Nowe funkcjonały gęstości do opisu własności jąder atomowych

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Seminarium „Struktura jądra atomowego”
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Plan seminarium

1. Wstęp - podstawy teorii funkcjonału gęstości.
2. Dopasowanie stałych sprzężenia funkcjonału do doświadczalnych energii jednocząstkowych.
3. Poszukiwanie nowych funkcjonałów gęstości o jakości spektroskopowej.
4. Uzależnienie wszystkich stałych sprzężenia funkcjonału od gęstości.
5. Nowe człony funkcjonału do szóstego rzędu w pochodnych.
6. Wnioski, perspektywy i plany.

Nuclear Energy Density Functional

We consider the EDF in the form,

$$\mathcal{E} = \int d^3r \mathcal{H}(r),$$

where the energy density $\mathcal{H}(r)$ can be represented as a sum of the kinetic energy and of the potential-energy isoscalar ($t = 0$) and isovector ($t = 1$) terms,

$$\mathcal{H}(r) = \frac{\hbar^2}{2m}\tau_0 + \mathcal{H}_0(r) + \mathcal{H}_1(r),$$

which for the time-reversal and spherical symmetries imposed read:

$$\mathcal{H}_t(r) = C_t^\rho \rho_t^2 + C_t^\tau \rho_t \tau_t + C_t^{\Delta\rho} \rho_t \Delta \rho_t + \tfrac{1}{2} C_t^J J_t^2 + C_t^{\nabla J} \rho_t \nabla \cdot J_t .$$

Following the parametrization used for the Skyrme forces, we assume the dependence of the coupling parameters C_t^ρ on the isoscalar density ρ_0 as:

$$C_t^\rho = C_{t0}^\rho + C_{t\mathrm{D}}^\rho \rho_0^\alpha .$$

The standard EDF depends linearly on 12 coupling constants,

$$C_{t0}^\rho, \quad C_{t\mathrm{D}}^\rho, \quad C_t^\tau, \quad C_t^{\Delta\rho}, \quad C_t^{\mathrm{J}}, \quad \text{and} \quad C_t^{\nabla \mathrm{J}},$$

for $t = 0$ and 1.

Mean-field equations

Mean-field potentials:

$$
\begin{aligned}
\Gamma_t^{\text{even}} &= -\vec{\nabla}\cdot M_t(\vec{r})\vec{\nabla} + U_t(\vec{r}) + \tfrac{1}{2i}\big(\overleftrightarrow{\nabla}\sigma \cdot \overleftrightarrow{B}_t(\vec{r}) + \overleftrightarrow{B}_t(\vec{r})\cdot \overleftrightarrow{\nabla}\sigma\big) \\
\Gamma_t^{\text{odd}} &= -\vec{\nabla}\cdot(\vec{\sigma}\cdot\vec{C}_t(\vec{r}))\vec{\nabla} + \vec{\sigma}\cdot\vec{\Sigma}_t(\vec{r}) + \tfrac{1}{2i}(\vec{\nabla}\cdot\vec{I}_t(\vec{r}) + \vec{I}_t(\vec{r})\cdot\vec{\nabla}) - \vec{\nabla}\cdot\vec{D}_t(\vec{r})\vec{\sigma}\cdot\vec{\nabla}
\end{aligned}
$$

where

$$
\begin{aligned}
U_t &= 2C_t^{\rho}\rho_t + 2C_t^{\Delta\rho}\Delta\rho_t + C_t^{\tau}\tau_t + C_t^{\nabla J}\vec{\nabla}\cdot\vec{J}_t, \\
\vec{\Sigma}_t &= 2C_t^{s}\vec{s}_t + 2C_t^{\Delta s}\Delta\vec{s}_t + C_t^{T}\vec{T}_t + C_t^{\nabla j}\vec{\nabla}\times\vec{j}_t, -2C_t^{\nabla s}\Delta\vec{s}_t + C_t^{F}\vec{F}_t - 2C_t^{\nabla s}\vec{\nabla}\times(\vec{\nabla}\times\vec{s}_t) \\
M_t &= C_t^{\tau}\rho_t, \\
\vec{C}_t &= C_t^{T}\vec{s}_t, \\
\overleftrightarrow{B}_t &= 2C_t^{J}\,\overleftrightarrow{J}_t - C_t^{\nabla J}\,\overleftrightarrow{\nabla}\,\rho_t, \\
\vec{I}_t &= 2C_t^{j}\vec{j}_t + C_t^{\nabla j}\vec{\nabla}\times\vec{s}_t, \\
\vec{D}_t &= C_t^{F}\vec{s}_t,
\end{aligned}
$$

Neutron and proton mean-field Hamiltonians:

$$
\begin{aligned}
h_n &= -\frac{\hbar^2}{2m}\Delta + \Gamma_0^{\text{even}} + \Gamma_0^{\text{odd}} + \Gamma_1^{\text{even}} + \Gamma_1^{\text{odd}}, \\
h_p &= -\frac{\hbar^2}{2m}\Delta + \Gamma_0^{\text{even}} + \Gamma_0^{\text{odd}} - \Gamma_1^{\text{even}} - \Gamma_1^{\text{odd}}.
\end{aligned}
$$

HF equation for single-particle wave functions:

$$h_\alpha\psi_{i,\alpha}(\vec{r}\sigma) = \epsilon_{i,\alpha}\psi_{i,\alpha}(\vec{r}\sigma),$$

where i numbers the neutron ($\alpha{=}n$) and proton ($\alpha{=}p$) eigenstates.

Phenomenological effective interactions

- Gogny force.*

$$\tilde{G}_{xyx'y'} = \delta(\vec{x}-\vec{x}')\delta(\vec{y}-\vec{y}')G(x,y),$$

where the tilde denotes a non-antisymmetrized matrix element ($G_{xyx'y'} = \tilde{G}_{xyx'y'} - \tilde{G}_{xyy'x'}$), and $G(x,y)$ is a sum of two Gaussians, plus a zero-range, density dependent part,

$$G(x,y) = \sum_{i=1,2} e^{-(\vec{x}-\vec{y})^2/\mu_i^2} \times (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)$$
$$+t_3(1+P_\sigma)\delta(\vec{x}-\vec{y})\rho^{1/3}\left[\tfrac{1}{2}(\vec{x}+\vec{y})\right].$$

In this Equation, $P_\sigma=\frac{1}{2}(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)$ and $P_\tau=\frac{1}{2}(1+\vec{\tau}_1\cdot\vec{\tau}_2)$ are, respectively, the spin and isospin exchange operators of particles 1 and 2, $\rho(\vec{r})$ is the total density of the system at point $\vec{r}$, and μ_i=0.7 and 1.2 fm, W_i, B_i, H_i, M_i, and t_3 are parameters.

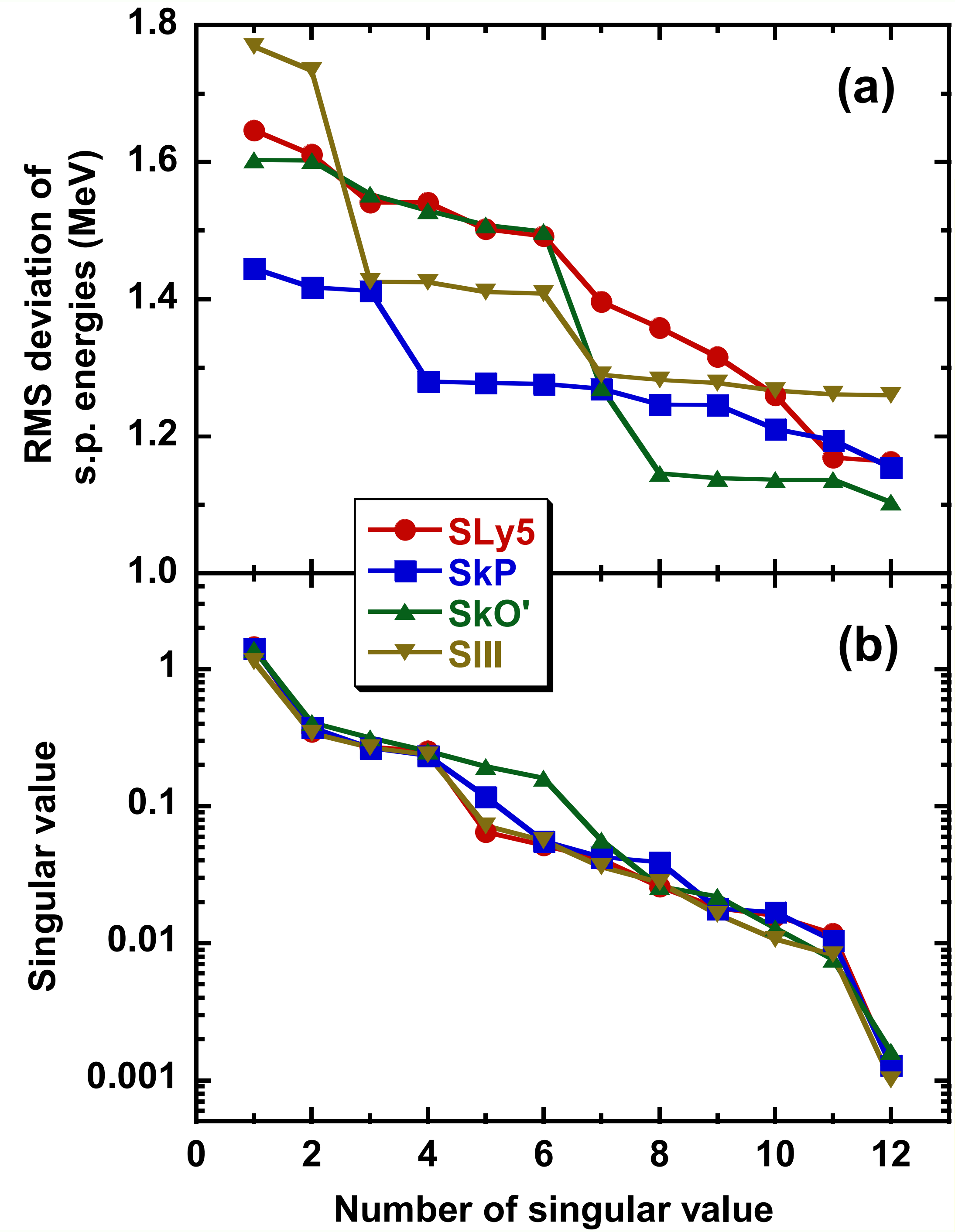
- Skyrme force.*

$$\tilde{G}_{xyx'y'} = \left\{t_0(1+x_0P^\sigma) + \tfrac{1}{6}t_3(1+x_3P^\sigma)\rho^\alpha\left(\tfrac{1}{2}(\vec{x}+\vec{y})\right)\right.$$
$$\left.+\tfrac{1}{2}t_1(1+x_1P^\sigma)[\vec{k}^2+\vec{k}'^2] + t_2(1+x_2P^\sigma)\vec{k}^*\cdot\vec{k}'\right\}\delta(\vec{x}-\vec{x}')\delta(\vec{y}-\vec{y}')\delta(\vec{x}-\vec{y}),$$

where the relative momentum operators read

$$\hat{\vec{k}} = \frac{1}{2i}\left(\vec{\nabla}_x - \vec{\nabla}_y\right), \qquad \hat{\vec{k}}' = \frac{1}{2i}\left(\vec{\nabla}'_x - \vec{\nabla}'_y\right).$$

*We omit the spin-orbit and tensor terms for simplicity.



M. Kortelainen *et al.*, Phys. Rev. C77, 064307 (2008)

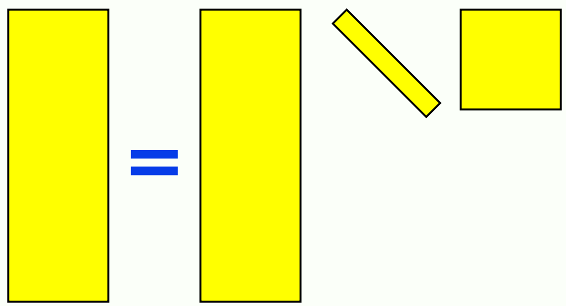
Fits of s.p. energies

$$\epsilon_i - \epsilon_i^{\mathrm{EXP}} = -\sum_m \beta_{im} \Delta C_m ,$$

XP: M.N. Schwierz, I. Wiedenhover,
d A. Volya, arXiv:0709.3525

Singular value decomposition

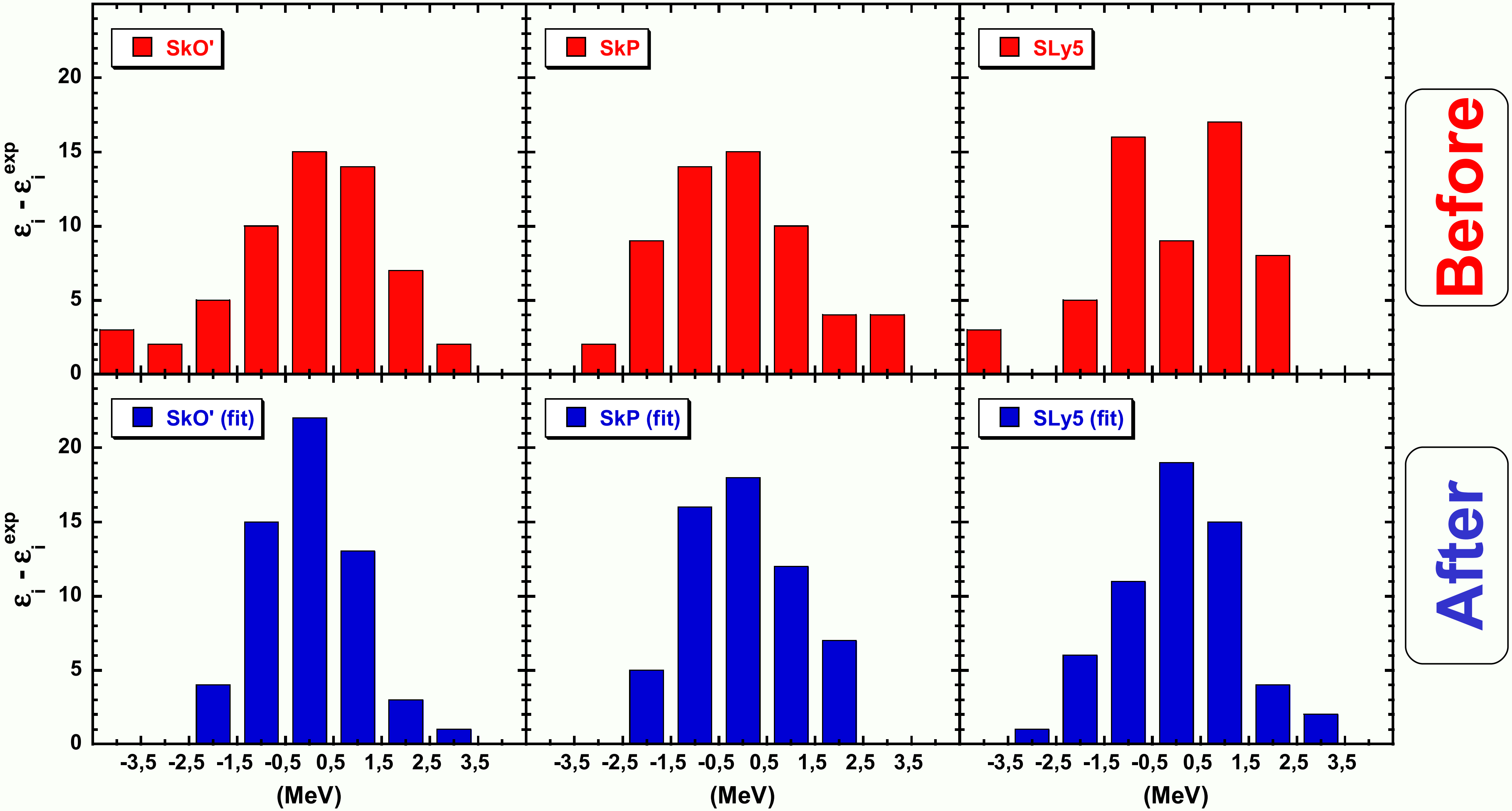
$$\beta_{im} = \sum_\mu V_{i\mu} d_\mu U^T_{\mu m} ,$$



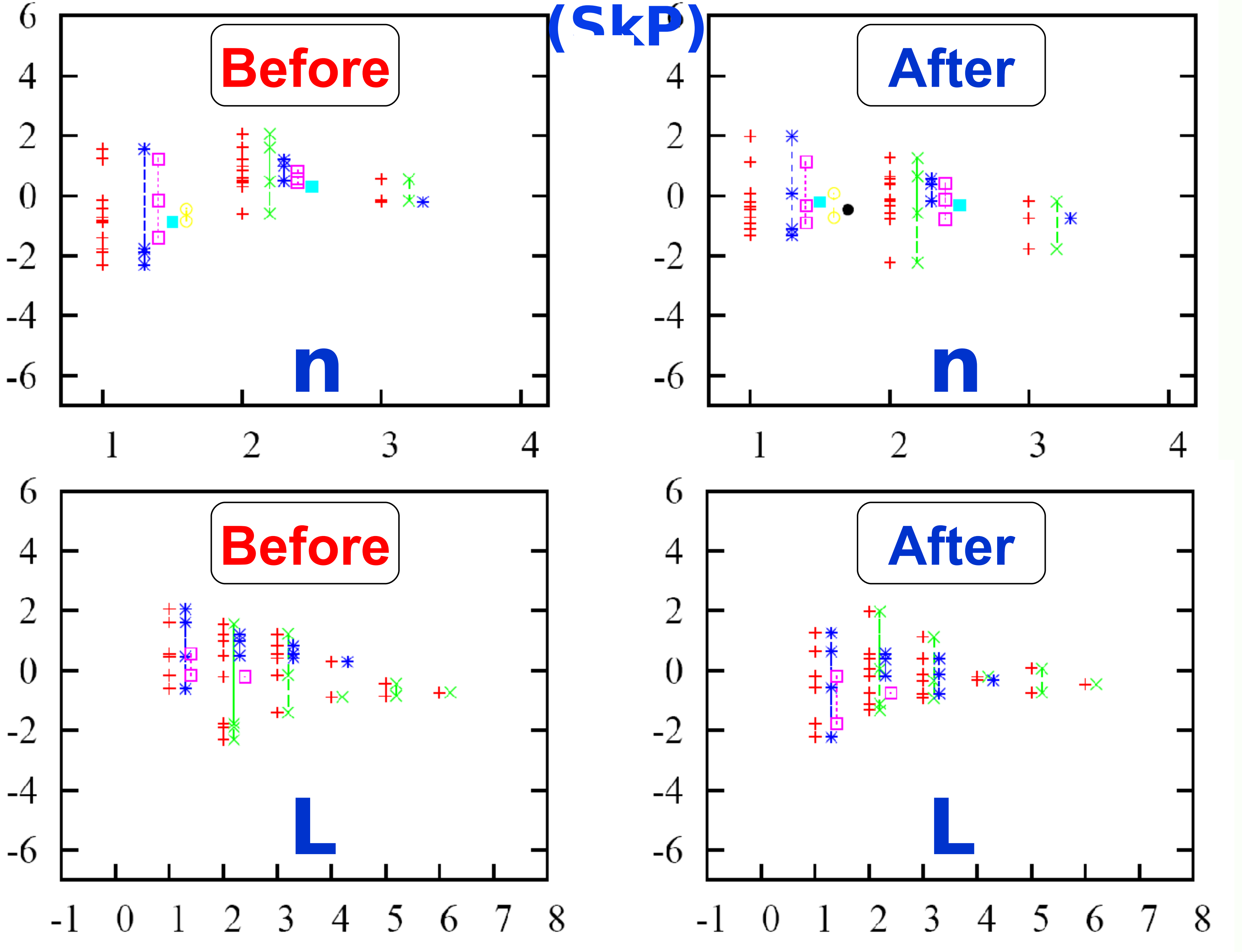
$$\sum_i V_{i\mu} V_{i\nu} = \delta_{\mu\nu} ,$$

$$\sum_m U_{m\mu} U_{m\nu} = \delta_{\mu\nu} ,$$

Fits of single-particle energies

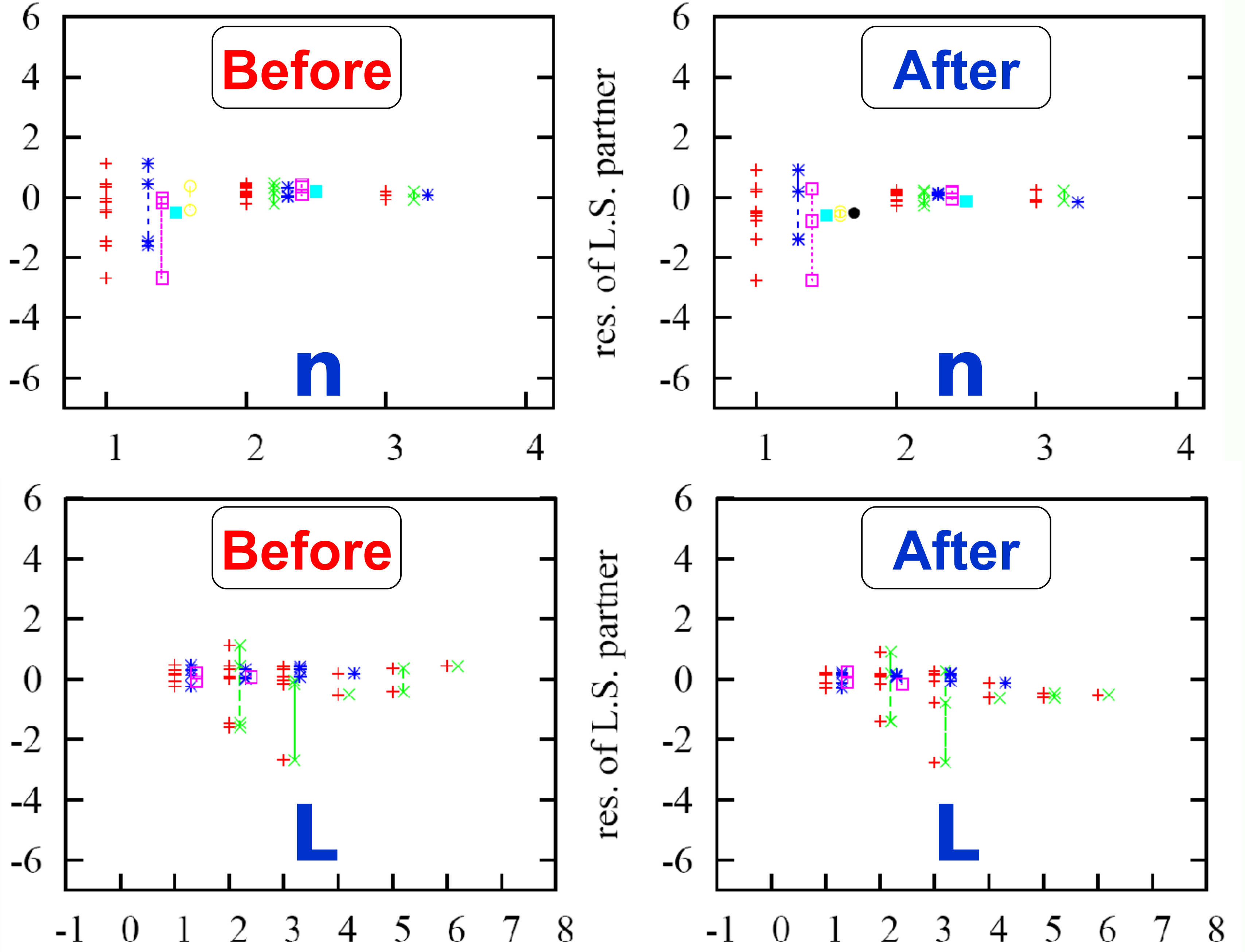


Fit residuals for centroids of SO partners (SkP)



M. Kortelainen *et al.*, to be published

Fit residuals for splittings of SO partners (SkP)



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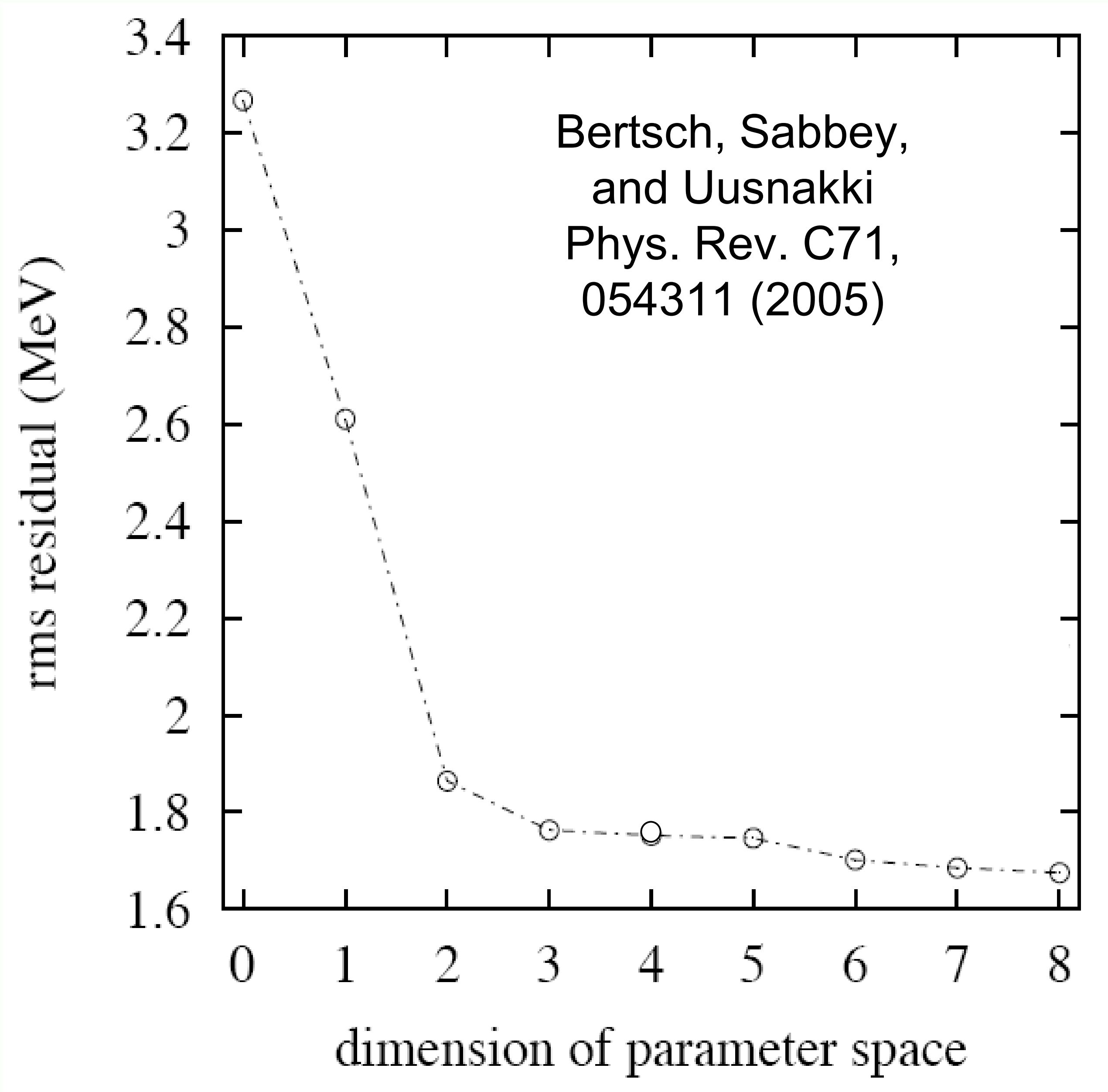
How many parameters are really needed?



Witek
19 marca 2008

Global (masses)

I. Density dependence of all the coupling constants

For the time-reversal and spherical symmetries imposed, the extended EDF reads

$$\mathcal{H}_t(r) = C_t^\rho \rho_t^2 + C_t^\tau \rho_t \tau_t + C_t^{\Delta\rho} \rho_t \Delta \rho_t + \tfrac{1}{2} C_t^J J_t^2 + C_t^{\nabla J} \rho_t \nabla \cdot J_t + C_t^{\nabla\rho} (\nabla \rho_t)^2 + C_t^{\nabla\rho'} (\nabla \rho_t) \cdot J_t$$

and depends linearly on 38 coupling constants,

$$C_t^\rho, \quad C_t^\tau, \quad C_t^{\Delta\rho}, \quad C_t^{\mathrm{J}}, \quad \text{and} \quad C_t^{\nabla \mathrm{J}},$$

$$\alpha_t^\rho, \quad \alpha_t^\tau, \quad \alpha_t^{\Delta\rho}, \quad \alpha_t^{\mathrm{J}}, \quad \alpha_t^{\nabla \mathrm{J}}, \quad \alpha_t^{\nabla\rho}, \quad \text{and} \quad \alpha_t^{\nabla\rho'},$$

$$\beta_t^\rho, \quad \beta_t^\tau, \quad \beta_t^{\Delta\rho}, \quad \beta_t^{\mathrm{J}}, \quad \beta_t^{\nabla \mathrm{J}}, \quad \beta_t^{\nabla\rho}, \quad \text{and} \quad \beta_t^{\nabla\rho'},$$

for $t = 0$ and 1, i.e.,

$$C_t^m(\rho_0, \rho_1) = C_t^m \left[1 + \alpha_t^m \left(1 - \left(\frac{\rho_0}{\rho_{\mathrm{sat}}} \right)^{\gamma_t^m} \right) + \beta_t^m \left(\left(\frac{\rho_1}{\rho_{\mathrm{sat}}} \right)^2 \right)^{\eta_t^m} \right]$$

and on 28 powers γ_t^m and η_t^m.

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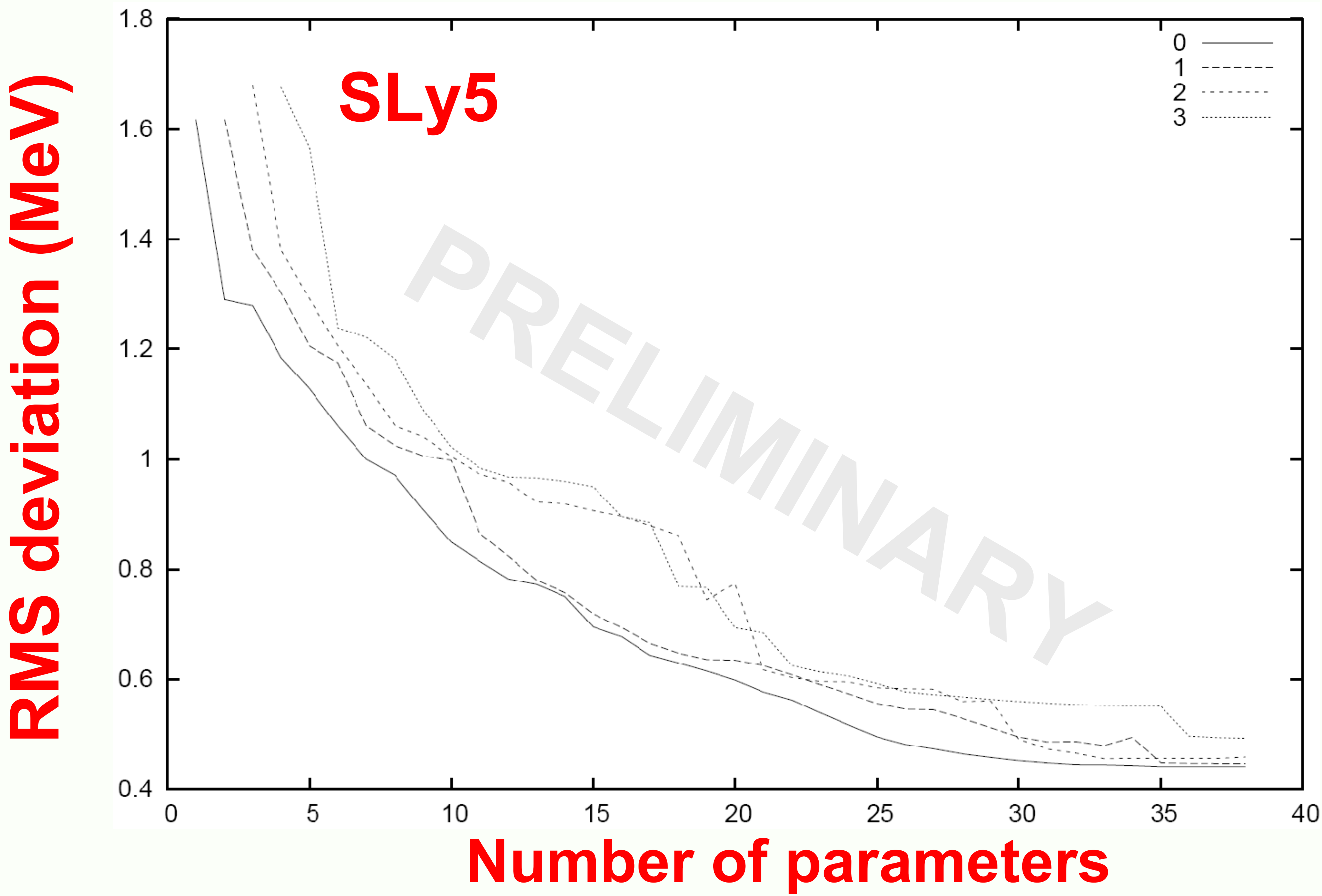
Fit to single-particle energies



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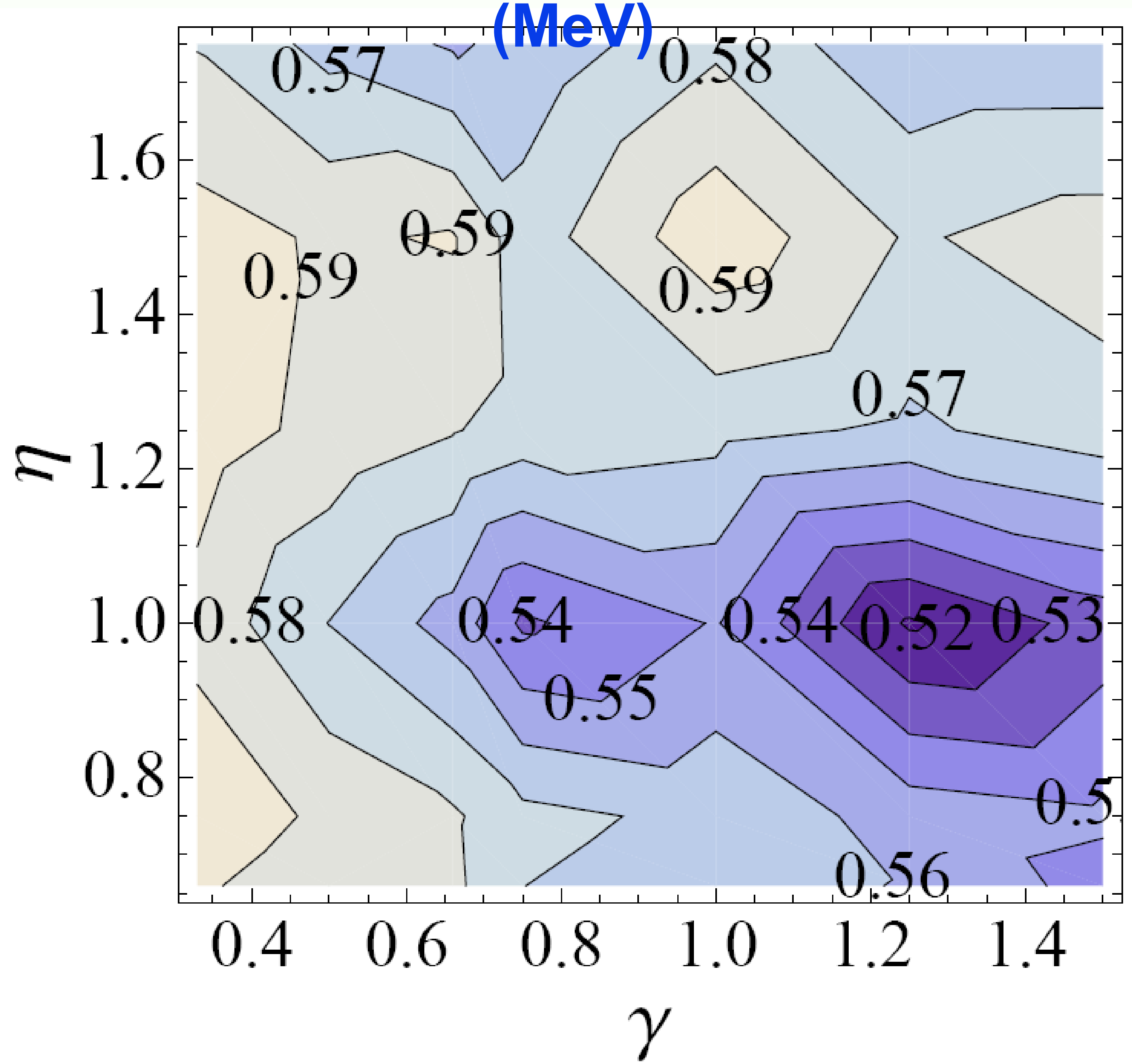
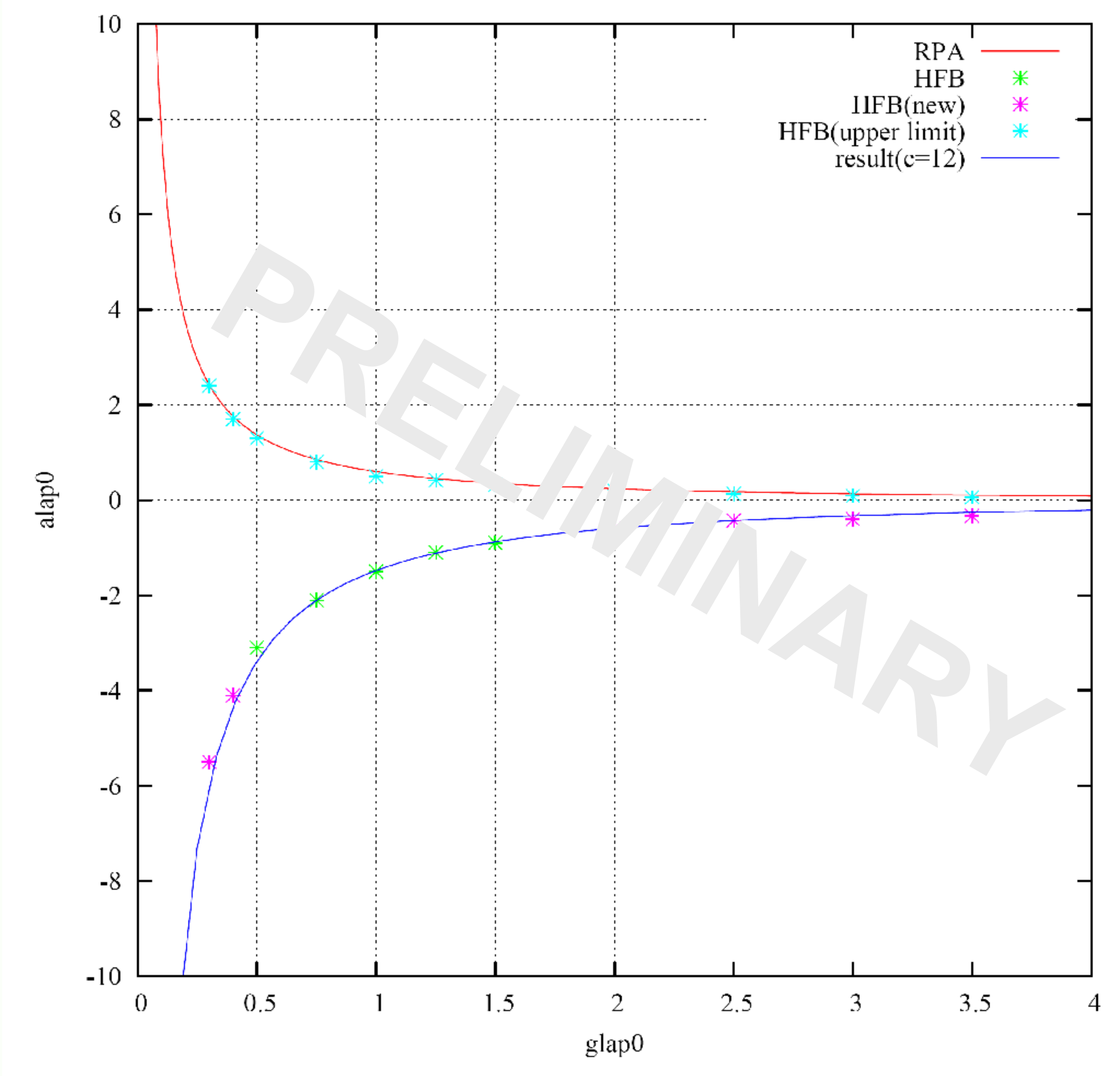
RMS deviation of single-particle energies (MeV)



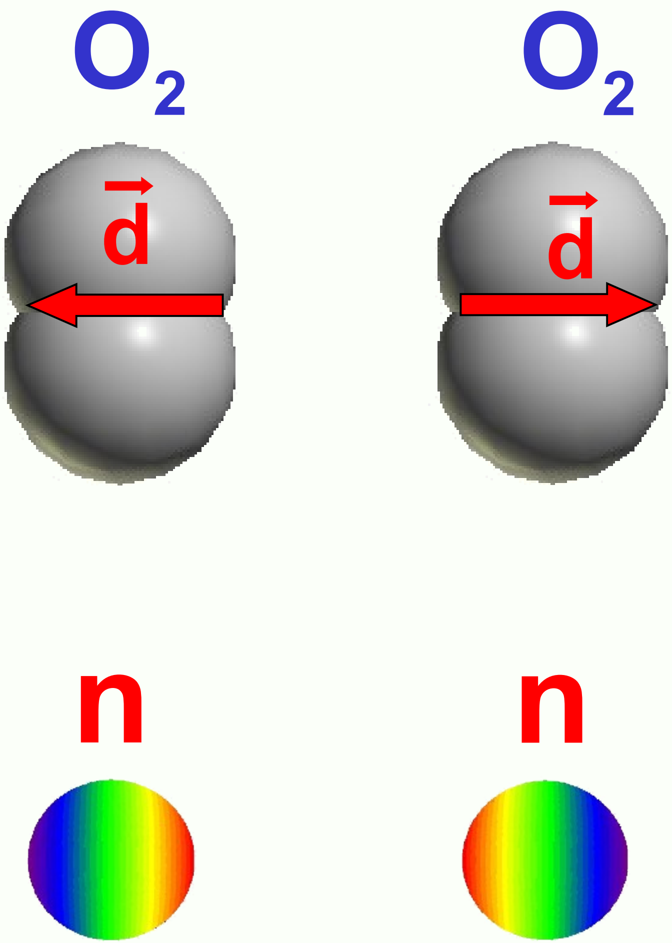
M. Kortelainen *et al.*, to be published

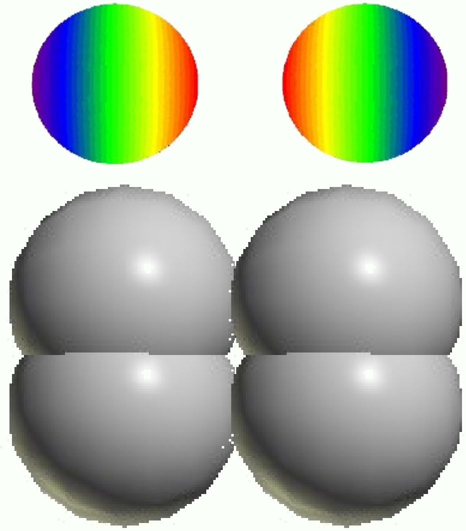
Stability of the Hartree-Fock solutions

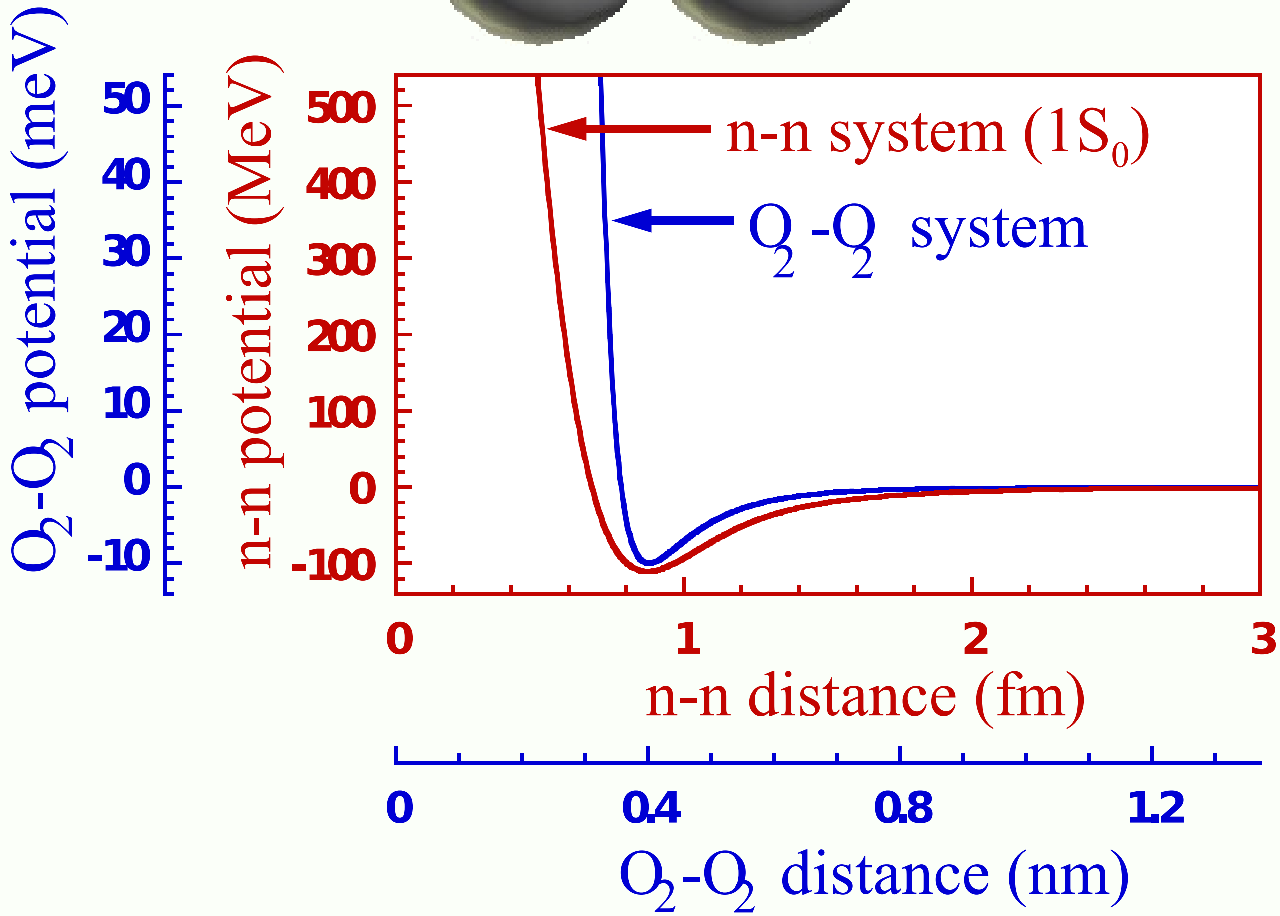


K. Mizuyama *et al.*, to be published

n-n versus O_2-O_2 interaction







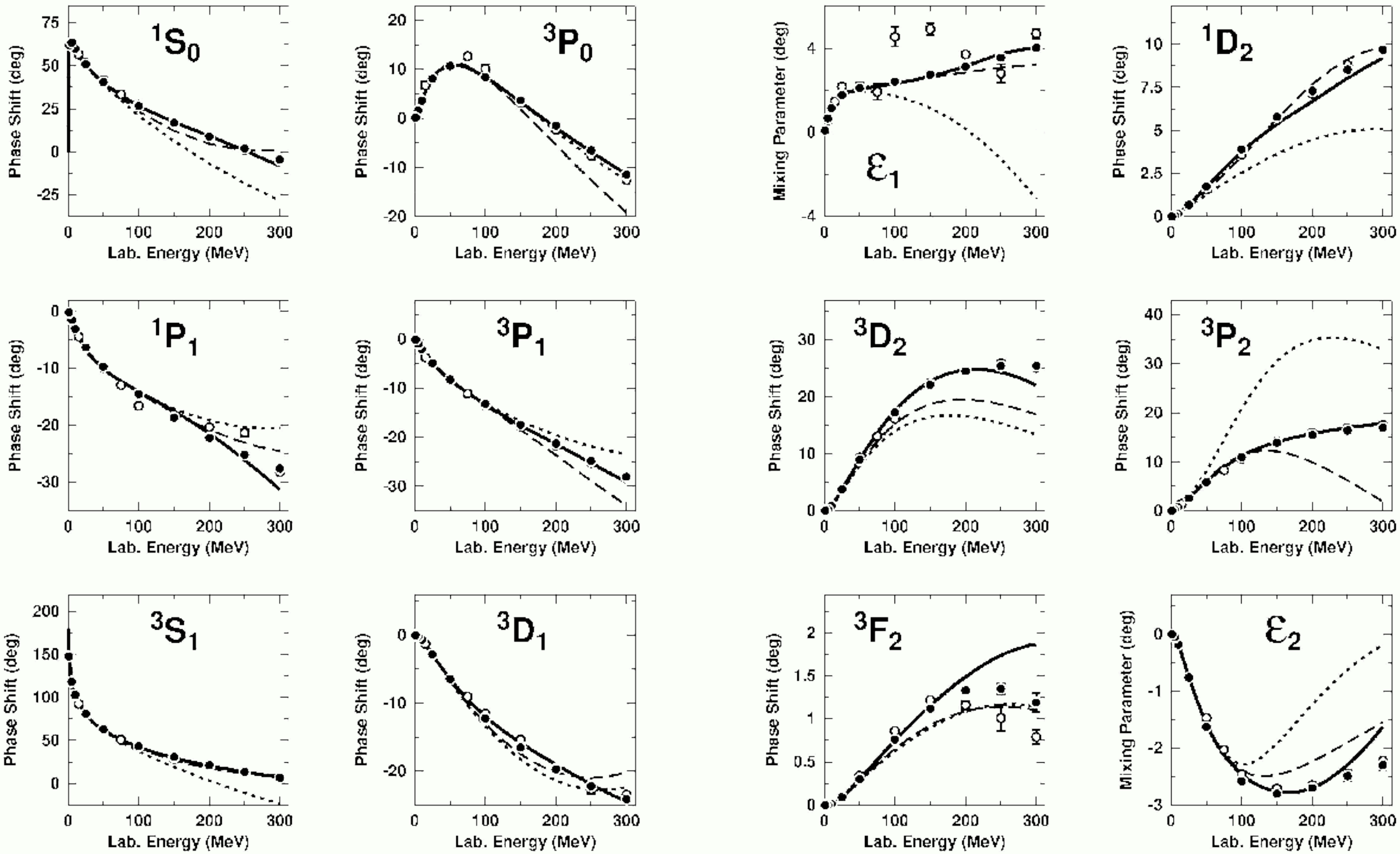
N^3LO in the chiral perturbation effective field theory

Table 1: Contact-gradient expansion for relative-coordinate two-particle matrix elements. Here $\overrightarrow{D}^2_M = (\overrightarrow{\nabla} \otimes \overrightarrow{\nabla})_{2M}$, $\overrightarrow{D}^0_0 = [(\sigma(1) \otimes \sigma(2))_2 \otimes D^2]_{00}$, $\overrightarrow{F}^3_M = (\overrightarrow{\nabla} \otimes \overrightarrow{D}^2)_{3M}$, $\overrightarrow{F}^1_M = [(\sigma(1) \otimes \sigma(2))_2 \otimes F^3]_{1M}$, $\overrightarrow{G}^4_M = (\overrightarrow{D}^2 \otimes \overrightarrow{D}^2)_{4M}$, $\overrightarrow{G}^2_M = [(\sigma(1) \otimes \sigma(2))_2 \otimes G^4]_{2M}$, and the scalar product of tensor operators is defined as $A^J \cdot B^J = \sum_{M=-J}^{M=J} (-1)^M A^J_M B^J_{-M}$.

Transitions	LO	NLO	NNLO	N^3LO
$^3S_1 \leftrightarrow {}^3S_1$ or $^1S_0 \leftrightarrow {}^1S_0$	$\delta(\mathrm{r}$	$\overleftarrow{\nabla}^2\ \delta(\mathrm{r}) + \delta(\mathrm{r})\ \overrightarrow{\nabla}^2$	$\overleftarrow{\nabla}^2\ \delta(\mathrm{r})\ \overrightarrow{\nabla}^2$ $\overleftarrow{\nabla}^4\ \delta(\mathrm{r}) + \delta(\mathrm{r})\ \overrightarrow{\nabla}^4$	$\overleftarrow{\nabla}^4\ \delta(\mathrm{r})\ \overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2\ \delta(\mathrm{r})\ \overrightarrow{\nabla}^4$ $\overleftarrow{\nabla}^6\ \delta(\mathrm{r}) + \delta(\mathrm{r})\ \overrightarrow{\nabla}^6)$
$^3S_1 \leftrightarrow {}^3D_1$		$\delta(\mathrm{r})\ \overrightarrow{D}^0 + \overleftarrow{D}^0\ \delta(\mathrm{r})$	$\overleftarrow{\nabla}^2\ \delta(\mathrm{r})\ \overrightarrow{D}^0 + \overleftarrow{D}^0\ \delta(\mathrm{r})\ \overrightarrow{\nabla}^2$ $\delta(\mathrm{r})\ \overrightarrow{\nabla}^2 \overrightarrow{D}^0 + \overleftarrow{D}^0 \overleftarrow{\nabla}^2\ \delta(\mathrm{r})$	$(\overleftarrow{\nabla}^4\ \delta(\mathrm{r})\ \overrightarrow{D}^0 + \overleftarrow{D}^0\ \delta(\mathrm{r})\ \overrightarrow{\nabla}^4$ $(\overleftarrow{\nabla}^2\ \delta(\mathrm{r})\ \overrightarrow{\nabla}^2 \overrightarrow{D}^0 + \overleftarrow{D}^0 \overleftarrow{\nabla}^2\ \delta(\mathrm{r})\ \overrightarrow{\nabla}^2$ $(\delta(\mathrm{r})\ \overrightarrow{\nabla}^4 \overrightarrow{D}^0 + \overleftarrow{D}^0 \overleftarrow{\nabla}^4\ \delta(\mathrm{r})$
$^1D_2 \leftrightarrow {}^1D_2$ or $^3D_J \leftrightarrow {}^3D_J$			$\overleftarrow{D}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{D}^2$	$\overleftarrow{D}^2 \overleftarrow{\nabla}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{D}^2 + \overleftarrow{D}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}^2 \overrightarrow{D}^2$
$^3D_3 \leftrightarrow {}^3G_3$				$(\overleftarrow{D}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{G}^2 + \overleftarrow{G}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{D}^2$
$^1P_1 \leftrightarrow {}^1P_1$ or $^3P_J \leftrightarrow {}^3P_J$		$\overleftarrow{\nabla} \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}$	$\overleftarrow{\nabla} \overleftarrow{\nabla}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla} + \overleftarrow{\nabla} \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}^2 \overrightarrow{\nabla}$	$\overleftarrow{\nabla} \overleftarrow{\nabla}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}^2 \overrightarrow{\nabla}$ $\overleftarrow{\nabla} \overleftarrow{\nabla}^4 \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla} + \overleftarrow{\nabla} \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}^4 \overrightarrow{\nabla}$
$^3P_2 \leftrightarrow {}^3F_2$			$\overleftarrow{\nabla} \cdot \delta(\mathrm{r})\ \overrightarrow{F}^1 + \overleftarrow{F}^1 \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}$	$\overleftarrow{\nabla} \overleftarrow{\nabla}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{F}^1 + \overleftarrow{F}^1 \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}^2 \overrightarrow{\nabla})$ $\overleftarrow{\nabla} \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}^2 \overrightarrow{F}^1 + \overleftarrow{F}^1 \overleftarrow{\nabla}^2 \cdot \delta(\mathrm{r})\ \overrightarrow{\nabla}$
$^1F_3 \leftrightarrow {}^1F_3$ or $^3F_J \leftrightarrow {}^3F_J$				$\overleftarrow{F}^3 \cdot \delta(\mathrm{r})\ \overrightarrow{F}^3$

W.C. Haxton, Phys. Rev. C77, 034005 (2008)

EFT phase-shift analysis



np phase parameters below 300 MeV lab. energy for partial waves with J=0,1,2. The solid line is the result at N³LO. The dotted and dashed lines are the phase shifts at NLO and NNLO, respectively, as obtained by Epelbaum *et al*. The solid dots show the Nijmegen multi-energy np phase shift analysis and the open circles are the VPI single-energy np analysis SM99.

D.R. Entem and R. Machleidt, Phys.Rev. C68 (2003) 041001

II. Derivatives of higher order up to N³LO

Nr	Tensor	order n	rank L
1	1	0	0
2	∇	1	1
3	Δ	2	0
4	$[\nabla\nabla]_2$	2	2
5	$\Delta\nabla$	3	1
6	$[\nabla[\nabla\nabla]_2]_3$	3	3
7	Δ^2	4	0
8	$\Delta[\nabla\nabla]_2$	4	2
9	$[\nabla[\nabla[\nabla\nabla]_2]_3]_4$	4	4
10	$\Delta^2\nabla$	5	1
11	$\Delta[\nabla[\nabla\nabla]_2]_3$	5	3
12	$[\nabla[\nabla[\nabla[\nabla\nabla]_2]_3]_4]_5$	5	5
13	Δ^3	6	0
14	$\Delta^2[\nabla\nabla]_2$	6	2
15	$\Delta[\nabla[\nabla[\nabla\nabla]_2]_3]_4$	6	4
16	$[\nabla[\nabla[\nabla[\nabla[\nabla\nabla]_2]_3]_4]_5]_6$	6	6

Total derivatives $(\vec{\nabla}^m)_I$ up to N^{3}LO

Nr	Tensor	order n	rank L
1	1	0	0
2	k	1	1
3	k^2	2	0
4	$[kk]_2$	2	2
5	k^2k	3	1
6	$[k[kk]_2]_3$	3	3
7	$(k^2)^2$	4	0
8	$k^2[kk]_2$	4	2
9	$[k[k[kk]_2]_3]_4$	4	4
10	$(k^2)^2k$	5	1
11	$k^2[k[kk]_2]_3$	5	3
12	$[k[k[k[kk]_2]_3]_4]_5$	5	5
13	$(k^2)^3$	6	0
14	$(k^2)^2[kk]_2$	6	2
15	$k^2[k[k[kk]_2]_3]_4$	6	4
16	$[k[k[k[k[kk]_2]_3]_4]_5]_6$	6	6

Relative derivatives $(\vec{k}^n)_L$ up to N^{3}LO

$$\nabla = \nabla_1 + \nabla_2, \qquad k = \frac{1}{2i}\left(\nabla_1 - \nabla_2\right),$$

$$\rho_{v=0} = \rho(r_1, r_2), \qquad \rho_{v=1} = \vec{s}(r_1, r_2),$$

$$\rho_{nLvJ} = \left((\vec{k}^n)_L \rho_v\right)_J \text{ (primary)}, \rho_{mInLvJQ} = \left((\vec{\nabla}^m)_I \left((\vec{k}^n)_L \rho_v\right)_J\right)_Q \text{ (secondary)}$$

B.G. Carlsson *et al.*, Phys. Rev. C 78, 044326 (2008)

Energy density functional up to N^3LO

order	from ρ	from $\vec{s}$	T-even	T-odd	total
0	1	1	1	1	2
1	1	3	3	1	4
2	2	4	2	4	6
3	2	6	6	2	8
4	2	5	2	5	7
5	1	4	4	1	5
6	1	2	1	2	3
total	10	25	19	16	35

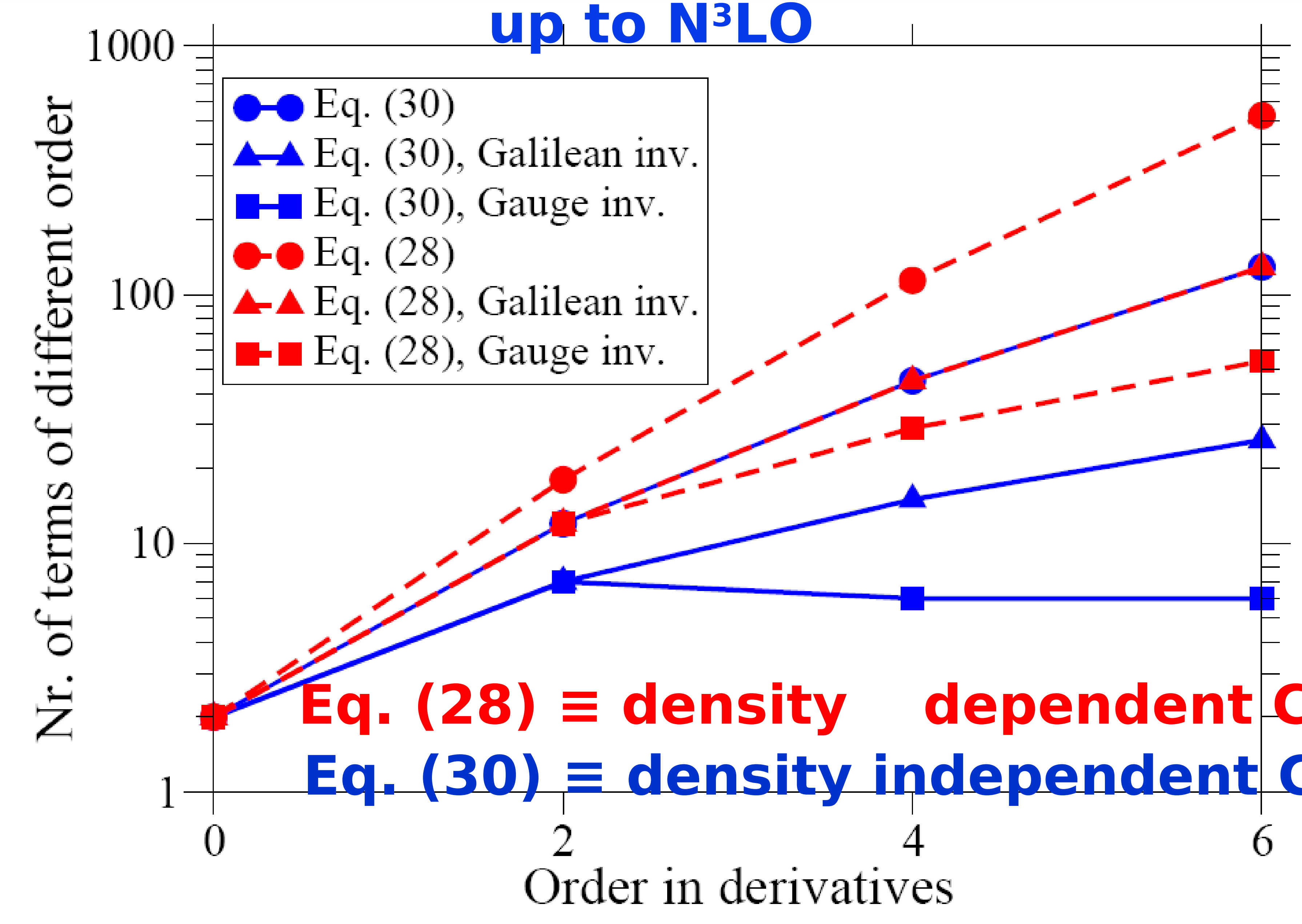
Numbers of primary ($m = 0$) local-densities up to N^3LO.

order	T-even	T-odd	total	Galilean invariant	Gauge invariant
0	1	1	2	2	2
2	6	6	12	7	7
4	22	23	45	15	6
6	64	65	129	26	6
N^3LO	93	95	188	50	21

Numbers of terms in the EDF up to N^3LO.

B.G. Carlsson *et al.*, Phys. Rev. C 78, 044326 (2008)

Numbers of terms in the density functional up to N^{3}LO



Eq. (28) ≡ density dependent C

Eq. (30) ≡ density independent C

B.G. Carlsson *et al.*, Phys. Rev. C 78, 044326 (2008)

Energy density functional for spherical nuclei (I)

For conserved spherical, space-inversion, and time-reversal symmetries, all non-zero densities can be defined as:

$$
\begin{aligned}
R_0 &= \rho, \\
R_2 &= \vec{k}^2\rho = \tau - \tfrac{1}{4}\Delta\rho, \\
\overleftrightarrow{R}_{2ab} &= \vec{k}_a\vec{k}_b\rho, \\
R_4 &= \vec{k}^4\rho, \\
\overleftrightarrow{R}_{4ab} &= \vec{k}^2\vec{k}_a\vec{k}_b\rho, \\
R_6 &= \vec{k}^6\rho,
\end{aligned}
$$

and

$$
\begin{aligned}
\vec{J}_{1a} &= (\vec{k}\times\vec{s})_a, \\
\vec{J}_{3a} &= \vec{k}^2(\vec{k}\times\vec{s})_a, \\
\overleftrightarrow{J}_{3abc} &= \vec{k}_a\vec{k}_b(\vec{k}\times\vec{s})_c + \vec{k}_b\vec{k}_c(\vec{k}\times\vec{s})_a \\
&\quad + \vec{k}_c\vec{k}_a(\vec{k}\times\vec{s})_b, \\
\vec{J}_{5a} &= \vec{k}^4(\vec{k}\times\vec{s})_a,
\end{aligned}
$$

where $\vec{k}^2 = \sum_a \vec{k}_a\vec{k}_a$ and the Cartesian indices are defined as $a, b, c = x, y, z$. To lighten the notation, in these definitions we have omitted the arguments of local densities, $\vec{r}$, and limits of $\vec{r}\,' = \vec{r}$.

Numbers of terms of different orders in the EDF up to N^3LO, evaluated for the conserved spherical, space-inversion, and time-reversal symmetries. The last two columns give numbers of terms when the Galilean or gauge invariance is assumed, respectively.

order	Total	Galilean	Gauge
0	1	1	1
2	4	4	4
4	13	9	3
6	32	16	3
N^3LO	50	30	11

B.G. Carlsson *et al.*, Phys. Rev. C 78, 044326 (2008)

Energy density functional for spherical nuclei (II)

We can write the N^{3}LO spherical energy density as a sum of contributions from zero, second, fourth, and sixth orders:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_2 + \mathcal{H}_4 + \mathcal{H}_6,$$

where

$$\mathcal{H}_0 = C^0_{00} R_0 R_0,$$

$$\begin{aligned}\mathcal{H}_2 &= C^0_{20} R_0 \Delta R_0 + C^0_{02} R_0 R_2 \\ &+ C^0_{11} R_0 \vec{\nabla}\cdot\vec{J}_1, + C^1_{01} \vec{J}_1^2,\end{aligned}$$

Energy densities $\mathcal{H}_0$ and $\mathcal{H}_2$ correspond, of course, to the standard Skyrme functional with $C^0_{00} = C^\rho$, $C^0_{20} = C^{\Delta\rho} + \frac{1}{4}C^\tau$, $C^0_{02} = C^\tau$, $C^0_{11} = C^{\nabla J}$, and $C^1_{01} = C^{J1}$. At fourth order, the energy density reads

$$\begin{aligned}\mathcal{H}_4 &= C^0_{40} R_0 \Delta^2 R_0 + C^0_{22} R_0 \Delta R_2 \\ &+ C^0_{04} R_0 R_4 + C^2_{02} R_2 R_2 \\ &+ D^0_{22} R_0 \sum_{ab} \vec{\nabla}_a \vec{\nabla}_b \overleftrightarrow{R}_{2ab} + D^2_{02} \sum_{ab} \overleftrightarrow{R}_{2ab} \overleftrightarrow{R}_{2ab} \\ &+ C^1_{21} \vec{J}_1 \cdot \Delta \vec{J}_1 + C^1_{03} \vec{J}_1 \cdot \vec{J}_3 \\ &+ D^1_{21} \vec{J}_1 \cdot \vec{\nabla}\left(\vec{\nabla}\cdot\vec{J}_1\right) \\ &+ C^0_{31} R_0 \Delta \left(\vec{\nabla}\cdot\vec{J}_1\right) + C^0_{13} R_0 \left(\vec{\nabla}\cdot\vec{J}_3\right) \\ &+ C^2_{11} R_2 \left(\vec{\nabla}\cdot\vec{J}_1\right) + D^2_{11} \sum_{ab} \overleftrightarrow{R}_{2ab} \vec{\nabla}_a \vec{J}_{1b},\end{aligned}$$

At sixth order, the energy density reads

$$\begin{aligned}\mathcal{H}_6 &= C^0_{60} R_0 \Delta^3 R_0 + C^0_{42} R_0 \Delta^2 R_2 \\ &+ C^0_{24} R_0 \Delta R_4 + C^0_{06} R_0 R_6 \\ &+ C^2_{22} R_2 \Delta R_2 + C^2_{04} R_2 R_4 \\ &+ D^0_{42} R_0 \Delta \sum_{ab} \vec{\nabla}_a \vec{\nabla}_b \overleftrightarrow{R}_{2ab} + D^0_{24} R_0 \sum_{ab} \vec{\nabla}_a \vec{\nabla}_b \overleftrightarrow{R}_{4ab} \\ &+ D^2_{22} R_2 \sum_{ab} \vec{\nabla}_a \vec{\nabla}_b \overleftrightarrow{R}_{2ab} + E^2_{22} \sum_{ab} \overleftrightarrow{R}_{2ab}\, \Delta\, \overleftrightarrow{R}_{2ab} \\ &+ F^2_{22} \sum_{abc} \overleftrightarrow{R}_{2ab} \vec{\nabla}_a \vec{\nabla}_c \overleftrightarrow{R}_{2cb} + E^2_{04} \sum_{ab} \overleftrightarrow{R}_{2ab} \overleftrightarrow{R}_{4ab} \\ &+ C^1_{41} \vec{J}_1 \cdot \Delta^2 \vec{J}_1 + C^1_{23} \vec{J}_1 \cdot \Delta \vec{J}_3 \\ &+ C^1_{05} \vec{J}_1 \cdot \vec{J}_5 + C^3_{03} \vec{J}_3 \cdot \vec{J}_3 \\ &+ D^1_{41} \vec{J}_1 \cdot \Delta \vec{\nabla}\left(\vec{\nabla}\cdot\vec{J}_1\right) + D^1_{23} \vec{J}_1 \cdot \vec{\nabla}\left(\vec{\nabla}\cdot\vec{J}_3\right) \\ &+ E^1_{23} \sum_{abc} \vec{J}_{1a} \vec{\nabla}_b \vec{\nabla}_c \overleftrightarrow{J}_{3abc} + D^3_{03} \sum_{abc} \overleftrightarrow{J}_{3abc} \overleftrightarrow{J}_{3abc} \\ &+ C^0_{51} R_0 \Delta^2 \left(\vec{\nabla}\cdot\vec{J}_1\right) + C^0_{33} R_0 \Delta \left(\vec{\nabla}\cdot\vec{J}_3\right) \\ &+ C^0_{15} R_0 \left(\vec{\nabla}\cdot\vec{J}_5\right) + C^2_{31} R_2 \Delta \left(\vec{\nabla}\cdot\vec{J}_1\right) \\ &+ C^2_{13} R_2 \left(\vec{\nabla}\cdot\vec{J}_3\right) + C^4_{11} R_4 \left(\vec{\nabla}\cdot\vec{J}_1\right) \\ &+ D^0_{33} R_0 \sum_{abc} \vec{\nabla}_a \vec{\nabla}_b \vec{\nabla}_c \overleftrightarrow{J}_{3abc} + D^2_{13} \sum_{abc} \overleftrightarrow{R}_{2ab} \vec{\nabla}_c \overleftrightarrow{J}_{3abc} \\ &+ D^2_{31} \sum_{ab} \overleftrightarrow{R}_{2ab} \Delta \vec{\nabla}_a \vec{J}_{1b} + E^2_{13} \sum_{ab} \overleftrightarrow{R}_{2ab} \vec{\nabla}_a \vec{J}_{3b} \\ &+ D^4_{11} \sum_{ab} \overleftrightarrow{R}_{4ab} \vec{\nabla}_a \vec{J}_{1b} \\ &+ E^2_{31} \sum_{ab} \overleftrightarrow{R}_{2ab} \vec{\nabla}_a \vec{\nabla}_b \left(\vec{\nabla}\cdot\vec{J}_1\right).\end{aligned}$$

The energy densities above are given in terms of 50 coupling constants $C^{n'}_{mn}$, $D^{n'}_{mn}$, $E^{n'}_{mn}$, and $F^{n'}_{mn}$.

B.G. Carlsson *et al.*, Phys. Rev. C 78, 044326 (2008)

Wnioski, perspektywy i plany

- Standardowa parametryzacja jądrowego funkcjonału ... rczająco bogata aby poprawnie ... energie jednocząstkowe.
- Konieczn ... rdowej parametryzacji i poszukiw ... ci o jakości spektros opowej.
- ... zacj ... rzez:
  - ... stałych sprzęże ... ia od gęstości
  - uwzglę ... dnych gęstości
  - uwzglę ... ga potęg gęstości

4. Implementacja nowej metody ... ń RPA i QRPA opartej na algor tmie ... gigantyczne i przejścia beta w jądrach deformowanych).
5. Parametry masowe i ruch kolektywny (ozszczepienie i pasma kolektywne)
6. Profesjonalne dopasowania stałych sprzężenia.

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Gillis Carlsson

Przemek Olbratowski

Jussi Toivanen

Andrzej Baran

Jorge Moré