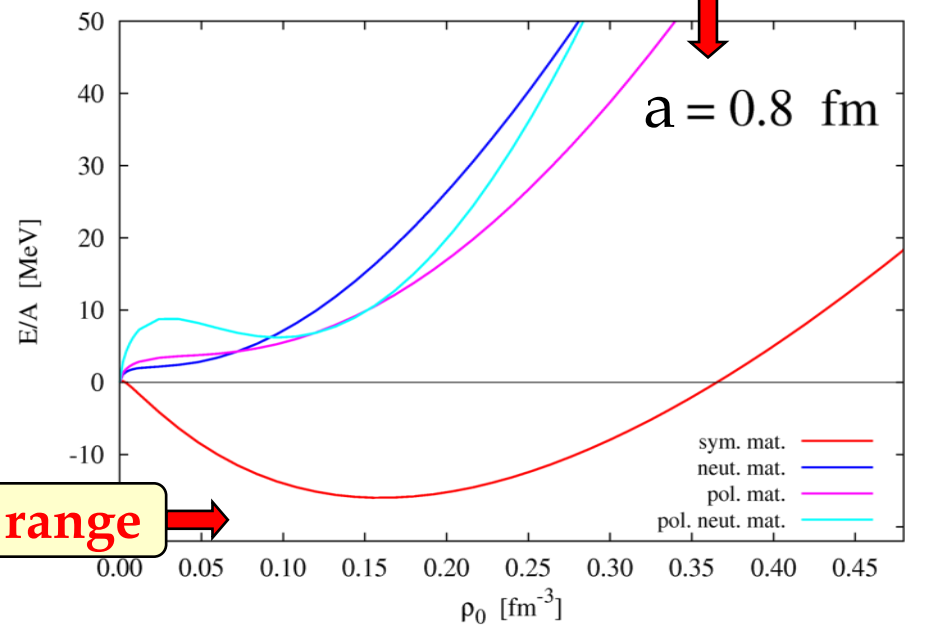


zero range

density-dependent

density-independent





finite range

Conclusions

1. We introduced a new class of energy density functionals that are based on regularized pseudopotentials. They allow for constructing density-independent functional generators.
2. In our opinion, future prospects for using the proposed regularized (pseudo)potentials are high.
   a. They may present better convergence properties than similar expansions based on the zero-range interactions.
   b. They allow for convergent summations of contributions from high single-particle momenta,
   c. They allow for formulating a consistent expansion in terms of the orders of derivatives, with the convergence properties gauged against the regularization scale.
   d. They allow for consistent beyond-mean field calculations.
3. The regularized pseudopotentials give us a possibility of building an order-by-order correctible theory.

Thank you

Rayleigh-Ritz Variational Method

$$\hat{H}|\Psi_i\rangle = E_i|\Psi_i\rangle$$

$$\Downarrow$$

$$|\Psi\rangle = a_0|\Psi_0\rangle + a_1|\Psi_1\rangle + a_2|\Psi_2\rangle + \ldots$$

$$\langle\Psi|\hat{H}|\Psi\rangle = E_0|a_0|^2 + E_1|a_1|^2 + E_2|a_2|^2 + \ldots$$

$$\Downarrow$$

$$\min_{|\Psi\rangle} \langle\Psi|\hat{H}|\Psi\rangle = E_0 \quad \Longleftarrow \text{Rayleigh-Ritz}$$

Energy density functional up to N^{3}LO

Local (primary) densities are defined by four quantum numbers $nLvJ$ as

$$\rho^t_{nLvJ}(\vec{r}) = \left\{[K_{nL}\rho^t_v(\vec{r},\vec{r}\,')]_J\right\}_{\vec{r}\,'=\vec{r}},$$

where the nth-order and rank-L relative derivative operators K_{nL} act on the scalar ($v=0$) or vector ($v=1$) isoscalar ($t=0$) or isovector ($t=1$) nonlocal densities.

We act on each of the local primary densities with mth-order and rank-I derivative operator D_{mI}, and then couple ranks I and J to the total rank J', which gives the local secondary densities, $\left[D_{mI}\rho^t_{nLvJ}(\vec{r})\right]_{J'}$. From primary and secondary densities we build terms of the EDF:

$$T^{n'L'v'J',t}_{mI,nLvJ}(\vec{r}) = [\rho^t_{n'L'v'J'}(\vec{r})[D_{mI}\rho^t_{nLvJ}(\vec{r})]_{J'}]_0,$$

Then, the total energy density reads

$$\mathcal{H}(\vec{r}) = \sum_{\substack{n'L'v'J',t\\ mI,nLvJ,J'}} C^{n'L'v'J',t}_{mI,nLvJ}\, T^{n'L'v'J',t}_{mI,nLvJ}(\vec{r}),$$

where $C^{n'L'v'J',t}_{mI,nLvJ}$ are coupling constants and the summation again runs over all allowed indices.

B.G. Carlsson *et al.*, Phys. Rev. C 78, 044326 (2008)

Energy density functional up to N^{3}LO

Numbers of local densities

Multiply by a factor of 2 for isoscalar and isovector densities

order	from ρ	from $\vec{s}$	T-even	T-odd	total
0	1	1	1	1	2
1	1	3	3	1	4
2	2	4	2	4	6
3	2	6	6	2	8
4	2	5	2	5	7
5	1	4	4	1	5
6	1	2	1	2	3
total	10	25	19	16	35

Numbers terms in the EDF

Multiply by a factor of 2 for isoscalar and isovector terms

order	T-even	T-odd	total	Galilean invariant	Gauge invariant
0	1	1	2	2	2
2	6	6	12	7	7
4	22	23	45	15	6
6	64	65	129	26	6
N^{3}LO	93	95	188	50	21

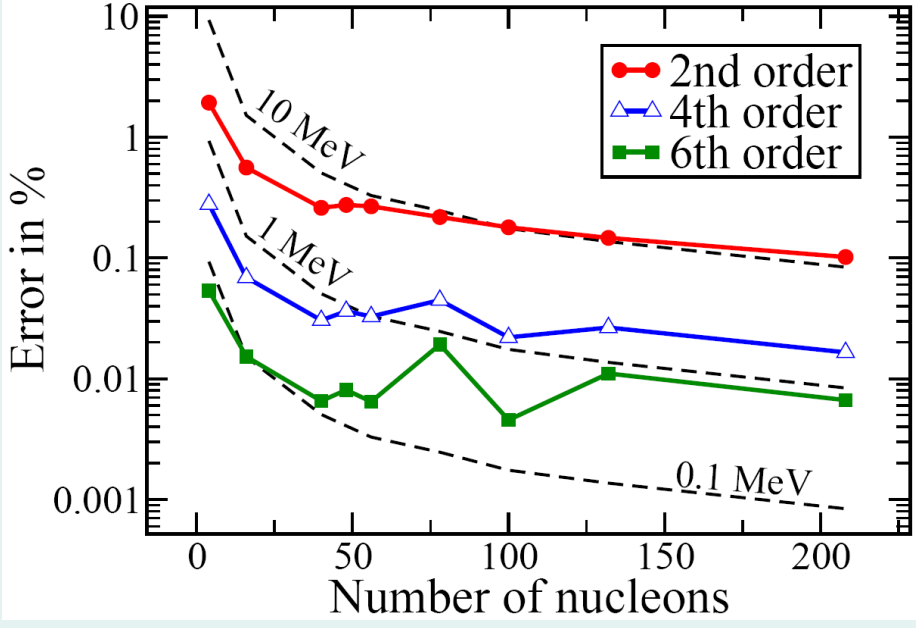
B.G. Carlsson *et al.*, Phys. Rev. C 78, 044326 (2008)

Convergence of the DME

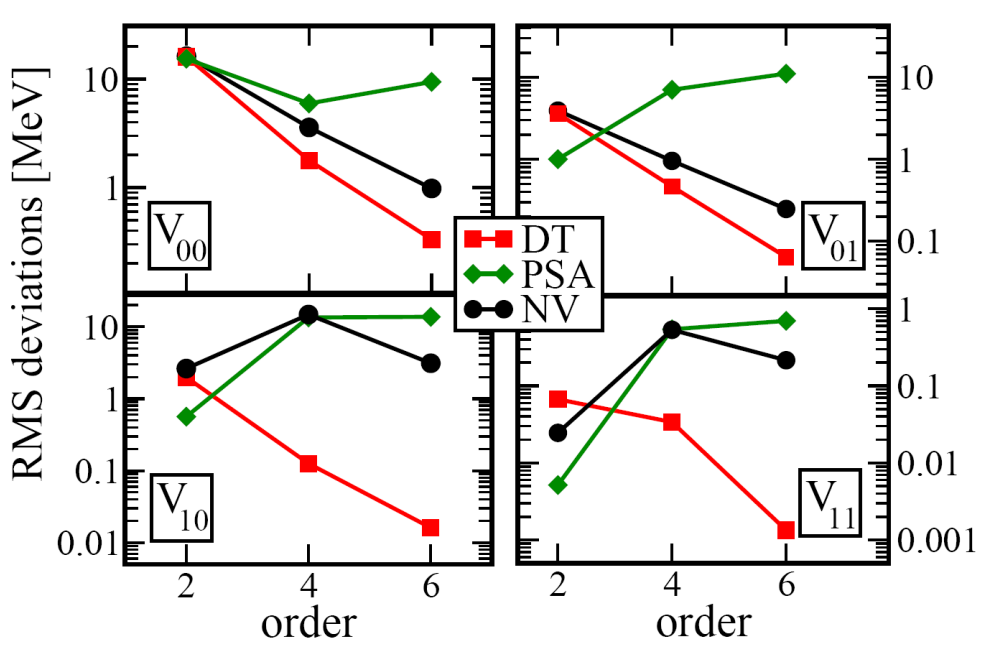
The success and convergence of the DME expansions relies on the fact that the finite-range nuclear effective interactions (G-matrix, Gogny, M3Y,…) are very short-range as compared to the spatial variations of nuclear densities. The quasi-local (gradient) expansion in nuclei works!

B.G. Carlsson & J.D. Phys. Rev. Lett. 105, 122501 (2010)

DME for the Gogny direct energy



DME for the Gogny exchange energy



Pseudopotentials vs. energy density functional

Number of terms in the pseudopotential					
order	$0th$	$2th$	$4th$	$6th$	N^{3}LO
Galilean	2	7	15	26	50
Gauge	2	7	6	6	21

At any order and symmetry, the number of terms in the pseudopotential is exactly equal to the number of the EDF terms for each isospin. Therefore, there are always twice as many EDF terms as there are pseudopotential terms

F. Raimondi *et al.*, Phys. Rev. C 83, 054311 (2011)

For the fourth-order gauge-invariant pseudopotential, the EDF isvector terms can be expressed as linear combinations of the isoscaler terms as:

$$C^{n'L'v'J',1}_{mI,nLvJ} = A(aC^{2202,0}_{00,2202} + bC^{2212,0}_{00,2212} + cC^{0011,0}_{00,4211} + dC^{0000,0}_{40,0000} + eC^{0011,0}_{40,0011} + fC^{0011,0}_{42,0011}).$$

	A	a	b	c	d	e	f
$C^{0000,1}_{40,0000}$	$\frac{1}{120}$	$-6\sqrt{15}$	$-18\sqrt{15}$	$-21\sqrt{5}$	$-40\sqrt{3}$	0	0
$C^{0011,1}_{40,0011}$	$\frac{1}{120}$	$-18\sqrt{5}$	$18\sqrt{5}$	$7\sqrt{15}$	0	$-40\sqrt{3}$	0
$C^{0011,1}_{42,0011}$	$-\frac{1}{\sqrt{3}}$	0	0	1	0	0	1
$C^{2202,1}_{00,2202}$	$\frac{1}{9}$	$-3\sqrt{3}$	0	0	$-4\sqrt{15}$	$-12\sqrt{5}$	0
$C^{0011,1}_{00,4211}$	$-\frac{1}{\sqrt{3}}$	0	0	1	0	0	4
$C^{2212,1}_{00,2212}$	$\frac{1}{9}$	0	$-3\sqrt{3}$	0	$-4\sqrt{15}$	$4\sqrt{5}$	14

Continuity equation

Within the Kohn-Sham approach, the total energy is the sum of the kinetic and potential-energy terms,

$$E\{\rho\} = E_k\{\rho\} + E_p\{\rho\},$$

Let us now assume that the potential energy is invariant with respect to a unitary transformation of the density matrix, $U = \exp(i\eta G)$,

$$E_p\{\rho\} = E_p\{U\rho U^+\}, \quad \Longrightarrow \quad \mathrm{Tr}\Gamma[G,\rho] \equiv \mathrm{Tr}G[\Gamma,\rho] = 0,$$

which allows us to derive the equation of motion for the average value of $\langle G\rangle = \mathrm{Tr}G\rho$.

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\langle G\rangle = i\hbar\mathrm{Tr}G\frac{\mathrm{d}}{\mathrm{d}t}\rho = \mathrm{Tr}G[h,\rho] = \mathrm{Tr}G[T,\rho],$$

that is, the time evolution of $\langle G\rangle$ is governed solely by the kinetic term of the mean-field Hamiltonian.

The CE now results from specifying ηG to the local gauge transformation, that is, $\psi'_\alpha(r\sigma\tau) \equiv (U\psi_\alpha)(r\sigma\tau) = e^{i\gamma(r)}\psi_\alpha(r\sigma\tau)$, which gives,

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{00}(r,t) = -\frac{\hbar}{m}\nabla\cdot j_{00}(r,t).$$

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What is the DFT good for?

$$\delta\langle \hat{H} - \lambda \hat{Q} \rangle = 0$$

$$\Downarrow$$

$$E = E(Q)$$

Energy E is a function(al) of Q

1) **Exact:** Minimization of E(Q) gives the exact E and exact Q
2) **Impractical:** Derivation of E(Q) requires the full variation δ (bigger effort than to find the exact ground state)
3) **Inspirational:** Can we build useful models E′(Q) of the exact E(Q)?
4) **Experiment-driven:** E′(Q) works better or worse depending on the physical input used to build it.

Pseudopotentials

In the central-like form, the pseudopotential is a sum of terms,

$$\hat{V} = \sum_{\tilde{n}'\tilde{L}'\tilde{n}\tilde{L},v_{12}S} C^{\tilde{n}'\tilde{L}'}_{\tilde{n}\tilde{L},v_{12}S} \hat{V}^{\tilde{n}'\tilde{L}'}_{\tilde{n}\tilde{L},v_{12}S}.$$

Each term in the sum is accompanied by the corresponding strength parameter $C^{\tilde{n}'\tilde{L}'}_{\tilde{n}\tilde{L},v_{12}S}$, and explicitly reads,

$$\begin{aligned}\hat{V}^{\tilde{n}'\tilde{L}'}_{\tilde{n}\tilde{L},v_{12}S} = \quad & \tfrac{1}{2} i^{v_{12}} \left(\left[\left[K'_{\tilde{n}'\tilde{L}'} K_{\tilde{n}\tilde{L}} \right]_S \hat{S}_{v_{12}S} \right]_0 + (-1)^{v_{12}+S} \left[\left[K'_{\tilde{n}\tilde{L}} K_{\tilde{n}'\tilde{L}'} \right]_S \hat{S}_{v_{12}S} \right]_0 \right) \\ & \times \left(1 - \hat{P}^M \hat{P}^\sigma \hat{P}^\tau \right) \hat{\delta}_{12}(r'_1 r'_2; r_1 r_2).\end{aligned}$$

$K_{\tilde{n}\tilde{L}}$ are the spherical tensor derivatives of order $\tilde{n}$ and rank $\tilde{L}$ are built of $k = (\nabla_1 - \nabla_2)/2i$, The two-body spin operators $\hat{S}_{v_{12}S}$ are defined as,

$$\hat{S}_{v_{12}S} = \left(1 - \tfrac{1}{2}\delta_{v_1,v_2} \right) \left([\sigma^{(1)}_{v_1} \sigma^{(2)}_{v_2}]_S + [\sigma^{(1)}_{v_2} \sigma^{(2)}_{v_1}]_S \right),$$

where $v_{12} = v_1 + v_2$ and $\sigma^{(i)}_{v\mu}$ are the spherical-tensor components of the rank-v Pauli matrices. The Dirac delta function,

$$\hat{\delta}_{12}(\vec{r}'_1 \vec{r}'_2, \vec{r}_1 \vec{r}_2) = \delta(\vec{r}'_1 - \vec{r}_1)\delta(\vec{r}'_2 - \vec{r}_2)\delta(\vec{r}_1 - \vec{r}_2),$$

ensures the locality and zero-range character of the pseudopotential.

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Regularized pseudopotentials

Below we determine coupling constants $V_{2n}^{(i)}$ in Eq. (??) by requiring that the lowest moments of the regularized and Gogny potentials are equal, that is,

$$M_{2m}^{(i)} \equiv \int r^{2m} G_i(r) \mathrm{d}^3 r = \int r^{2m} V_i(r) \mathrm{d}^3 r,$$

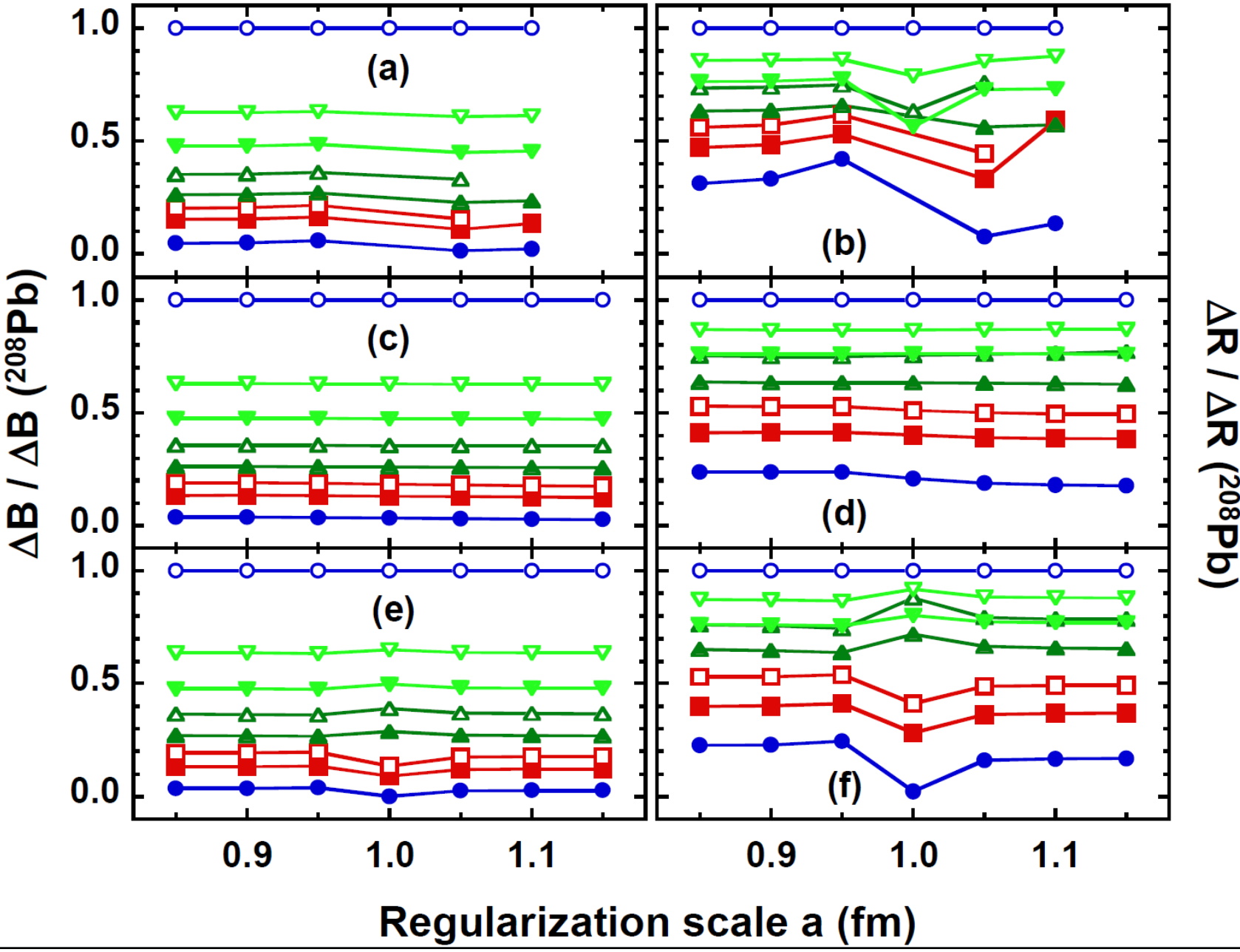
for $m = 0, 1, \ldots, n_{max}$. This conditions gives the coupling constants of the regularized potential in simple analytical forms,

$$\begin{aligned} V_{2n}^{(i)} &= \sum_{m=0}^{n} \left(-\frac{a^2}{4}\right)^{n-m} \frac{M_{2m}^{(i)}}{(n-m)!(2m+1)!} \\ &= \frac{1}{4^n n!} \sum_{k=1,2} G_k^{(i)} \left(a_k^2 - a^2\right)^n, \end{aligned}$$

where $G_k^{(i)}$ and a_k are the parameters of the Gogny interaction.

J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Regularized pseudopotentials vs. Gogny



J.D, K. Bennaceur, F. Raimondi, J. Phys. G. 39, 125103 (2012)

Regularized pseudopotentials vs. Gogny

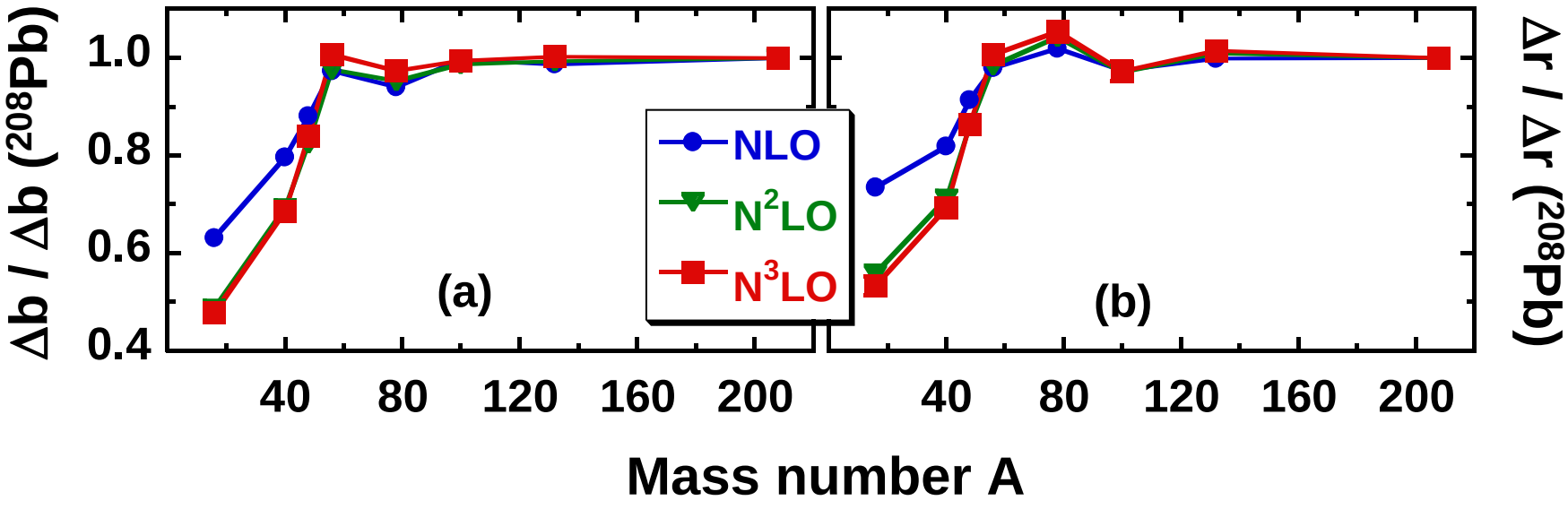


Figure 3. The same as in figure 2, but for the deviations of binding energies scaled by the numbers of particles, $\Delta b = \Delta B/A$, and those of radii scaled by the nuclear sizes, $\Delta r = \Delta R/A^{1/3}$. Note that the N^{2}LO results almost perfectly hide behind those obtained at N^{3}LO.

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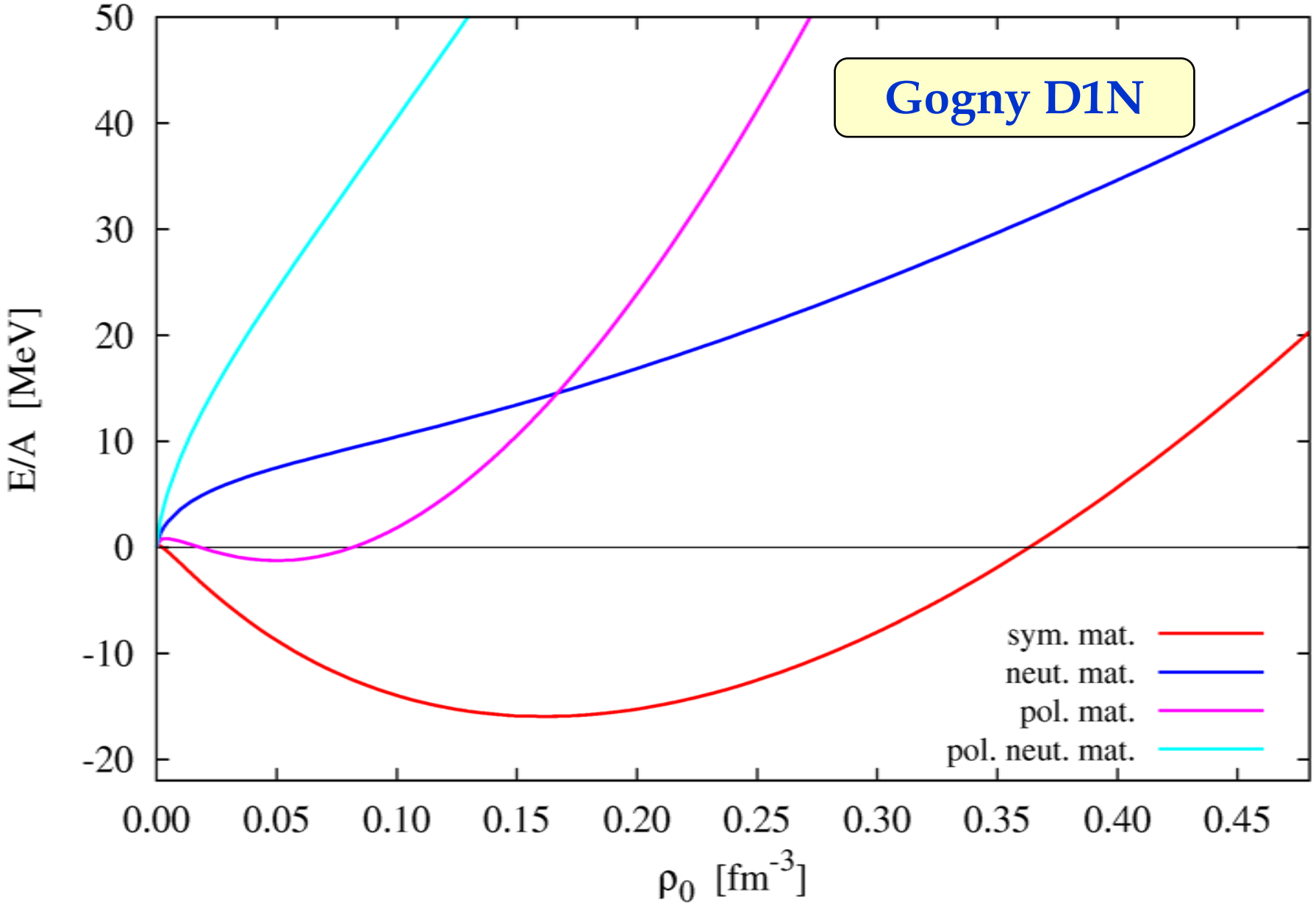
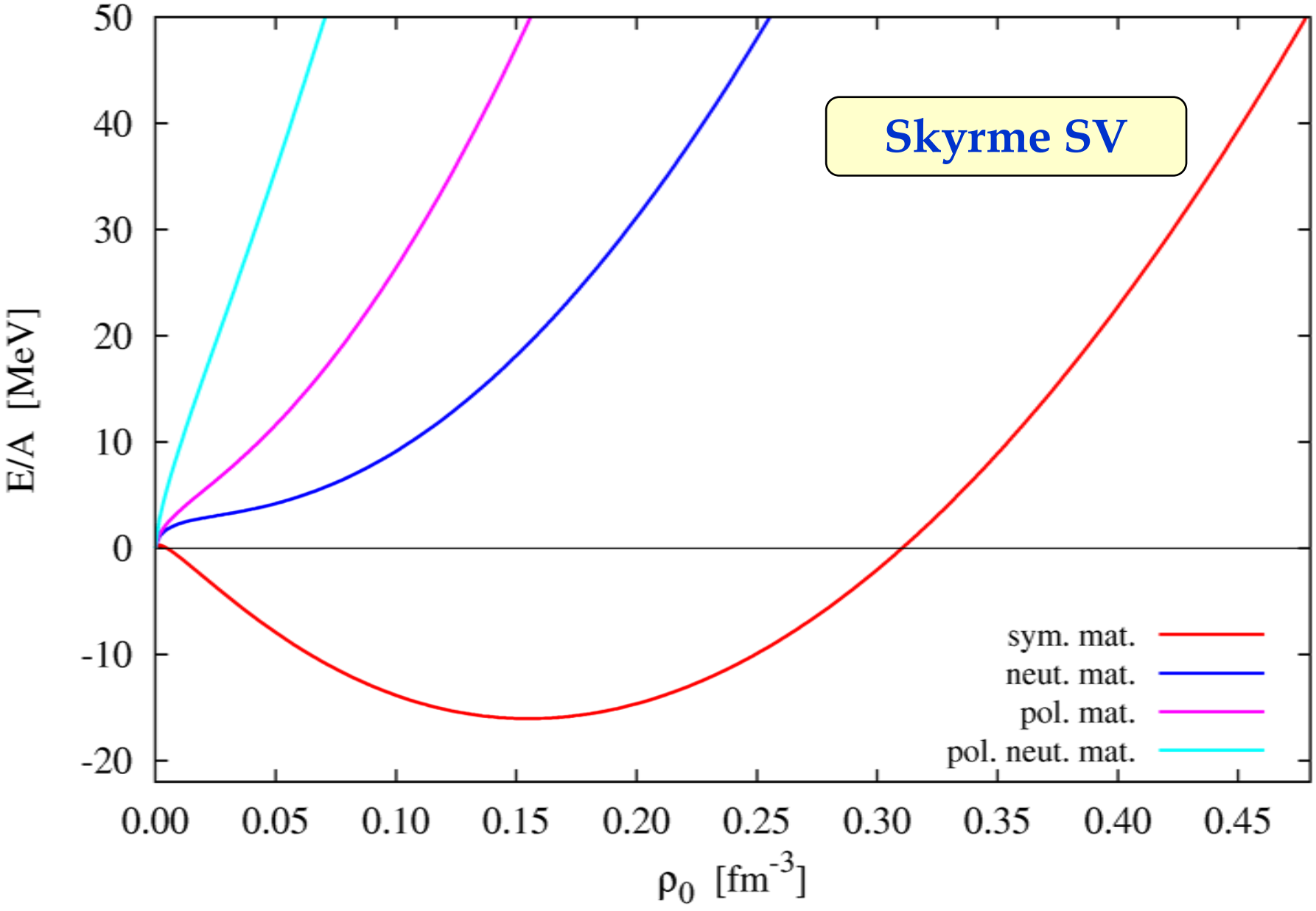
Equations of state



Equations of state



Equations of state