Rozpad α zjonizowanych i neutralnych atomów

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Plan wykładu

1. Motywacja pracy o rozpadach α dla zupełnie zjonizowanych i neutralnych atomów.
2. Elementy teorii rozpadu α.
3. Metoda obliczania wpływu chmury elektronowej na prawdopodobieństwo rozpadu α.
4. Wyniki obliczeń i perspektywy ich doświadczalnej weryfikacji.
5. Analiza rozpadu α atomów zupełnie zjonizowanych jest częścią badań poświęconych pomiarom czasów życia i mas jąder atomowych w laboratorium FAIR-Darmstadt /ILIMA/.

Motywacja

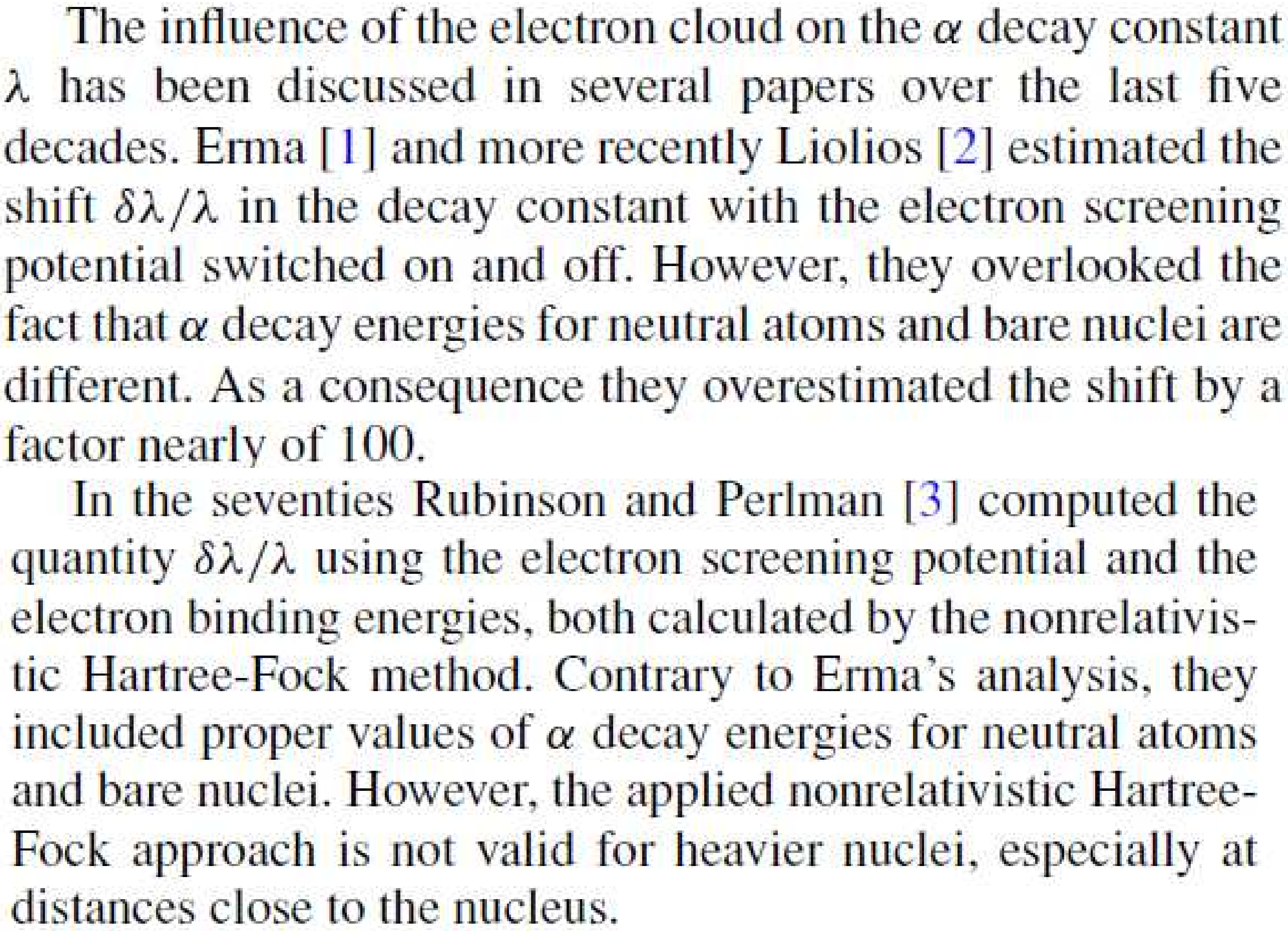
1. **Zastosowania astrofizyczne: zmiany czasu życia w rozpadzie α we wnętrzach i atmosferach gwiazd.**

2. **Test pomiarów czasów życia w pierścieniu akumulacyjnym ESR (np. testowanie istnienia oscylacji ilości rozpadających się jonów w funkcji czasu).**

3. **Czy zmiana chemicznego otoczenia (np. ochłodzenie) długo-życiowych atomów (powstałych np. w elektrowniach atomowych) powoduje istotne zmiany ich czasu życia na rozpad α?**

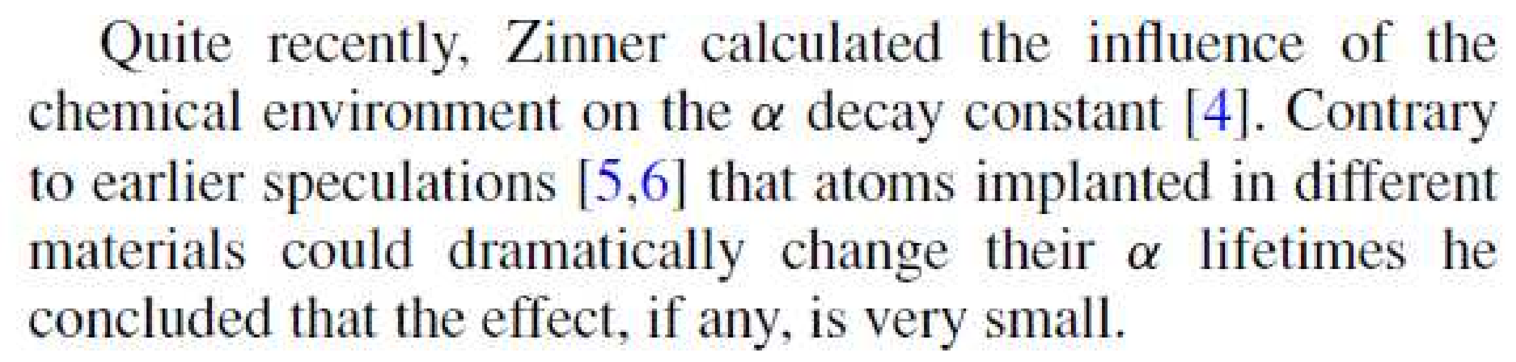
C. Rolfs, Frankfurter Allgemeine Zeitung, 13.08.2006, p. 56

Short history of the problem



The influence of the electron cloud on the α decay constant λ has been discussed in several papers over the last five decades. Erma [1] and more recently Liolios [2] estimated the shift $\delta\lambda/\lambda$ in the decay constant with the electron screening potential switched on and off. However, they overlooked the fact that α decay energies for neutral atoms and bare nuclei are different. As a consequence they overestimated the shift by a factor nearly of 100.

In the seventies Rubinson and Perlman [3] computed the quantity $\delta\lambda/\lambda$ using the electron screening potential and the electron binding energies, both calculated by the nonrelativistic Hartree-Fock method. Contrary to Erma's analysis, they included proper values of α decay energies for neutral atoms and bare nuclei. However, the applied nonrelativistic Hartree-Fock approach is not valid for heavier nuclei, especially at distances close to the nucleus.



Quite recently, Zinner calculated the influence of the chemical environment on the α decay constant [4]. Contrary to earlier speculations [5,6] that atoms implanted in different materials could dramatically change their α lifetimes he concluded that the effect, if any, is very small.

The technique involves embedding the nuclear waste in a metal and cooling it to ultra-low temperatures. This speeds up the rate of decay of the adioactive materials potentially cutting their half lives by a factor of 100 or more.

Professor Rolfs added “We are currently investigating radium-226, a hazardous component of spent nuclear fuel with a half-life of 1600 years. I calculate that using this technique could reduce the half-life to 100 years. At best, I have calculated that it could be reduced to as little as two years. This would avoid the need to bury nuclear waste in deep repositories - a hugely expensive and difficult process.”

Eelementy teorii rozpadu α

$$\lambda \propto Q \exp\left\{-\frac{2\sqrt{2m}}{\hbar}\int_{r_1}^{r_2}\sqrt{P(r)}dr\right\} \qquad P(r) \equiv V(r) - Q$$

Atom neutralny $V(r) = V_N(r) + V_C(r) + 2V_e(Z,r)$

Jądro atomowe (bez elektronów) $V(r) = V_N(r) + V_C(r)$

$$Q_N = Q_B + \delta B(Z) \qquad V_e(Z,r) = V_e(Z,0) + \delta V_e(Z,r)$$

$$P_N(r) = P_B(r) - [\delta B(Z) - 2V_e(Z,0)] + 2\delta V_e(Z,r)$$

$$\delta B(Z) \equiv B(Z,Z) - B(Z-2,Z-2) - B(2,2)$$

Hellman-Feynman theorem

It was proved independently by Hellmann (1937) and Feynman (1939). The theorem states

$$\frac{\partial E}{\partial \lambda} = \int \psi^*(\lambda) \frac{\partial \hat{H}_\lambda}{\partial \lambda} \psi(\lambda)\, d\tau,$$

where H is a Hamiltonian operator depending upon a continuous parameter λ, E(λ) is the energy (eigenvalue) of the normalized wavefunction,

Hans Gustav Adolf Hellmann (1903-1938) was a German theoretical physicist. After the Nazi came to power, Hellmann was dismissed on 1933 as ‘undesirable’ because of his Jewish wife. He immigrated to the Soviet Union, taking up a position in Moscow. However, he was later denounced during the Great Purge, imprisoned on 10 May 1938, and executed on 29 May 1938. His son, Hans Hellmann, Jr., was only allowed to leave the former Soviet Union in 1991.

$$V_e(Z,r) = V_e(Z,0) + \delta V_e(Z,r)$$

Potencjał w środku atomu oszacowany został z twierdzenia Hellmana-Feynmana (H-F) i energii wiązań elektronów*:

$$\frac{\partial B(Z,N_e)}{\partial Z}\Big|_{N_e=Z} = V_e(Z,0)$$

Część potencjału zależną od odległości r oszacowano na podstawie obliczeń gęstości elektronowej I. Tupitsyna /2007/ (r – fm, energia-eV)

$$2\delta V_e(Z,r) = 0.10277(Z/54)^{6.45} r + 0.00626(Z/54)^{4.06} r^2$$

Test $V_e(Z,0)$:

Xe	**Rn**	
−9110 eV	**−18970 eV**	**z twierdzenia. H-F i energii wiązania elektronów ***
−9130 eV	**−19030 eV**	**z gęstości chmury elektronowej /I. Tupitsyn 2007/**

•Energie wiązania B(Z,N) dla N elektronów w polu jądra o Z protonach opublikowano w **G. C. Rodrigues *et al.* At. DataNucl. Data Tables 86, 117 (2004).**

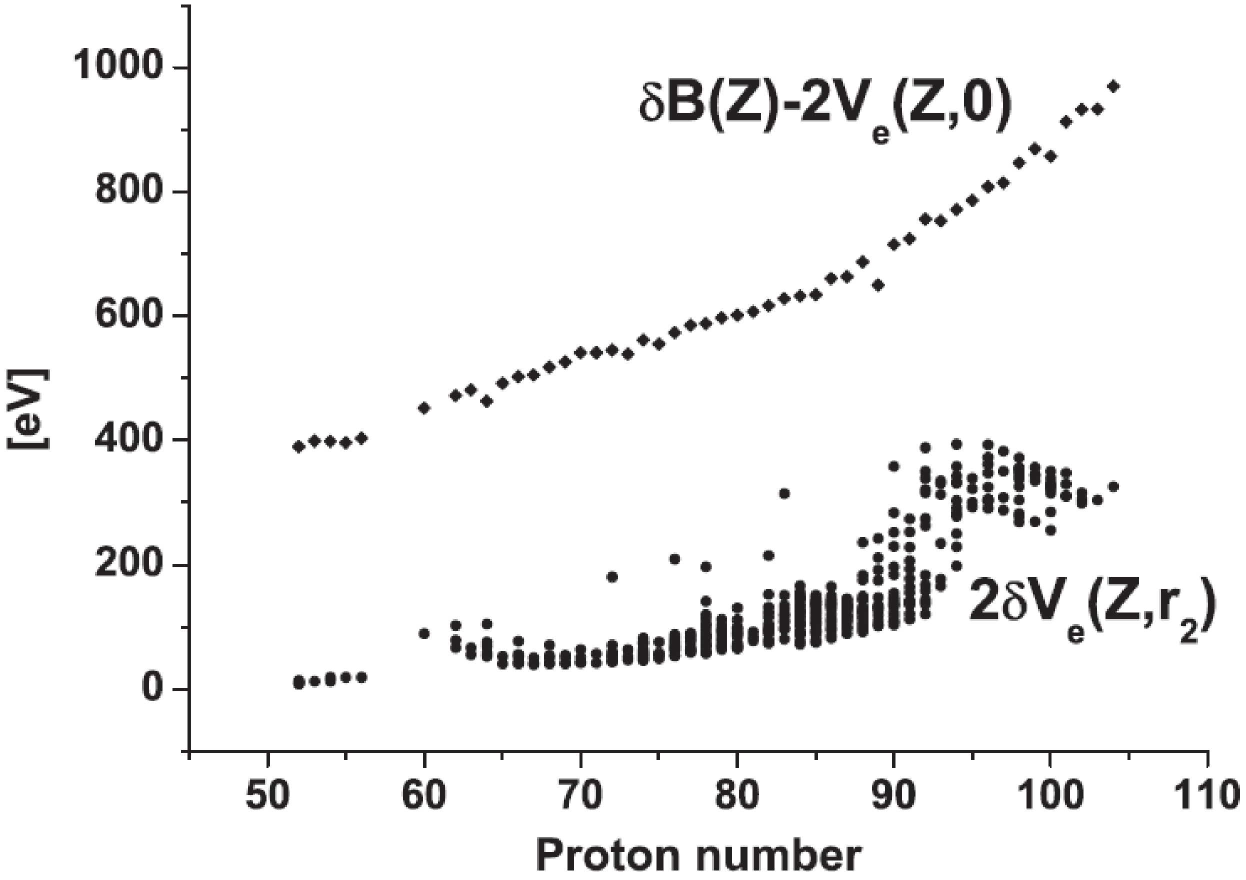


FIG. 1. In the upper part the difference between electron binding energy $\delta B(Z)$ and the α potential at atomic nucleus $2V_e(Z,0)$ in neutral atoms is plotted. The lower part illustrates the values of residual electron potential $2\delta V_e(Z,r_2)$ at the exit point, r_2 of the α particle from the potential barrier in neutral atom.

$$P_N(r) = P_B(r) - [\delta B(Z) - 2V_e(Z,0)] + 2\delta V_e(Z,r)$$

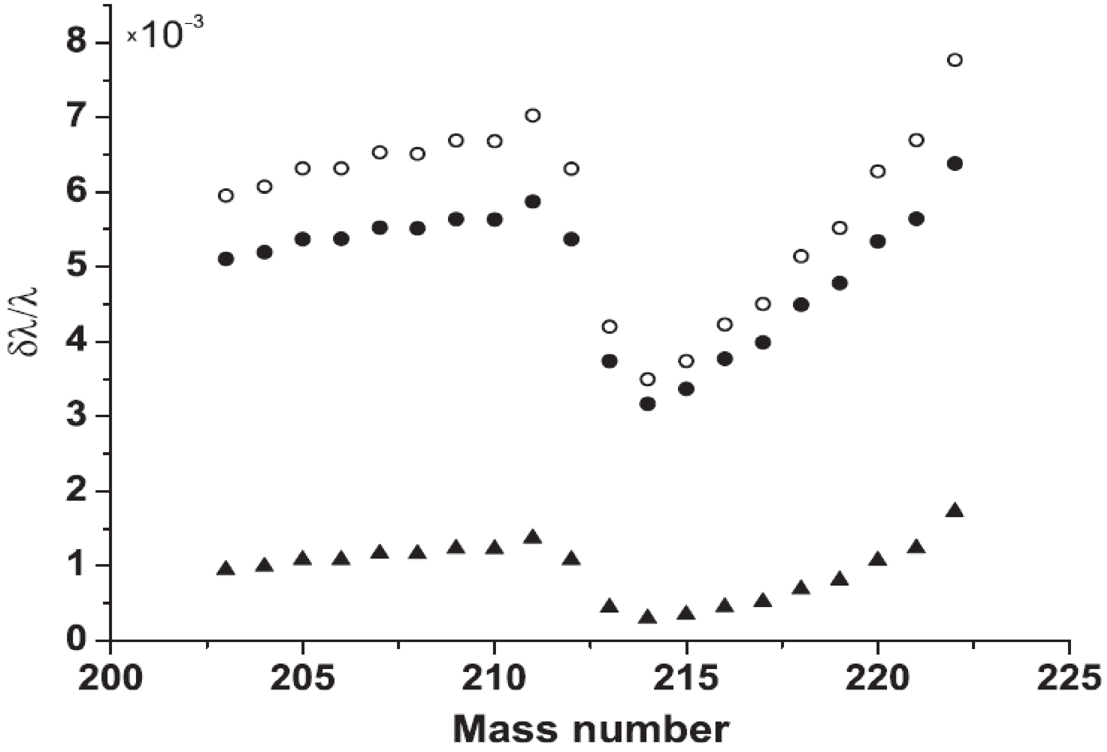


FIG. 2. α decay constant's shift (multiplied by 1000) between bare nucleus and neutral atom in radon isotopes. If the distance dependance in the electron screening potential is switched on then the λ shifts are denoted by full circles, in opposite case by open ones. The triangles refer to the formula of Rubinson and Perlman [3] with the relativistic electron density [16] and their values equal approximately to differences between the shifts denoted by open and full circles.

W. Rubinson and M. L. Perlman (1972) założyli, że

$\delta B(Z) - 2Ve(Z, 0) = 0$,

natomiast $2Ve(Z, r)$ obliczali używając relatywistycznej gęstości elektronów.

Wpływ niecentralnego potencjału cząstki α na potencjał pochodzący od chmury elektronowej nie powoduje większej zmiany w oszacowaniu $\delta\lambda/\lambda$ niż 0.1 promila

W. Rubinson and M. L. Perlman, Phys. Lett. **B40**, 352 (1972).

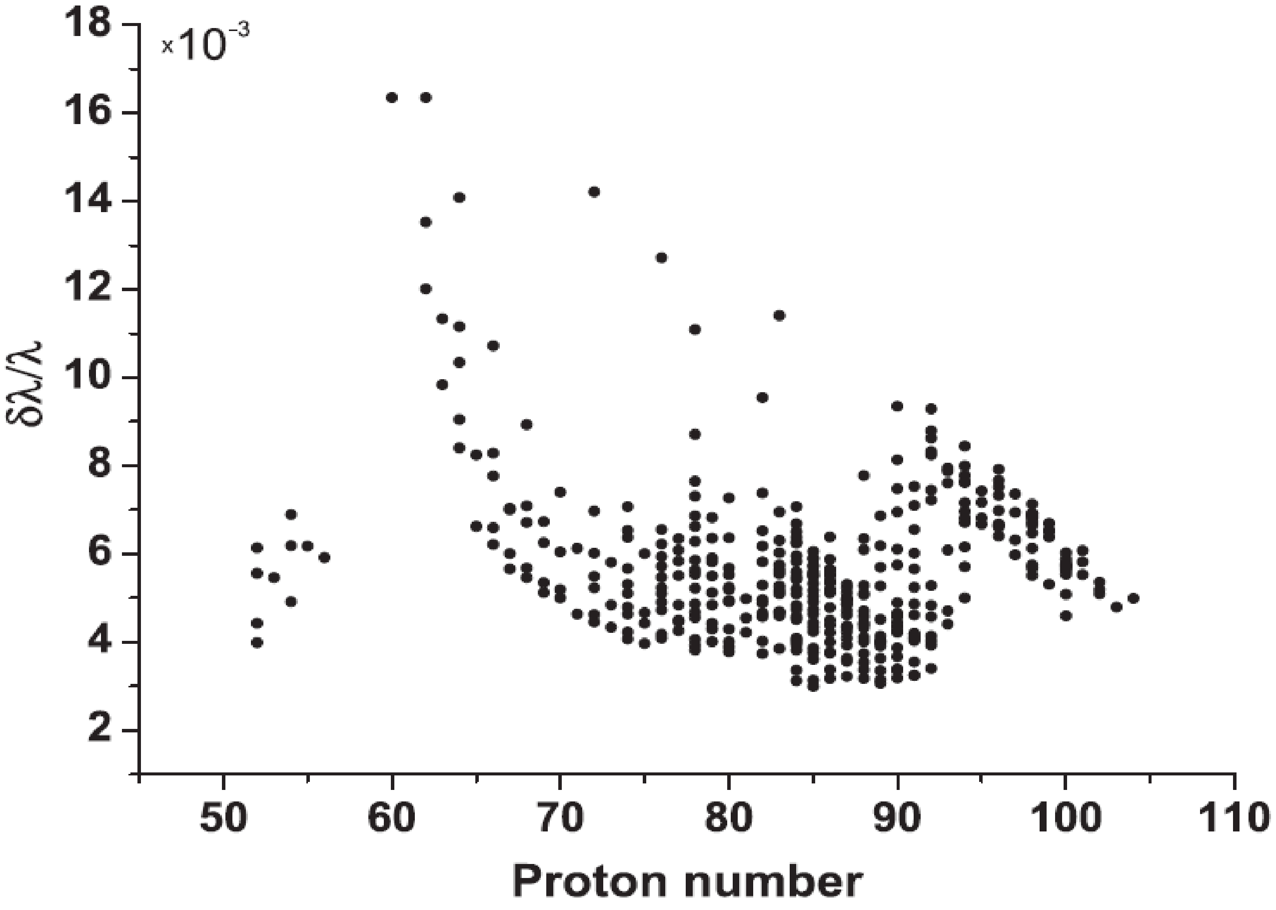


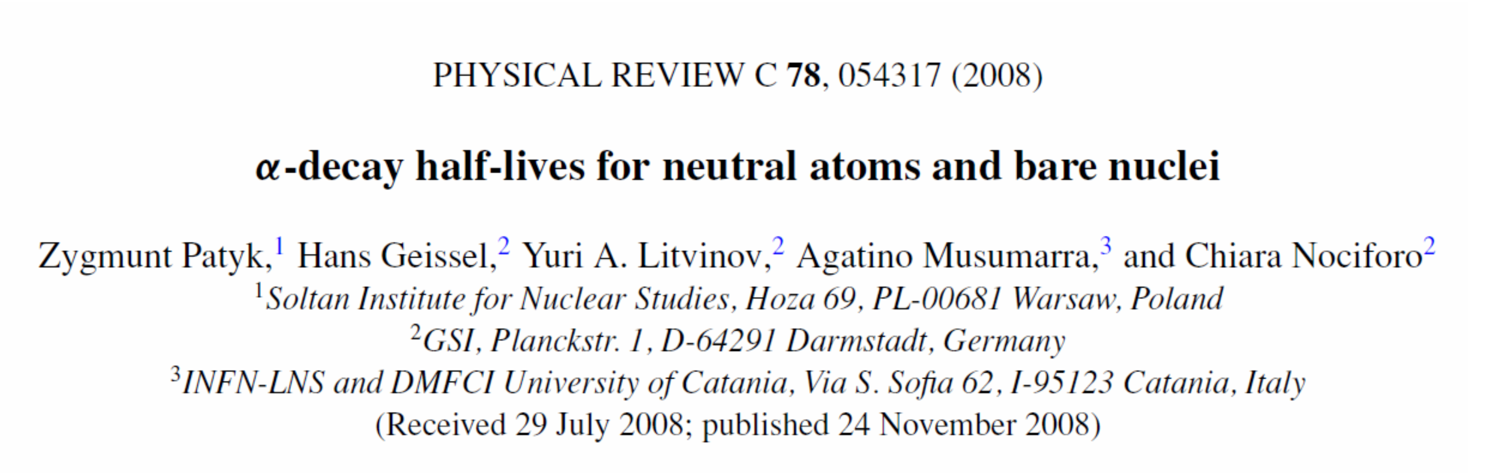
FIG. 3. α decay constant's shift (multiplied by 1000) for nuclei with available experimental α decay energy [14]. Only for a few nuclei the shift is larger than 1%. Two largest values are analyzed in detail in Table I (fourth and fifth rows).

$$\delta\lambda/\lambda \equiv 2(\lambda_N - \lambda_B)/(\lambda_N + \lambda_B)$$

Wybrane rozpady α

TABLE I. A few selected alpha emitters with an estimation of the λ shift are tabulated. The experimental α decay constants, λ_{exp} (or $T_{1/2}$) with uncertainties $\Delta\lambda_{\mathrm{exp}}$ for neutral atoms can be found in Ref. [14].

Z	A	Q_α [MeV]	$-2V_e(Z,0)$ [keV]	r_2 [fm]	$\delta\lambda/\lambda \times 1000$	$\Delta\lambda_{\mathrm{exp}}/\lambda_{\mathrm{exp}} \times 1000$	$\log_{10}(T_{1/2})$ $T_{1/2}$ in s
86	222	5.59	37.9	43.0	6	0.08	5.52
84	212	8.95	36.5	26.3	3	7	−6.52
62	147	2.31	22.4	74.1	14	10	18.5
62	148	1.99	22.4	86.0	16	428	23.3
60	144	1.91	21.3	86.7	16	70	22.9
87	213	6.91	38.7	35.3	5	9	1.54
87	220	6.80	38.7	35.8	5	11	1.44
86	222	6.39	37.9	37.7	5	50	3.16



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α-decay half-lives for neutral atoms and bare nuclei

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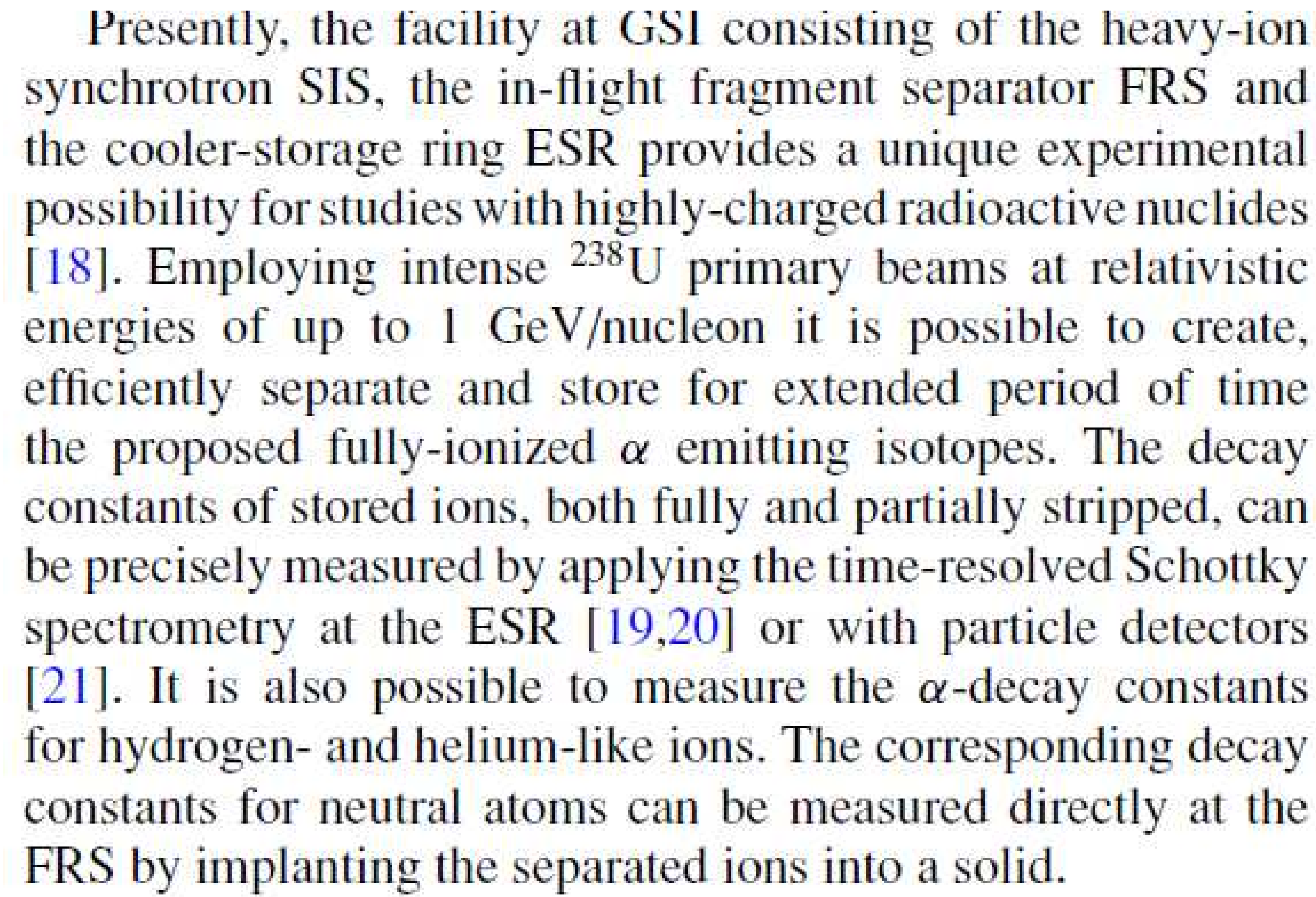
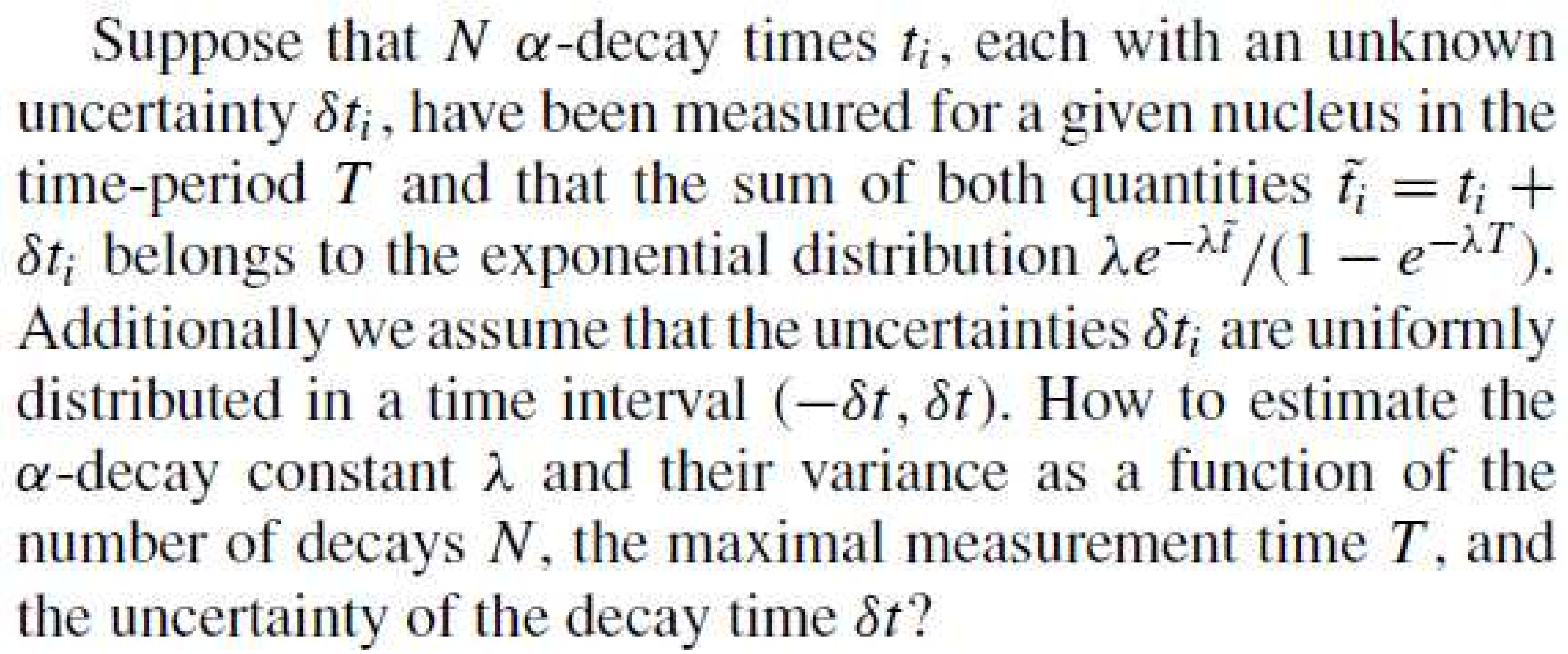
Experimental test

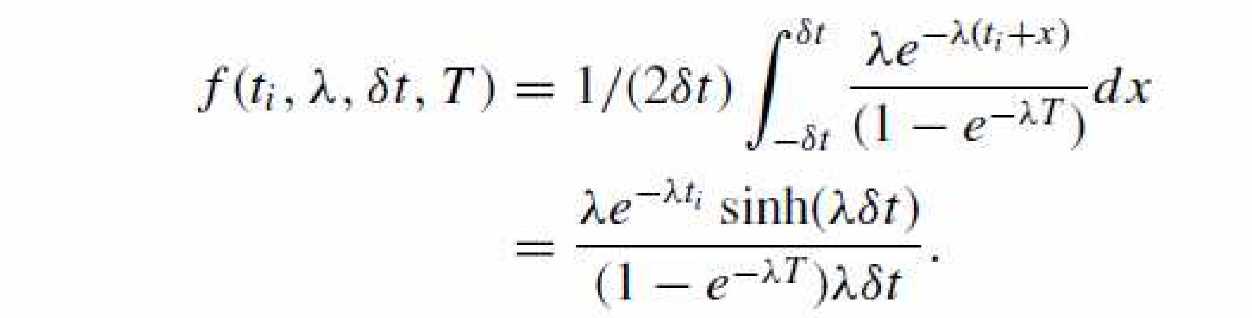


Presently, the facility at GSI consisting of the heavy-ion synchrotron SIS, the in-flight fragment separator FRS and the cooler-storage ring ESR provides a unique experimental possibility for studies with highly-charged radioactive nuclides [18]. Employing intense ^{238}U primary beams at relativistic energies of up to 1 GeV/nucleon it is possible to create, efficiently separate and store for extended period of time the proposed fully-ionized α emitting isotopes. The decay constants of stored ions, both fully and partially stripped, can be precisely measured by applying the time-resolved Schottky spectrometry at the ESR [19,20] or with particle detectors [21]. It is also possible to measure the α-decay constants for hydrogen- and helium-like ions. The corresponding decay constants for neutral atoms can be measured directly at the FRS by implanting the separated ions into a solid.

Oszacowanie błędu stałej rozpadu λ



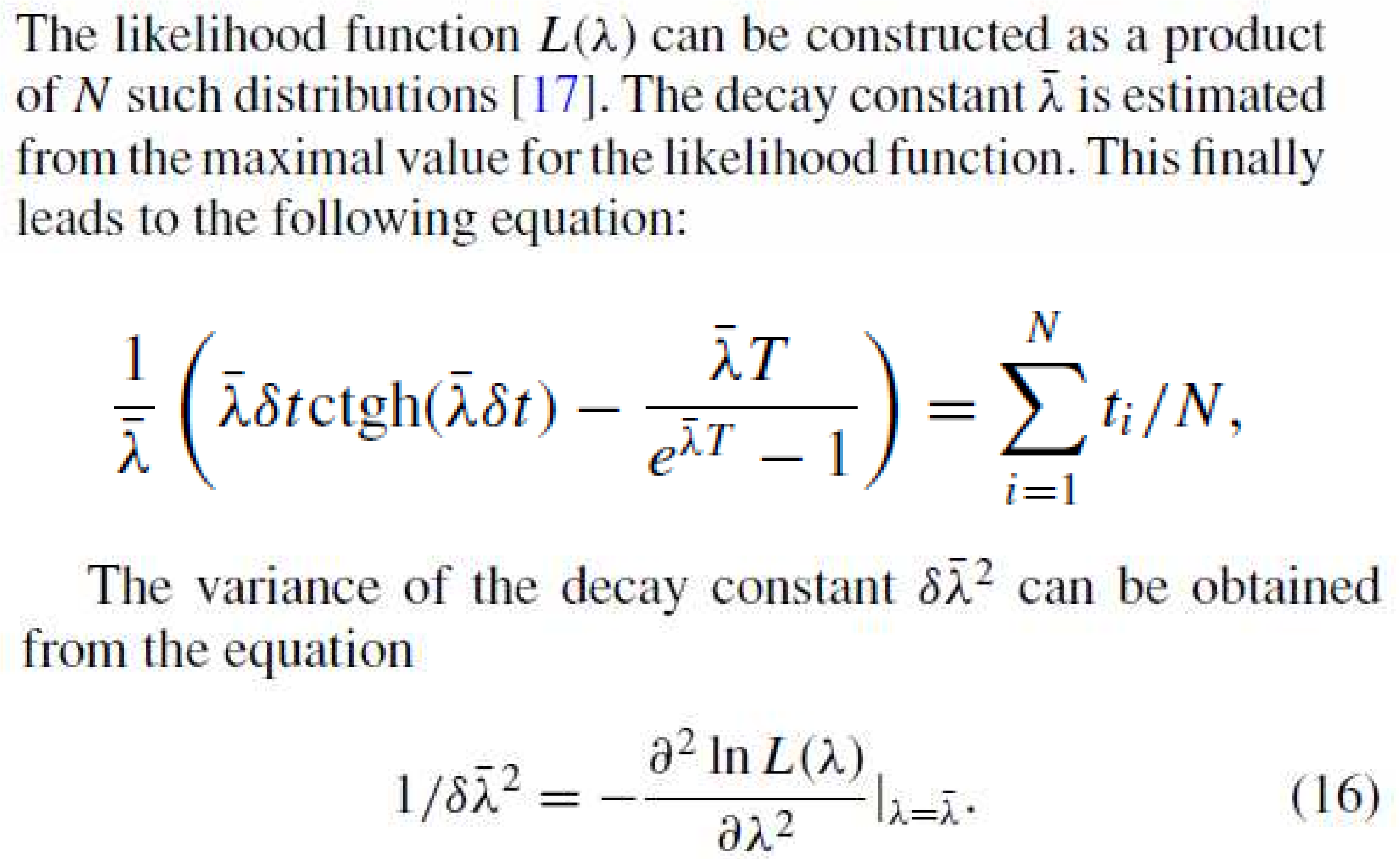
Suppose that N α-decay times t_i, each with an unknown uncertainty δt_i, have been measured for a given nucleus in the time-period T and that the sum of both quantities $\tilde{t}_i = t_i + \delta t_i$ belongs to the exponential distribution $\lambda e^{-\lambda \tilde{t}}/(1 - e^{-\lambda T})$. Additionally we assume that the uncertainties δt_i are uniformly distributed in a time interval $(-\delta t, \delta t)$. How to estimate the α-decay constant λ and their variance as a function of the number of decays N, the maximal measurement time T, and the uncertainty of the decay time δt?



$$f(t_i, \lambda, \delta t, T) = 1/(2\delta t) \int_{-\delta t}^{\delta t} \frac{\lambda e^{-\lambda(t_i + x)}}{(1 - e^{-\lambda T})} dx$$

$$= \frac{\lambda e^{-\lambda t_i} \sinh(\lambda \delta t)}{(1 - e^{-\lambda T}) \lambda \delta t}.$$

Oszacowanie wartości średniej λ i błedu



The likelihood function $L(\lambda)$ can be constructed as a product of N such distributions [17]. The decay constant $\bar{\lambda}$ is estimated from the maximal value for the likelihood function. This finally leads to the following equation:

$$\frac{1}{\bar{\lambda}}\left(\bar{\lambda}\delta t \operatorname{ctgh}(\bar{\lambda}\delta t) - \frac{\bar{\lambda}T}{e^{\bar{\lambda}T}-1}\right) = \sum_{i=1}^{N} t_i/N,$$

The variance of the decay constant $\delta\bar{\lambda}^2$ can be obtained from the equation

$$1/\delta\bar{\lambda}^2 = -\frac{\partial^2 \ln L(\lambda)}{\partial\lambda^2}\Big|_{\lambda=\bar{\lambda}}. \qquad (16)$$

Oszacowanie dokładności pomiaru stałej rozpadu λ

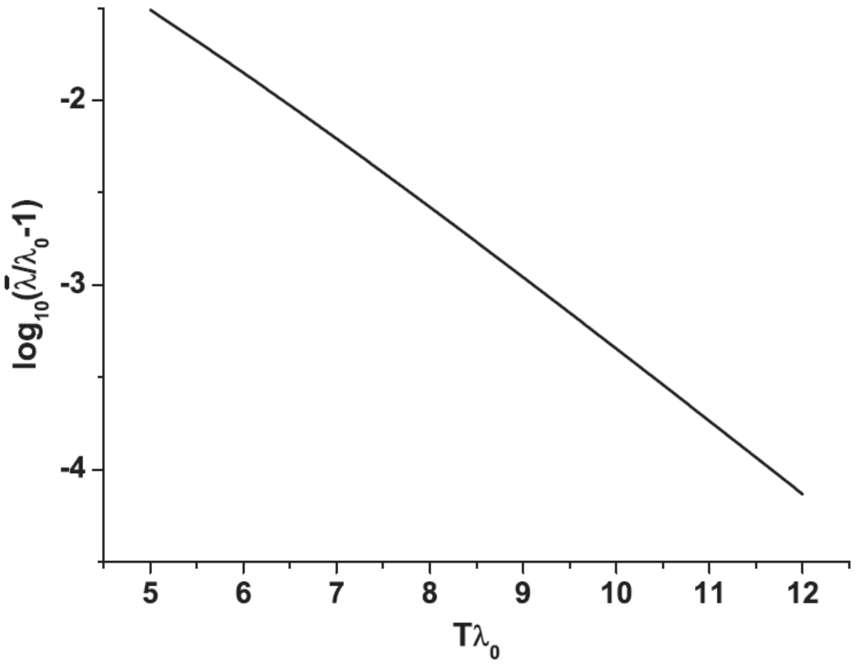


FIG. 4. The solution of Eq. (14) $\bar{\lambda}$, expressed in λ_0 units, as a function of the maximal time T, measured in $1/\lambda_0$ units. One can see that the quantity $\bar{\lambda}/\lambda_0 - 1$ is less then 1 per mil for $T\lambda_0$ greater then 9.

$$\delta\bar{\lambda} = \frac{\bar{\lambda}}{\sqrt{N}}\frac{1}{\sqrt{g(\bar{\lambda}\delta t) - g(\bar{\lambda}T/2)}}$$

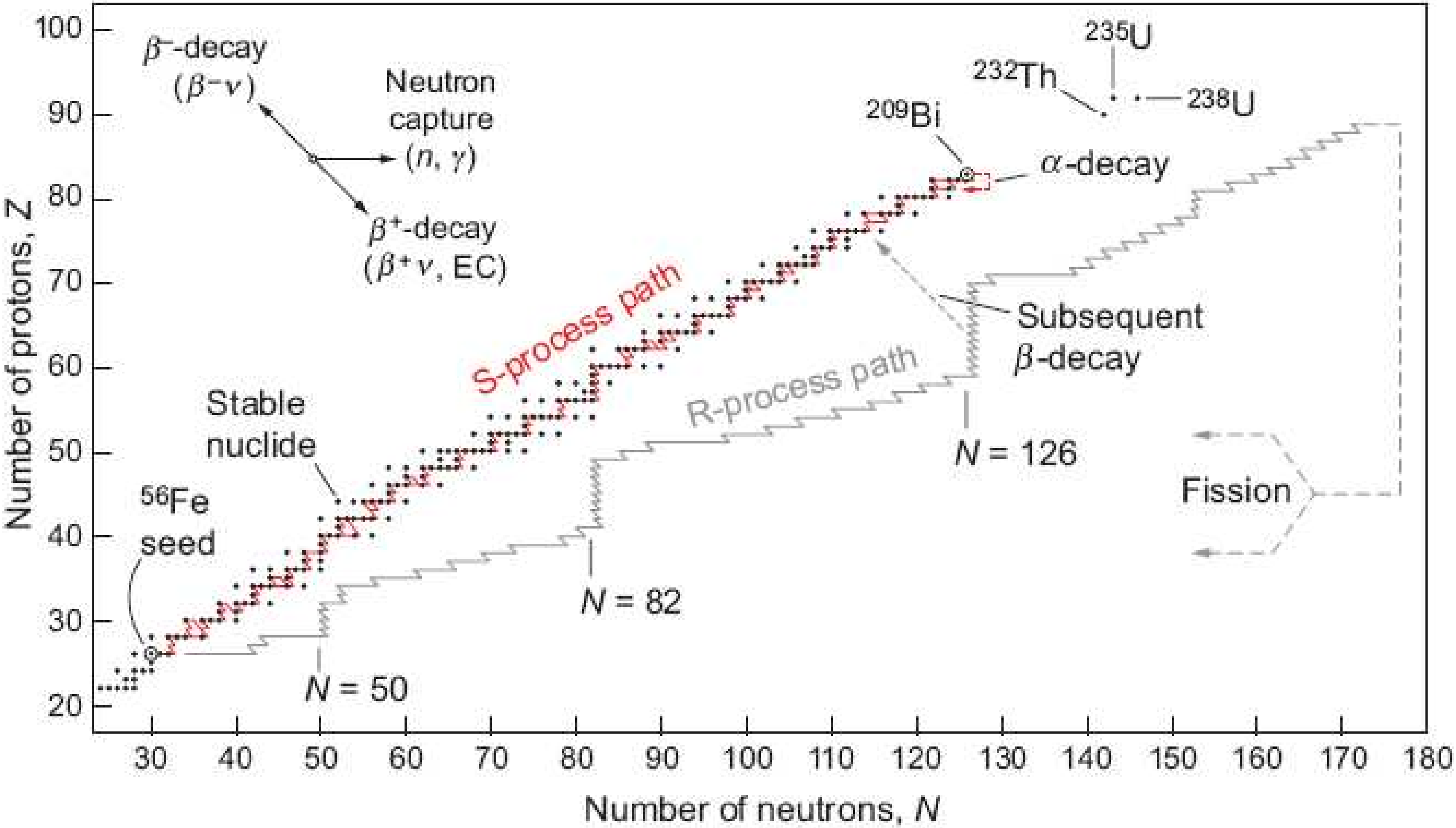
- λ_0 oznacza odwrotność średniej zmierzonych czasów rozpadu.
- W celu osiągnięcia dokładności $\delta\lambda/\lambda$ rzędu 10^{-3} należy dokonać ok. 10^6 pomiarów czasów rozpadu.
- Minimalny czas oczekiwania potrzebny do osiągnięcia tej dokładności wynosi $13T_{1/2}$

$$g(x) = x^2/\sinh^2(x)$$

$$g(0) = 1$$

$$x \to \infty,\ g(x) \to 0$$

Procesy R(apid) i S(low) syntezy jąder cięższych od żelaza ^{56}Fe, zachodzące w gwiazdach



Streszczenie wykładu

1. Różnice czasów życia na rozpad α pomiędzy zupełnie zjonizowanymi i neutralnymi atomami sięgają kilku promili.
2. Oszacowanie niniejsze wyklucza możliwość „unieszkodliwiania” odpadów radioaktywnych metodą chłodzenia.
3. Wykonanie pomiarów λ o dokładności 1 promila wymaga minimalnego czasu T obserwacji rzędu $T\lambda > 9$.
4. Analiza rozpadu α atomów zupełnie zjonizowanych i neutralnych będzie przeprowadzona w GSI-Darmstadt a w przyszłości w FAIR-Darmstadt /ILIMA/.

Mass measurements at NuSTAR/FAIR (ILIMA)