ISOSPIN MIXING AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS TO THE SUPERALLOWED BETA DECAY

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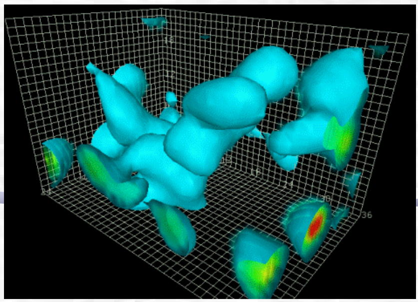
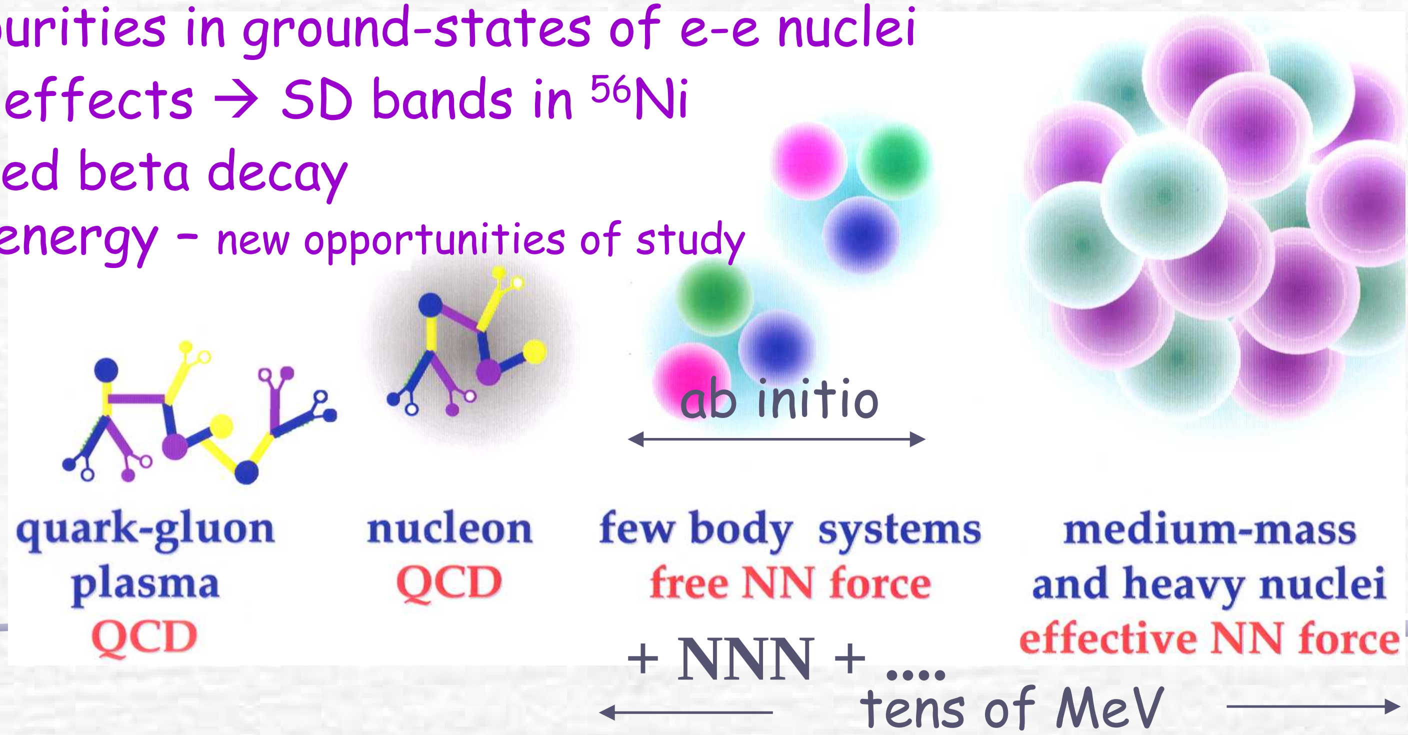
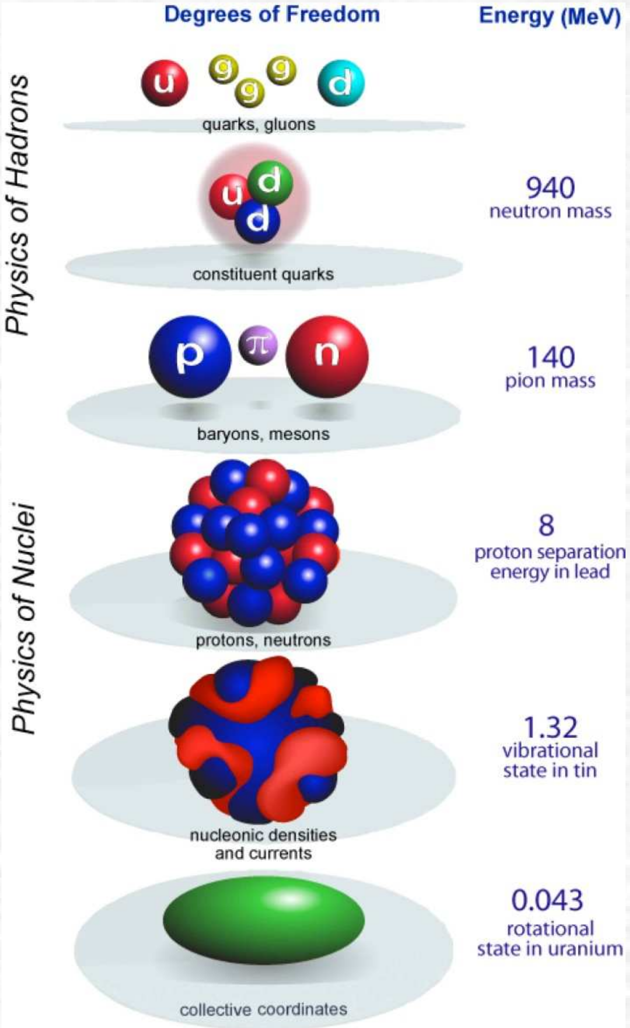
in collaboration with J. Dobaczewski, W. Nazarewicz & M. Rafalski

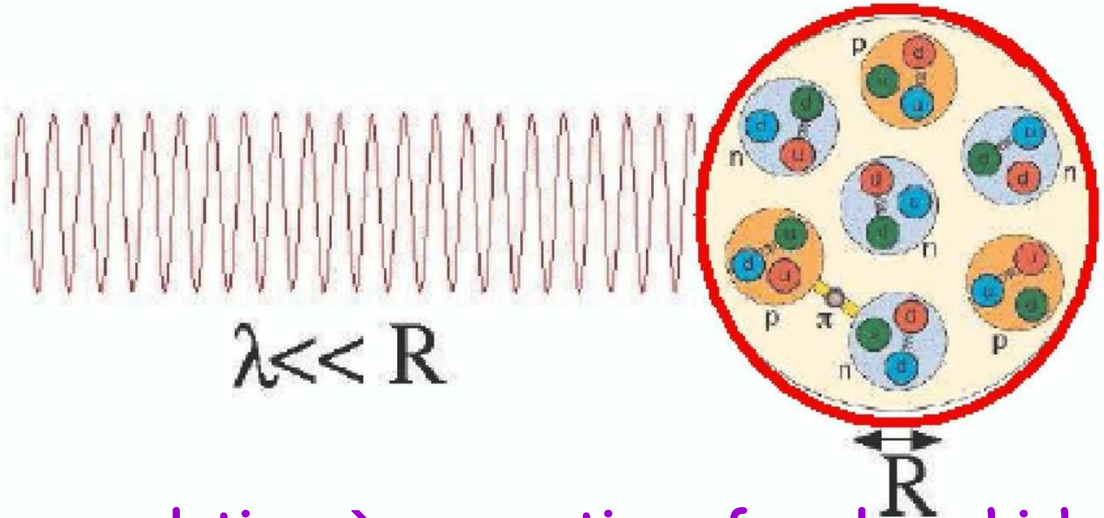
- Intro: effective low-energy theory for medium mass and heavy nuclei → mean-field (or nuclear DFT) → beyond mean-field (projection)
- Symmetry (isospin) violation and restoration:
  - → unphysical symmetry violation → isospin projection
  - → Coulomb rediagonalization (explicit symmetry violation)
  - isospin impurities in ground-states of e-e nuclei
  - structural effects → SD bands in ^{56}Ni
  - superallowed beta decay
  - symmetry energy – new opportunities of study
- Summary



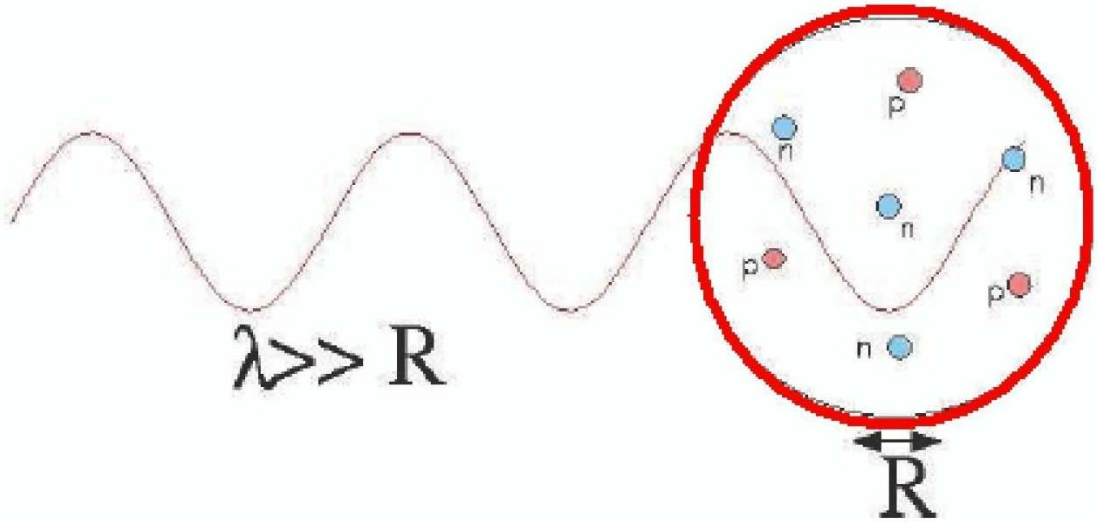


Effective theories for low-energy (low-resolution) nuclear physics (I):





Low-resolution → separation of scales which is a cornerstone of all effective theories



The nuclear effective theory

is based on a simple and very intuitive assumption that low-energy nuclear theory is independent on high-energy dynamics

ultraviolet cut-off →

$$v_S(q^2) \approx v_S(0) + v_S^{(1)}(0)q^2 + v_S^{(2)}(0)q^4 \ldots ,$$

Fourier

regularization

Coulomb

Long-range part of the NN interaction (must be treated exactly!!!)

hierarchy of scales: $v_{eff}(\boldsymbol{r}) \approx v_{long}(\boldsymbol{r})$

$$+ ca^2\delta_a(\boldsymbol{r})$$
$$+ d_1 a^4 \boldsymbol{\nabla}^2 \delta_a(\boldsymbol{r}) + d_2 a^4 \boldsymbol{\nabla} \delta_a(\boldsymbol{r}) \boldsymbol{\nabla}$$
$$+ \ldots$$
$$+ g_1 a^{n+2} \boldsymbol{\nabla}^n \delta_a(\boldsymbol{r}) + \ldots ,$$

local correcting potential

$\frac{2r_o A^{1/3}}{r_o}$ ~ $2A^{1/3}$ ~ 10

where $\delta_a(\boldsymbol{r})$ denotes an arbitrary Dirac-delta model

przykład

$$\delta_a(\boldsymbol{r}) \equiv \frac{e^{-\boldsymbol{r}^2/2a^2}}{(2\pi)^{3/2}a^3} .$$

Gogny interaction

There exist an „infinite" number of equivalent realizations of effective theories

Skyrme interaction - specific (local) realization of the nuclear effective interaction: $\lim_{a \to 0} \delta_a$

10(11) parameters

$$
\begin{aligned}
v(1,2) = \; & t_0 (1 + x_0 \hat{P}_\sigma)\delta(\boldsymbol{r}_{12}) && \text{LO} \\
& + \frac{1}{2} t_1 (1 + x_1 \hat{P}_\sigma)\left(\hat{\boldsymbol{k}}'^2 \delta(\boldsymbol{r}_{12}) + \delta(\boldsymbol{r}_{12})\hat{\boldsymbol{k}}^2\right) && \text{NLO} \\
& + t_2 (1 + x_2 \hat{P}_\sigma)\hat{\boldsymbol{k}}' \delta(\boldsymbol{r}_{12})\hat{\boldsymbol{k}} && \text{NLO} \\
& + \frac{1}{6} t_3 (1 + x_3 \hat{P}_\sigma)\rho_0^\gamma(\boldsymbol{R})\delta(\boldsymbol{r}_{12}) && \text{density dependence} \\
& + i W_0 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\left(\hat{\boldsymbol{k}}' \times \delta(\boldsymbol{r}_{12})\hat{\boldsymbol{k}}\right), && \text{spin-orbit}
\end{aligned}
$$

SV

$\boldsymbol{r}_{12} = \boldsymbol{r}_1 - \boldsymbol{r}_2$; $\boldsymbol{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$;

$\hat{\boldsymbol{k}} = \frac{1}{2i}(\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)$ $\hat{\boldsymbol{k}}' = -\frac{1}{2i}(\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2)$ relative momenta

$\hat{P}_\sigma = \frac{1}{2}(1 + \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2)$ spin exchange

Skyrme-force-inspired local energy density functional

$$\langle \Psi \mid v(1,2) \mid \Psi \rangle$$

Slater determinant
(s.p. HF states are equivalent to the Kohn-Sham states)

$$\mathcal{E}^{Skyrme} = \sum_{t=0,1} \int d^3\boldsymbol{r} \left[\mathcal{H}_t^{(TE)}(\boldsymbol{r}) + \mathcal{H}_t^{(TO)}(\boldsymbol{r}) \right]$$

local energy density functional

Skyrme (nuclear) interaction conserves such symmetries like:

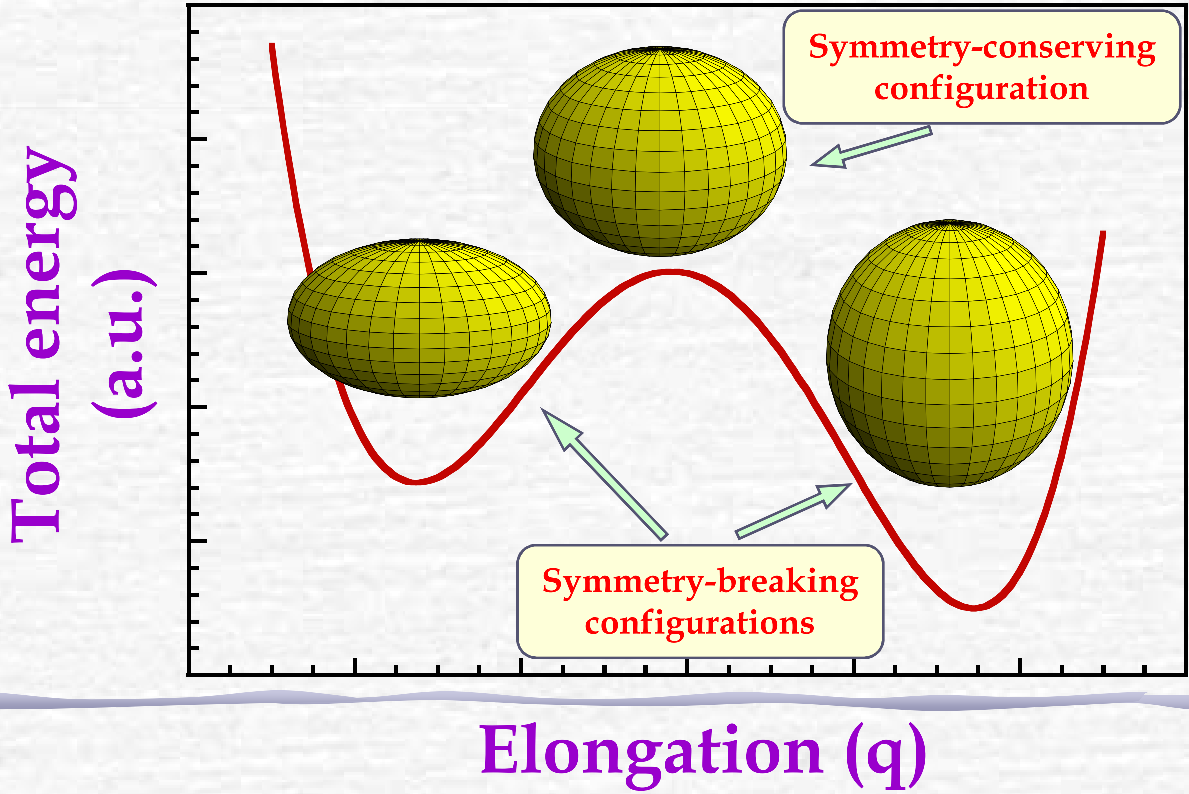
→ rotational (spherical) symmetry

→ isospin symmetry: $V_{nn}^{LS} = V_{pp}^{LS} = V_{np}^{LS}$ (in reality approximate)

→ parity…

Mean-field solutions (Slater determinants) break (spontaneously) these symmetries

$$\hat{R}^{\dagger}(\boldsymbol{\Omega})\Phi_{SL} \neq \Phi_{SL} \quad \text{and} \quad \hat{R}^{\dagger}(\boldsymbol{\Omega})\hat{H}_{HF}[\boldsymbol{\rho}_0]\hat{R}(\boldsymbol{\Omega}) \neq \hat{H}_{HF}[\boldsymbol{\rho}_0]$$



Restoration of broken symmetry

Beyond mean-field → multi-reference density functional theory

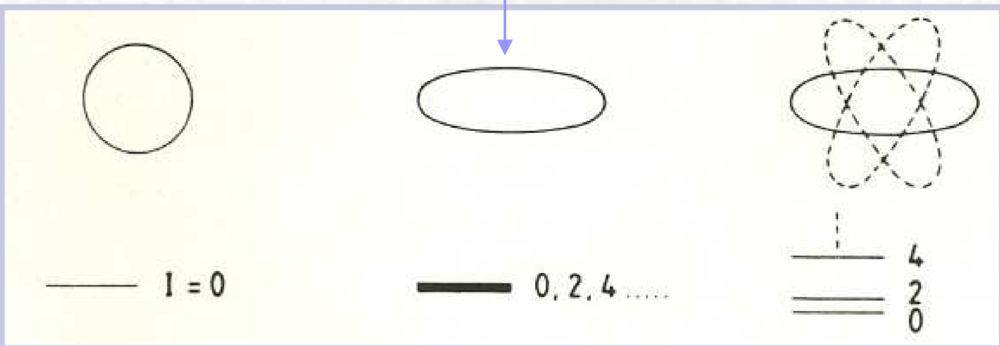
$$\boldsymbol{Q}_0 \equiv \boldsymbol{\Omega}_0, \varphi_0$$

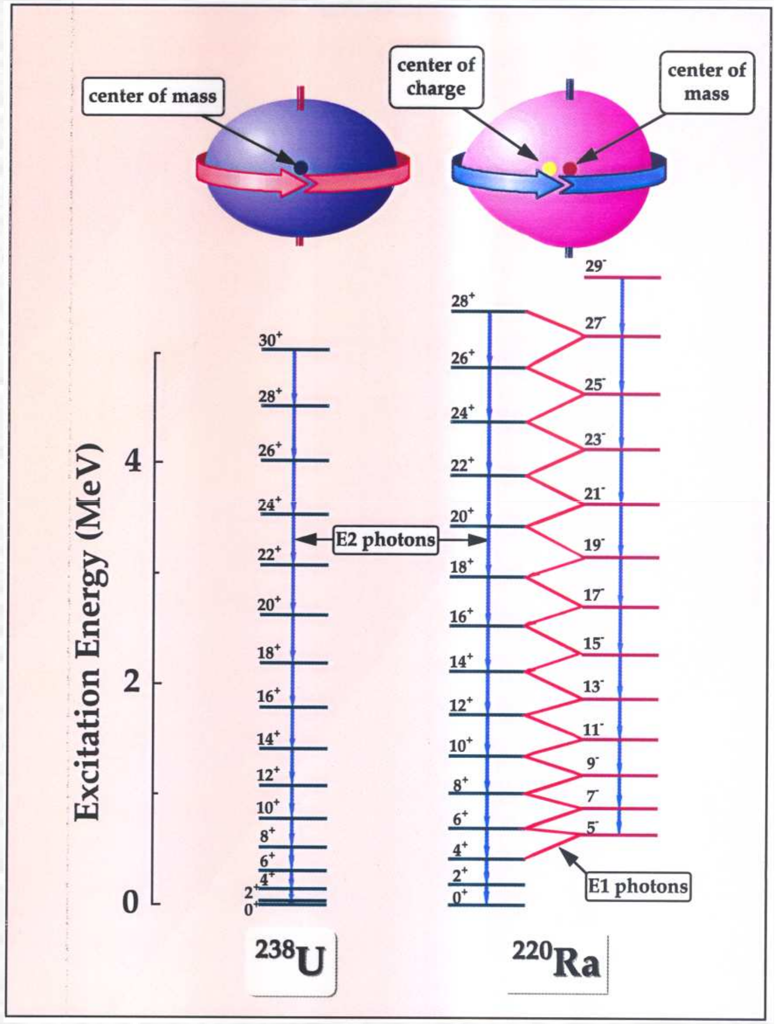
Euler angles → $\boldsymbol{\Omega}_0$; gauge angle → φ_0

rotated Slater determinants are equivalent solutions

$$\hat{R}(\boldsymbol{Q})|\varphi(\boldsymbol{Q}_0)\rangle = |\varphi(\boldsymbol{Q}')\rangle$$

$$\langle\varphi|\hat{H}|\varphi\rangle = \langle\varphi|\hat{R}^\dagger(\boldsymbol{Q})\hat{H}\hat{R}(\boldsymbol{Q})|\varphi\rangle$$





$$|\Psi_Q\rangle \equiv \int d\boldsymbol{q}\, g(\boldsymbol{q}) \hat{P}_Q |\varphi(\boldsymbol{q})\rangle \equiv \int d\boldsymbol{q}\, g(\boldsymbol{q}) \left[\int d\boldsymbol{Q}\, f_Q(\boldsymbol{Q}) \hat{R}_Q(\boldsymbol{Q}) |\varphi(\boldsymbol{q})\rangle \right]$$

$$\hat{P}_{IM} = \sum_K g_K \hat{P}^I_{MK} = \sum_K g_K \frac{2I+1}{8\pi^2} \int d\boldsymbol{\Omega}\, D^{I*}_{MK}(\boldsymbol{\Omega}) \hat{R}(\boldsymbol{\Omega}) \quad \text{and} \quad \hat{P}_I = \sum_M g^*_M \hat{P}_{IM}$$

$$E_Q = \frac{\int d\boldsymbol{Q}\, f_Q(\boldsymbol{Q}) h(\boldsymbol{Q})}{\int d\boldsymbol{Q}\, f_Q(\boldsymbol{Q}) n(\boldsymbol{Q})} \quad \text{where} \quad \left\{ \begin{array}{c} h(\boldsymbol{Q}) \\ n(\boldsymbol{Q}) \end{array} \right\} = \langle\varphi| \left\{ \begin{array}{c} \hat{H}\hat{R}(\boldsymbol{Q}) \\ \hat{R}(\boldsymbol{Q}) \end{array} \right\} |\varphi\rangle$$

Isospin symmetry restoration

There are two sources of the isospin symmetry breaking:

- **unphysical**, caused solely by the HF approximation → Engelbrecht & Lemmer, PRL24, (1970) 607
- **physical**, caused mostly by Coulomb interaction
  (also, but to much lesser extent, by the strong force isospin non-invariance)

- Find self-consistent HF solution (including Coulomb) → deformed Slater determinant |HF>:

$$|\mathrm{HF}\rangle = \sum_{T\geq|T_z|} b_{T,T_z}|\alpha;T,T_z\rangle$$

See: Caurier, Poves & Zucker, PL 96B, (1980) 11; 15

- Apply the isospin projector:

$$\hat{P}^T_{T_zT_z} = \frac{2T+1}{2}\int_0^\pi d\beta \sin\beta d^{T*}_{T_zT_z}(\beta)\hat{R}(\beta)$$

in order to create good isospin „basis":

$$|\alpha;T,T_z\rangle = \frac{1}{b_{T,T_z}}\hat{P}^T_{T_zT_z}|\mathrm{HF}\rangle$$

- Calculate the projected energy and the Coulomb mixing **Before Rediagonalization**:

$$E^T_{\mathbf{BR}} = \frac{\langle\mathrm{HF}|\hat{P}^{T\dagger}_{T_zT_z}\hat{H}\hat{P}^T_{T_zT_z}|\mathrm{HF}\rangle}{\langle\mathrm{HF}|\hat{P}^{T\dagger}_{T_zT_z}\hat{P}^T_{T_zT_z}|\mathrm{HF}\rangle}$$

$$\alpha_C^{\mathbf{BR}} = 1 - |b_{T=|T_z|}|^2$$

- Diagonalize total Hamiltonian in „good isospin basis" $|\alpha,T,T_z\rangle$
→ takes physical isospin mixing

$$\sum_{T'\geq|T_z|}\langle\alpha;T,T_z|\hat{H}|\alpha;T',T_z\rangle a^n_{T',T_z} = E^{\rm AR}_{n,T_z}\, a^n_{T,T_z}$$

$$|\alpha;n,T_z\rangle = \sum_{T\geq|T_z|} a^n_{T,T_z}|\alpha;T,T_z\rangle,$$

$$\alpha_C^{AR} = 1 - |a^{n=1}_{T=T_z}|^2$$

$$\hat{H} = \hat{H}^S + \hat{V}^C$$

$$\hat{H}^S = \hat{T} + \hat{V}^S$$

Isospin invariant

Isospin breaking: isoscalar, isovector & isotensor

$$\langle{\rm HF}|\hat{H}^S\hat{P}^T_{T_zT_z}|{\rm HF}\rangle = \int_0^\pi d\beta\sin\beta\, d^T_{T_zT_z}(\beta)\langle{\rm HF}|\hat{H}^S\hat{R}(\beta)|{\rm HF}\rangle.$$

$$\langle{\rm HF}|\hat{P}^T_{T_zT_z}\hat{V}^C_{\lambda 0}\hat{P}^{T'}_{T_zT_z}|{\rm HF}\rangle = C^{TT_z}_{T'T_z\,\lambda 0}\sum_{\mu'=-\lambda}^{\lambda} C^{TT_z}_{T'T'_z\,\lambda\mu'}$$

$$\frac{2T'+1}{2}\int_0^\pi d\beta\;\sin\beta\; d^{T'}_{T'_z,T_z}(\beta)\;\langle{\rm HF}|\hat{V}^C_{\lambda\mu'}\hat{R}(\beta)|{\rm HF}\rangle,$$

$$|\widetilde{\rm HF}(\beta)\rangle = \hat{R}(\beta)|{\rm HF}\rangle$$

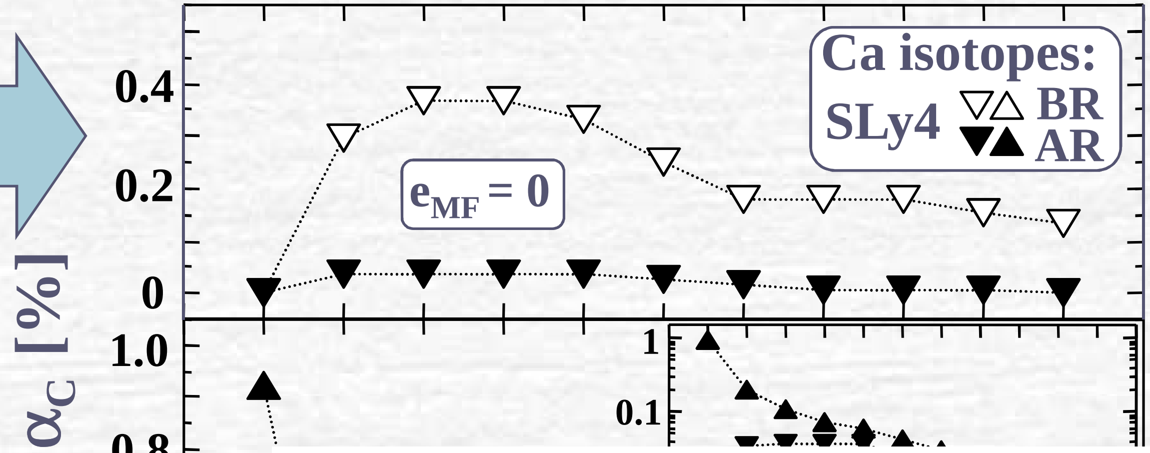
Numerical results:
(I) Isospin impurities in ground states of e-e nuclei

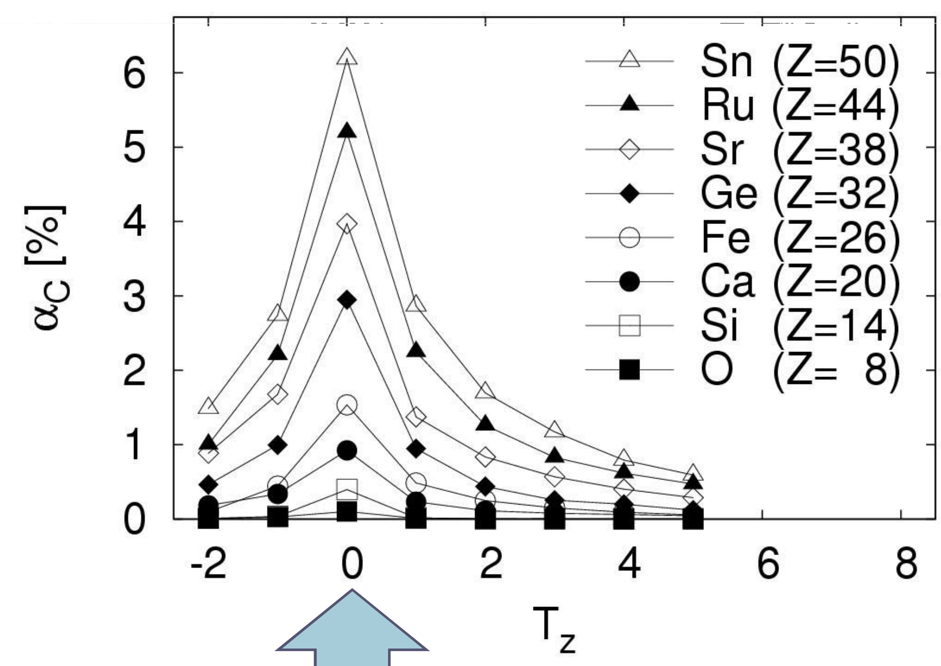
W.Satuła, J.Dobaczewski, W.Nazarewicz, M.Rafalski, PRL103 (2009) 012502

Here the HF is solved without Coulomb $|HF;e_{MF}=0\rangle$.

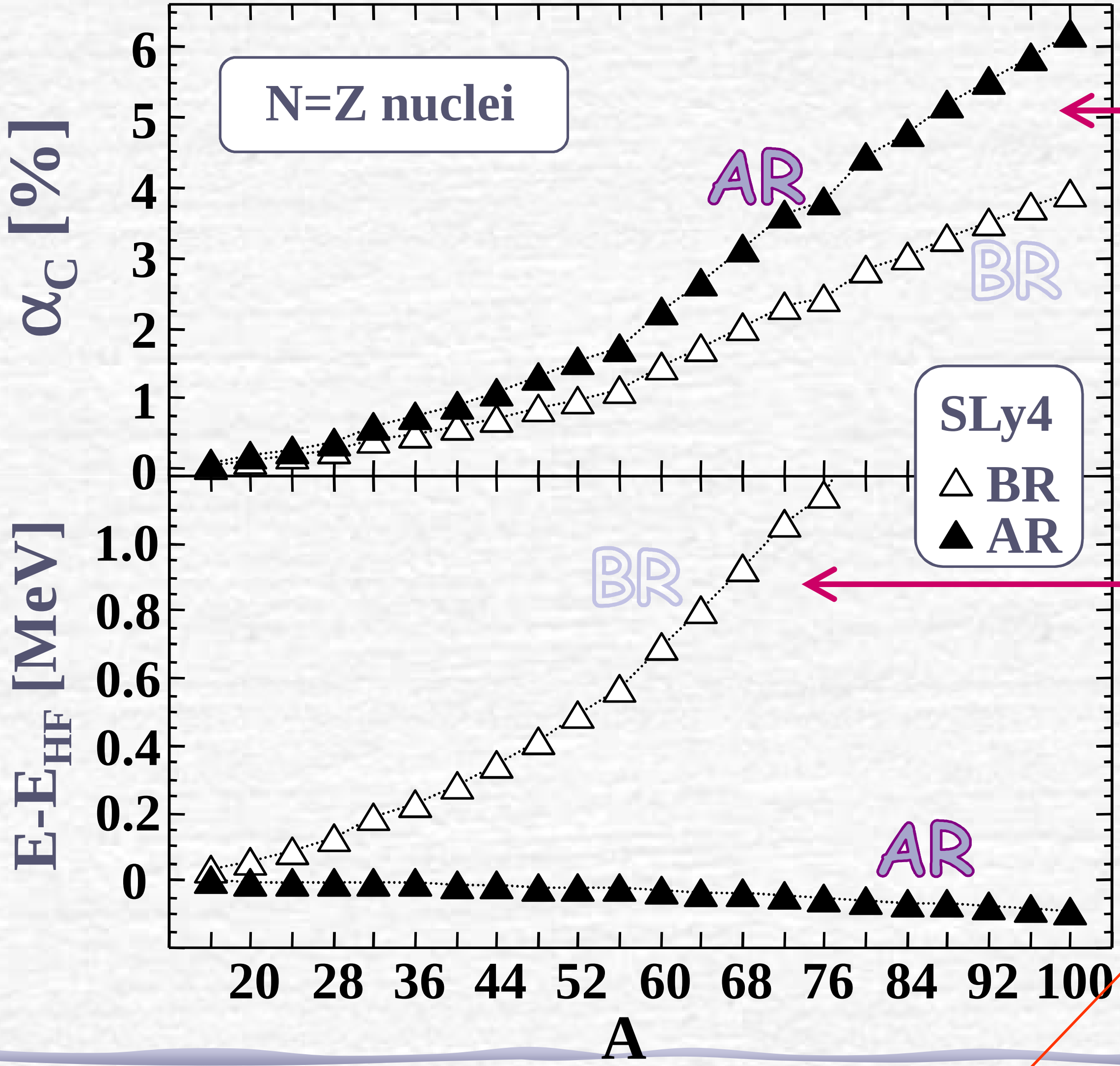
Here the HF is solved with Coulomb $|HF;e_{MF}=e\rangle$.

In both cases rediagonalization is performed for the total Hamiltonian including Coulomb





(II) Isospin mixing & energy in the ground states of e-e N=Z nuclei:



HF tries to reduce the isospin mixing by: $\Delta\alpha_C$ ~30% in order to minimize the total energy

Projection increases the ground state energy
(the Coulomb and symmetry energies are repulsive)

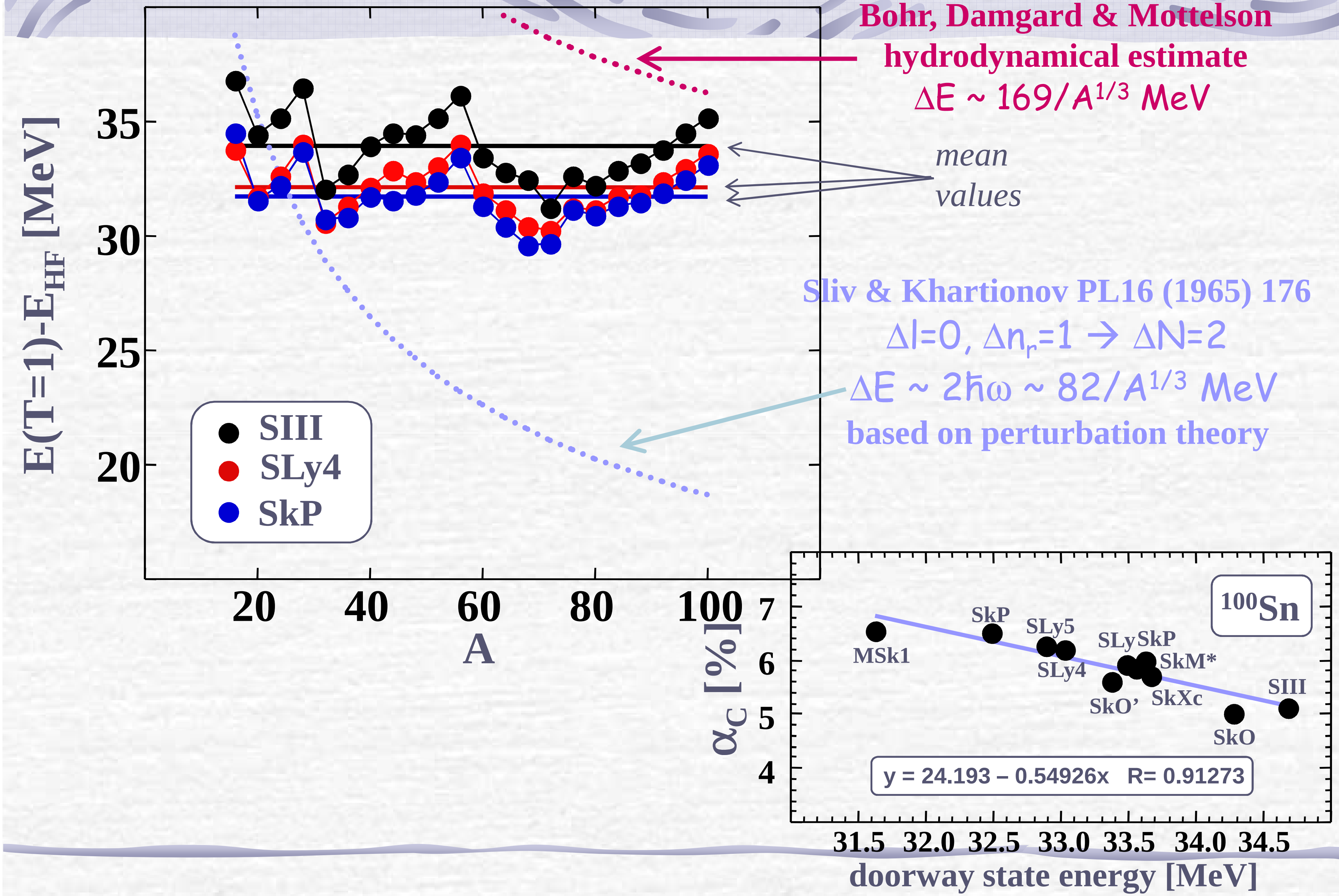
Rediagonalization (GCM)

$$|\alpha; n, T_z\rangle = \sum_{T\geq|T_z|} a^n_{T,T_z} |\alpha; T, T_z\rangle,$$

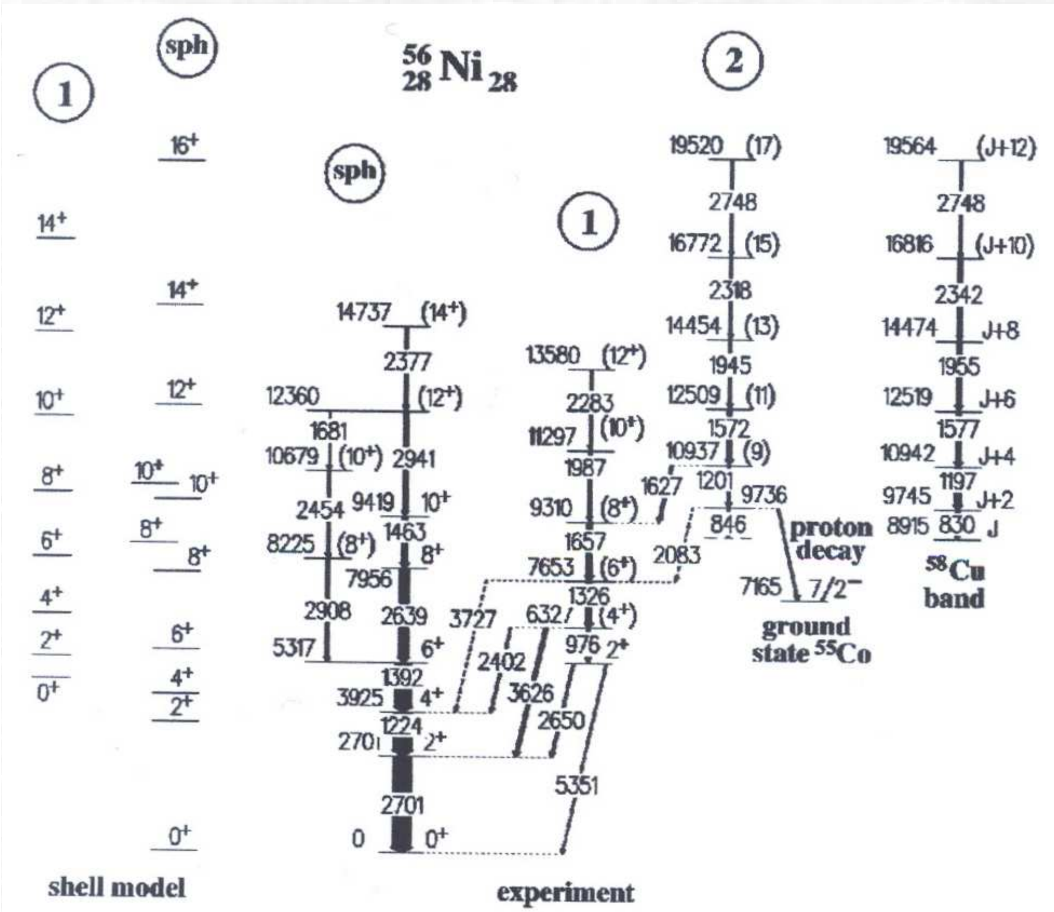
lowers the ground state energy but only slightly below the HF

This is not a single Slater determinat
There are no constraints on mixing coefficients

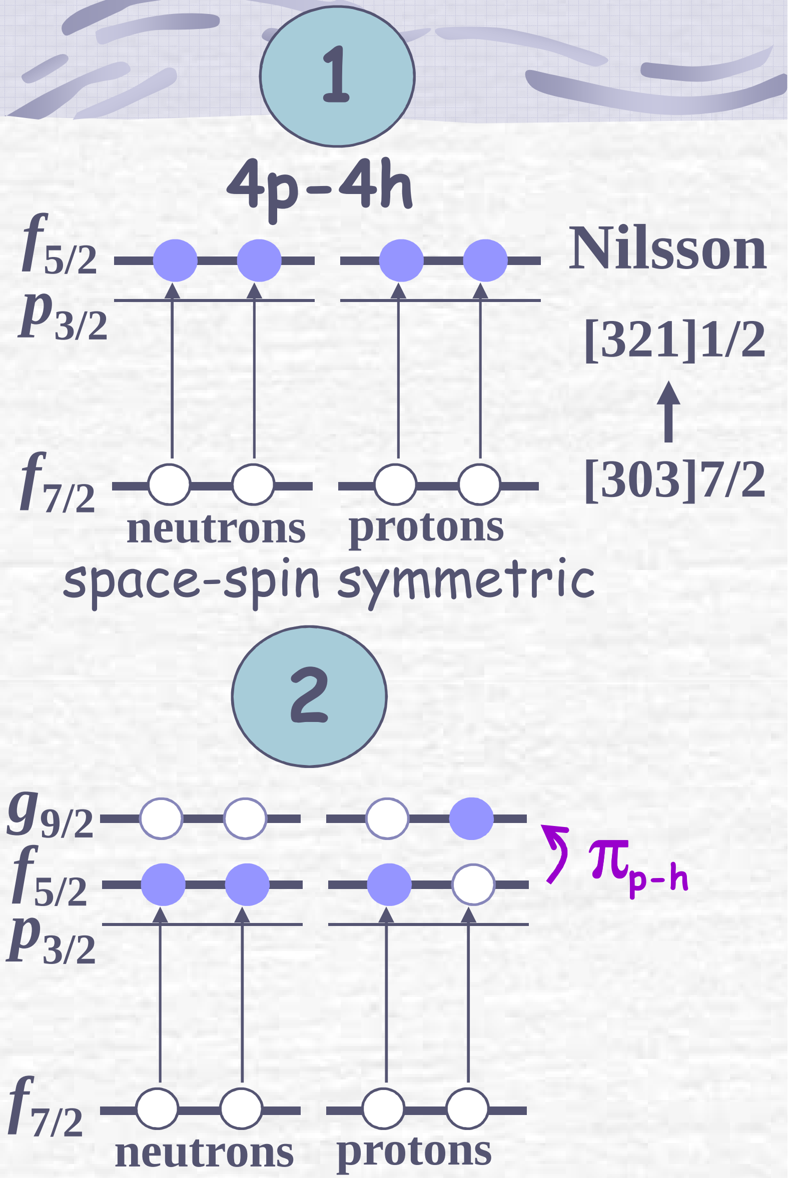
Position of the T=1 doorway state in N=Z nuclei



Isospin symmetry violation in superdeformed bands in ^{56}Ni



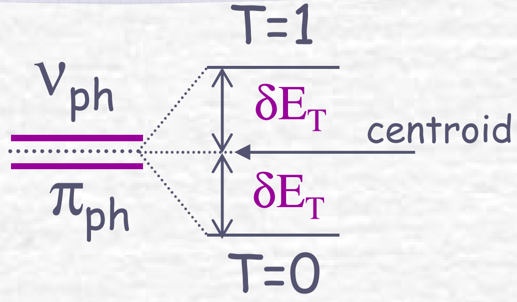
D. Rudolph et al. PRL82, 3763 (1999)



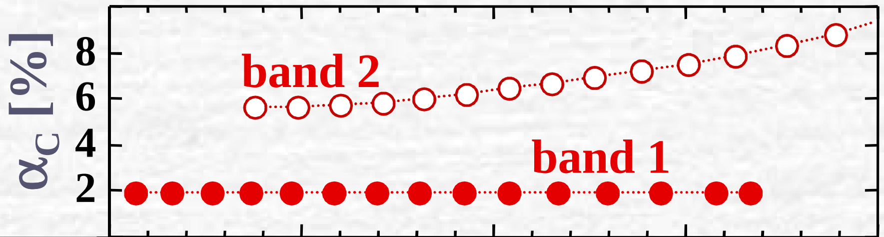
space-spin symmetric

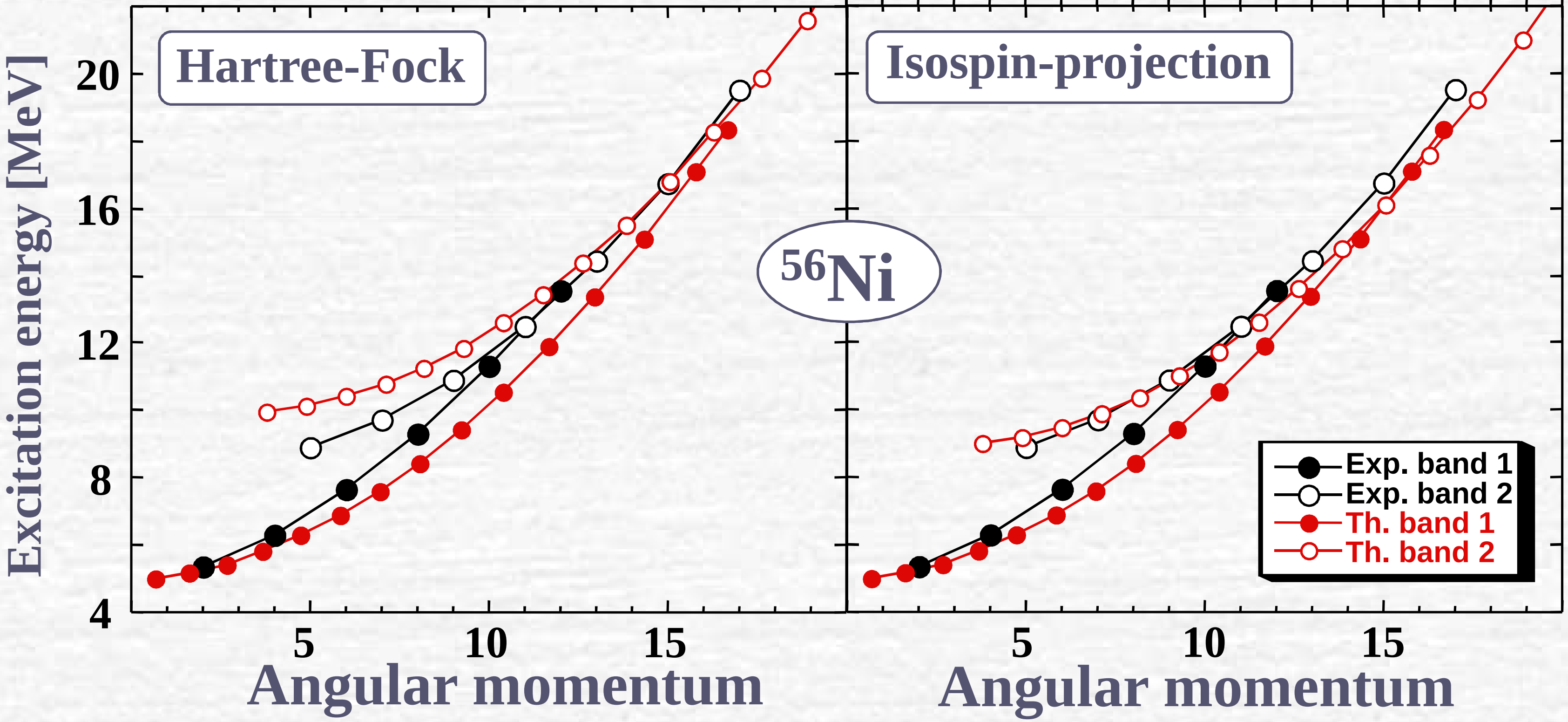
two isospin asymmetric degenerate solutions

Mean-field



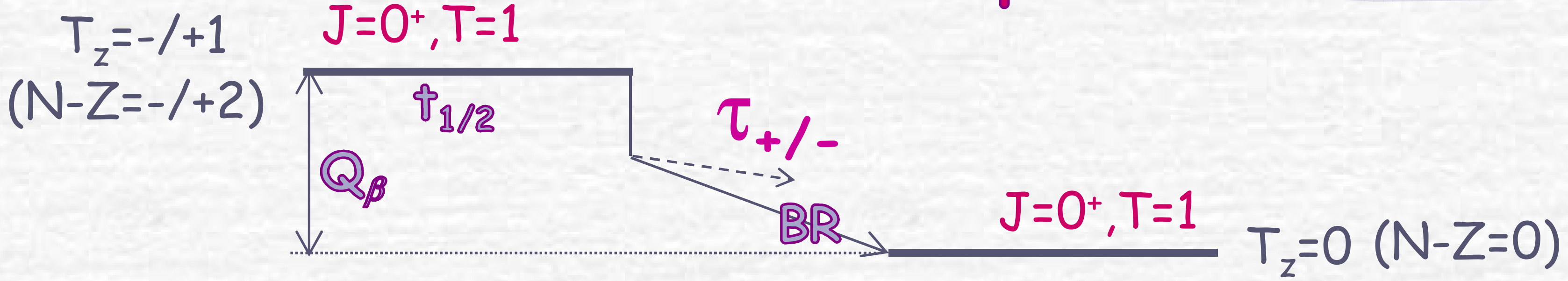
Isospin projection

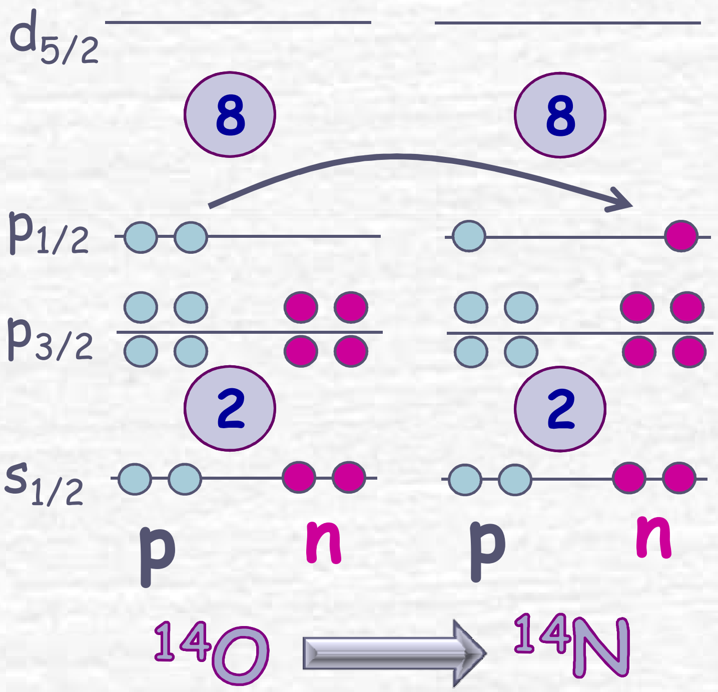




W.Satuła, J.Dobaczewski, W.Nazarewicz, M.Rafalski, PRC81 (2010) 054310

Primary motivation of the project → isospin corrections for superallowed beta decay





Hartree-Fock

Experiment:

Fermi beta decay:

$$ft = \frac{K}{G_V^2 \langle \tau \rangle^2}$$

f → statistical rate function $f(Z,Q_\beta)$

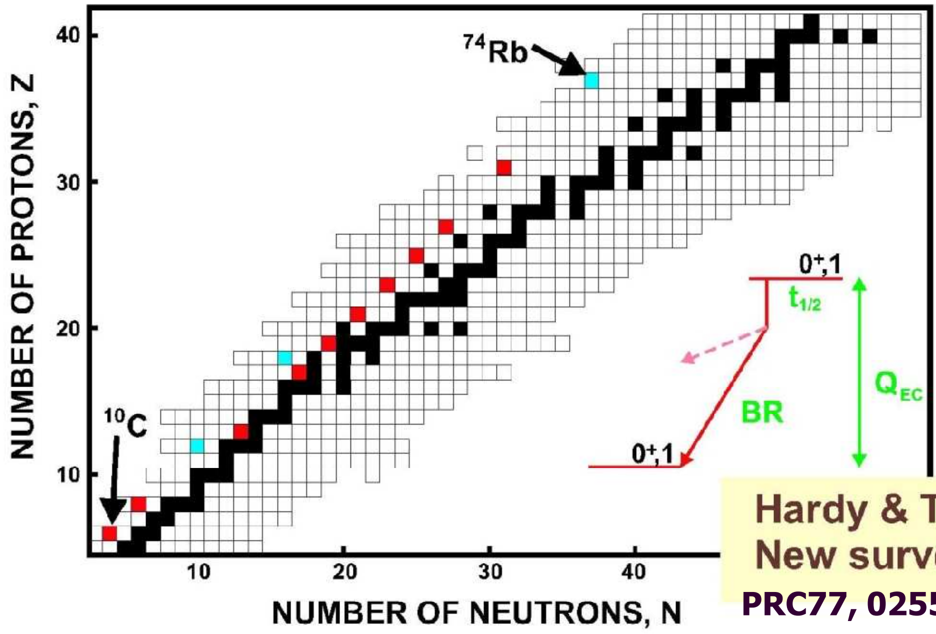
t → partial half-life $f(t_{1/2},BR)$

G_V → vector (Fermi) coupling constant

$\langle \tau_{+/-} \rangle$ → Fermi (vector) matrix element

$$|\langle \tau_{+/-} \rangle|^2 = 2(1-\delta_C)$$

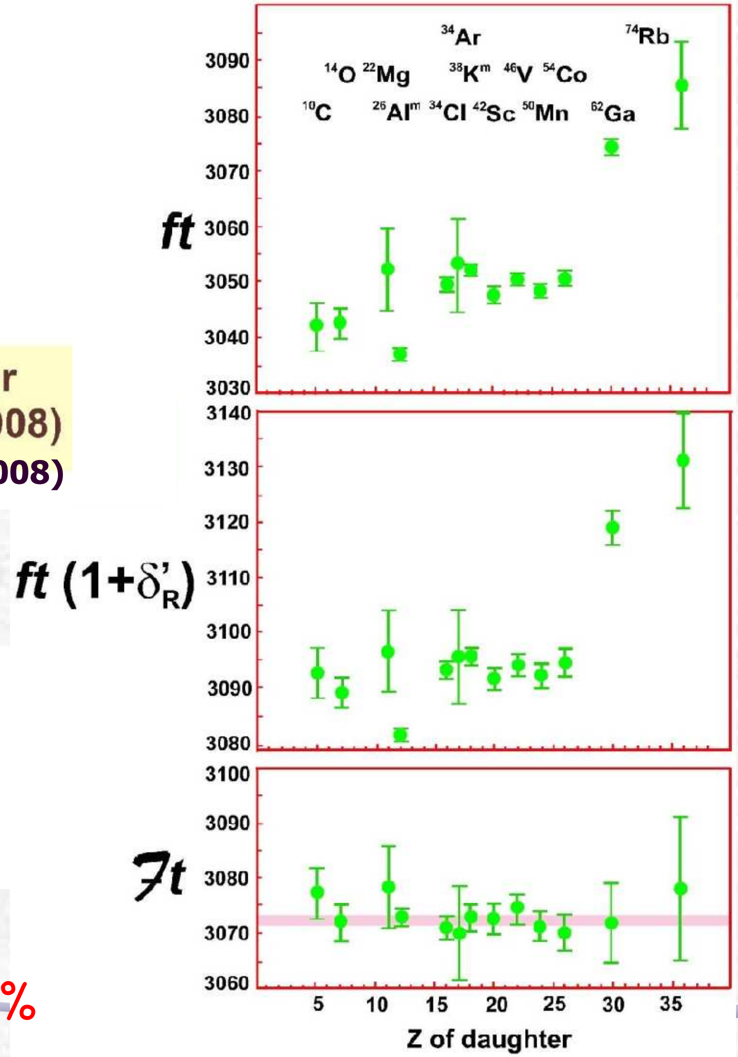
Experiment → world data survey'08



Hardy & Towner
New survey (2008)
PRC77, 025501 (2008)

10 cases measured with accuracy ***ft*** ~0.1%

3 cases measured with accuracy ***ft*** ~0.3%



INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \delta'_R)[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

nucleus-independent

~1.5%

0.3% - 1.5%

~2.4%

Marciano & Sirlin, PRL96 032002 (2006)

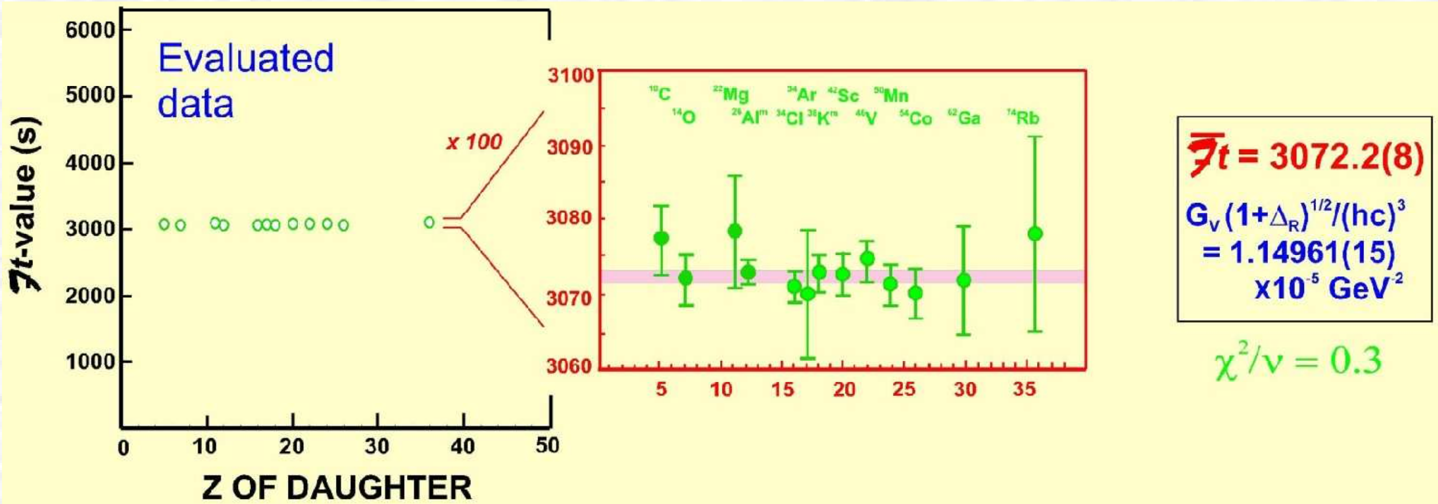
What can we learn out of it?

- From a single transiton we can determine experimentally:

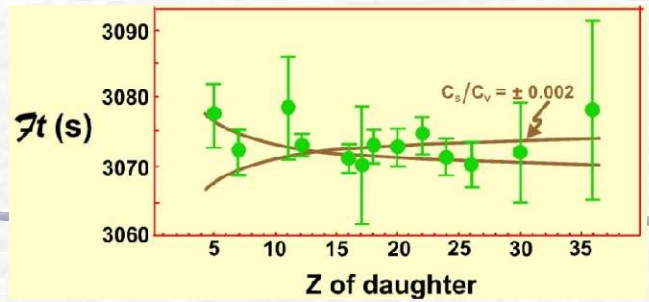
  $G_V^2(1+\Delta_R) \rightarrow G_V$=const. ✔ verified to ± 0.013%

- From many transitions we can:

  → test of the CVC hypothesis ←→ **$\mathcal{F}t$ values constant**
  (Conserved Vector Current)



  → exotic decays
  Test for presence of a Scalar Current



With the CVC being verified and knowing G_μ (muon decay) one can determine

$$V_{ud} = G_V / G_\mu$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates — CKM (Cabibbo-Kobayashi-Maskawa) — mass eigenstates

$$|V_{ud}| = 0.97425 \pm 0.00023$$

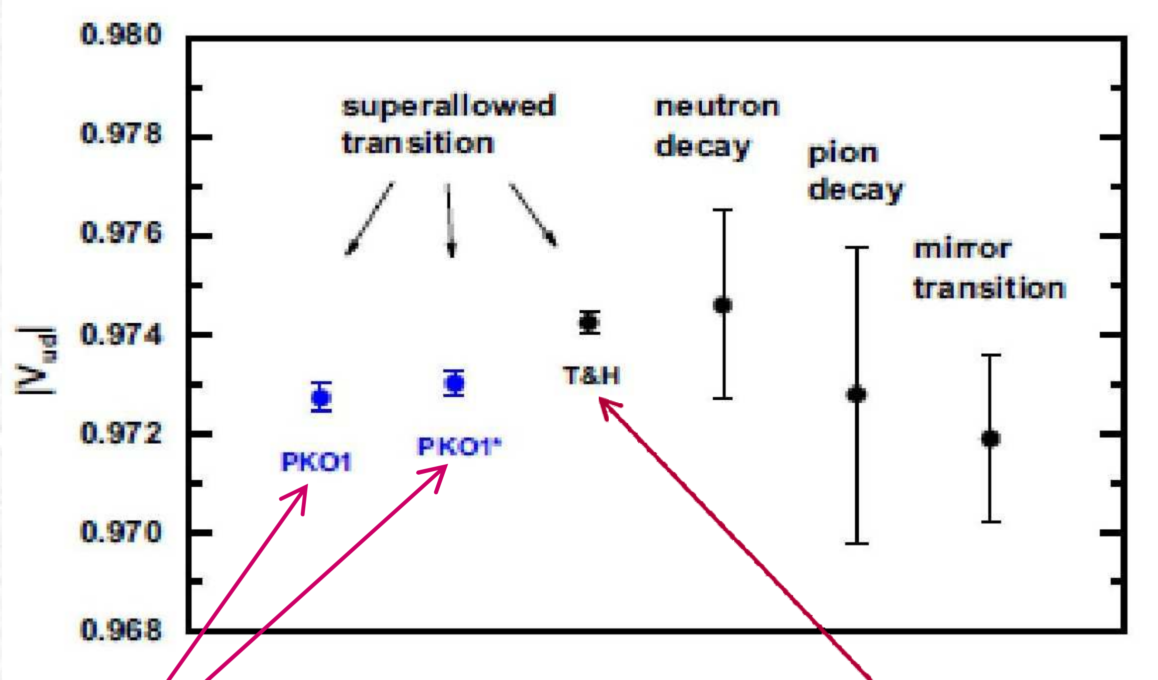
→ test unitarity of the CKM matrix

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9996(7)$$

0.9491(4) 0.0504(6) <0.0001

test of three generation quark Standard Model of electroweak interactions

Model dependence



Hardy &Towner
Phys. Rev. C77, 025501 (2008)

$\delta_C = \delta_{C1} + \delta_{C2}$

Liang & Giai & Meng
Phys. Rev. C79, 064316 (2009)

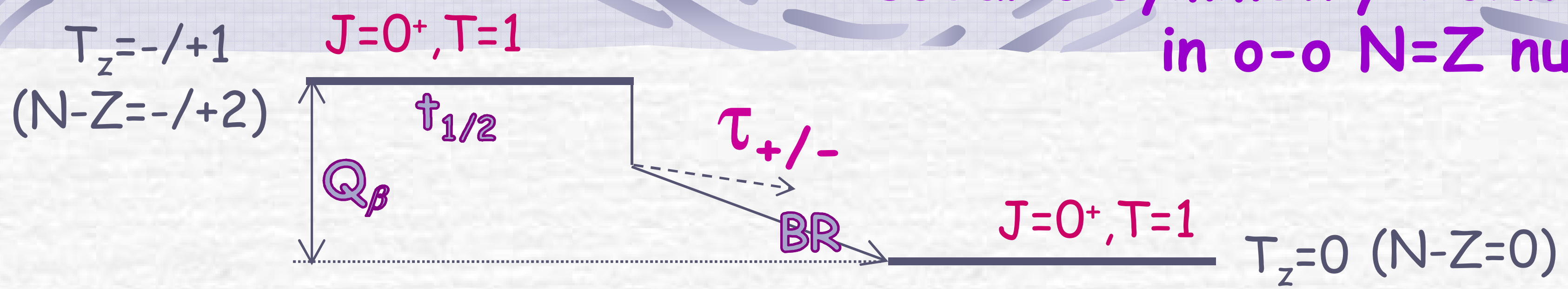
spherical RPA
Coulomb exchange treated in the Slater approxiamtion

mean field
radial mismatch of the wave functions

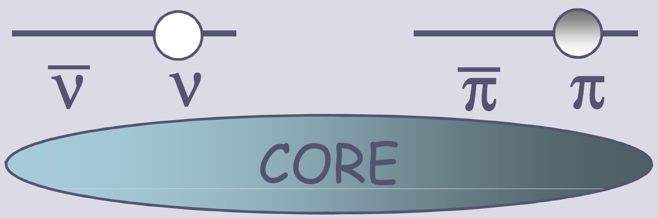
shell model
configuration mixing

Miller & Schwenk
Phys. Rev. C78 (2008) 035501;C80 (2009) 064319

Isobaric symmetry violation in o-o N=Z nuclei

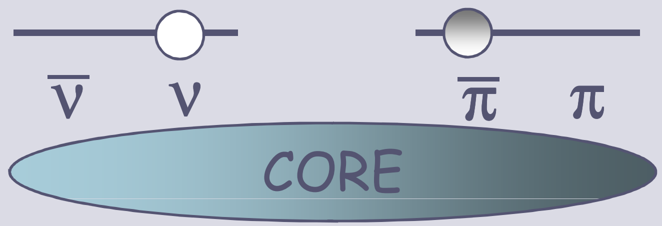


MEAN FIELD



aligned configurations

$\nu\otimes\pi$ or $\bar{\nu}\otimes\bar{\pi}$



anti-aligned configurations

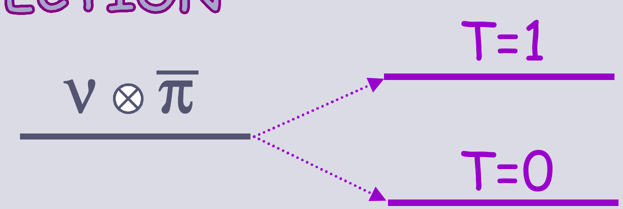
$\nu\otimes\bar{\pi}$ or $\bar{\nu}\otimes\pi$

ISOSPIN PROJECTION

Mean-field can differentiate between

$\nu\otimes\pi$ and $\nu\otimes\bar{\pi}$

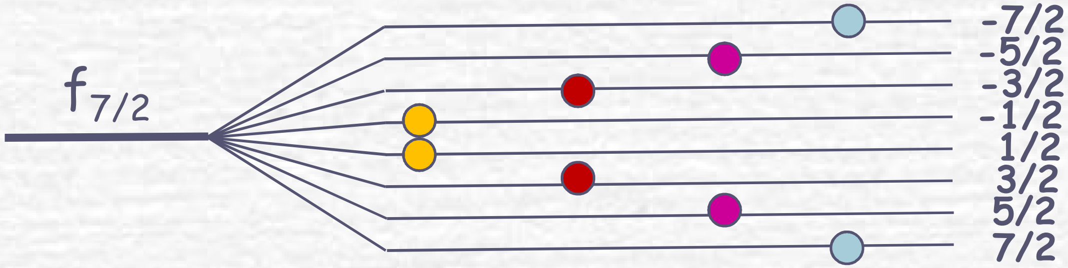
only through time-odd polarizations!

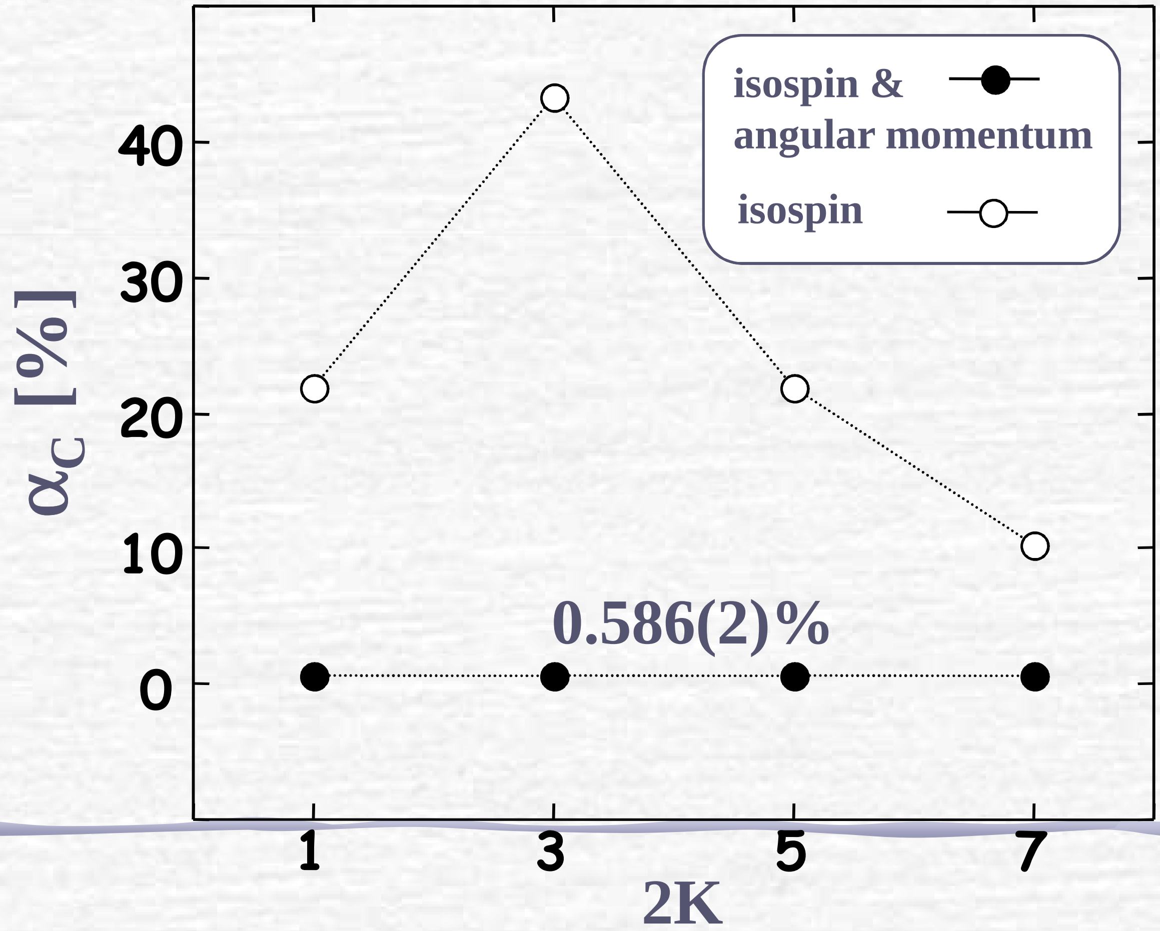


ground state

is beyond mean-field!

^{42}Sc – isospin projection from [K,-K] configurations with K=1/2,…,7/2





Hartree-Fock

ground state
in N-Z=+/-2 (e-e) nucleus

↓

Project on good isospin
(T=1) and angular
momentum (I=0)
(and perform Coulomb
rediagonalization)

CPU

~ few h

↓

~ few years

antialigned state
in N=Z (o-o) nucleus

↓

Project on good isospin
(T=1) and angular
momentum (I=0)
(and perform Coulomb
rediagonalization)

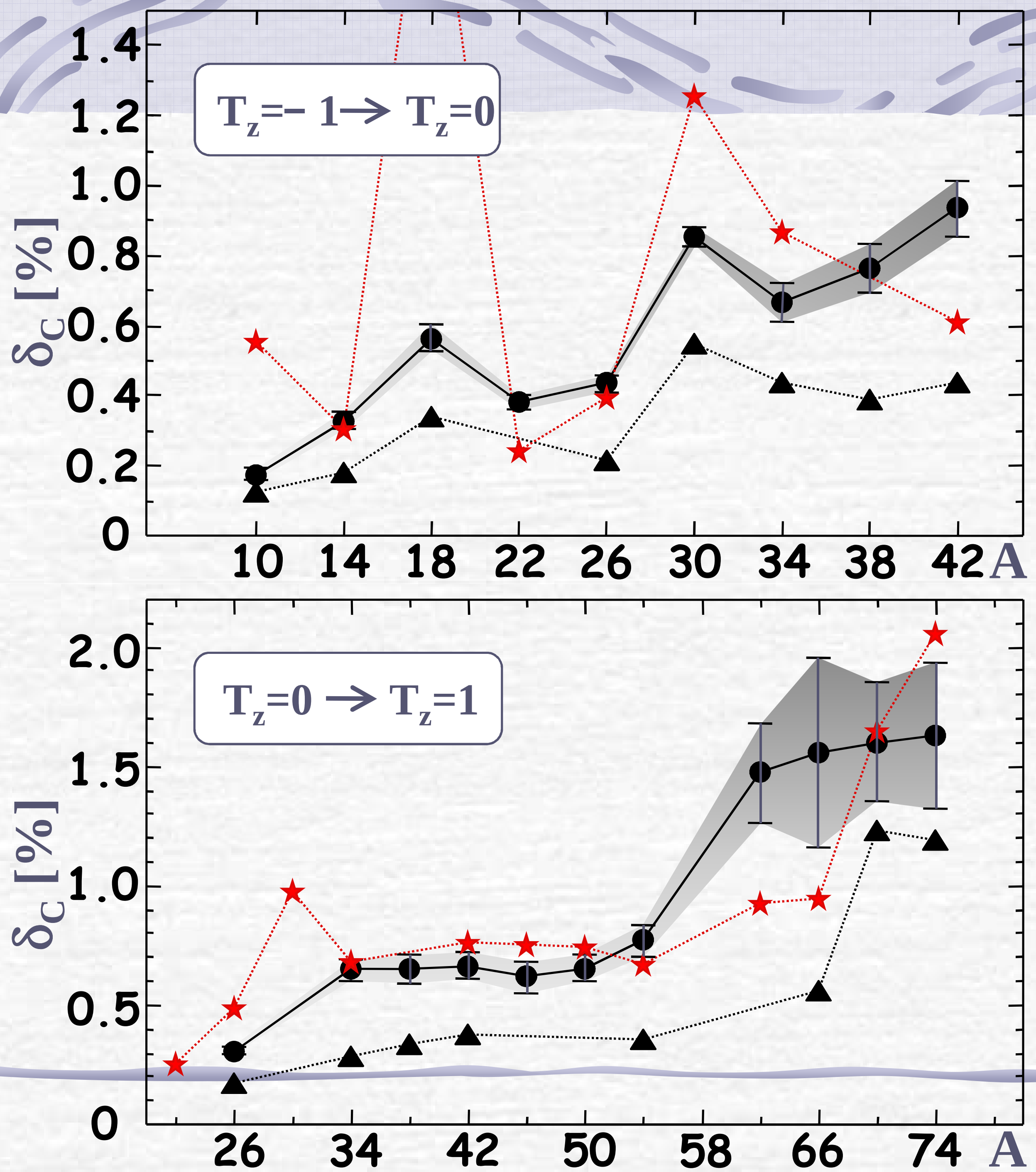
$$\langle T\approx 1, T_z=+/-1, I=0 | T_{+/-} | I=0, T\approx 1, T_z=0 \rangle$$

$^{14}O \Longrightarrow {}^{14}N$

H&T → δ_C=0.330%

L&G&M → δ_C=0.181%

our: → δ_C=0.303% (Skyrme-V; N=12)



H&T:

Ft=3071.4(8)+0.85(85)

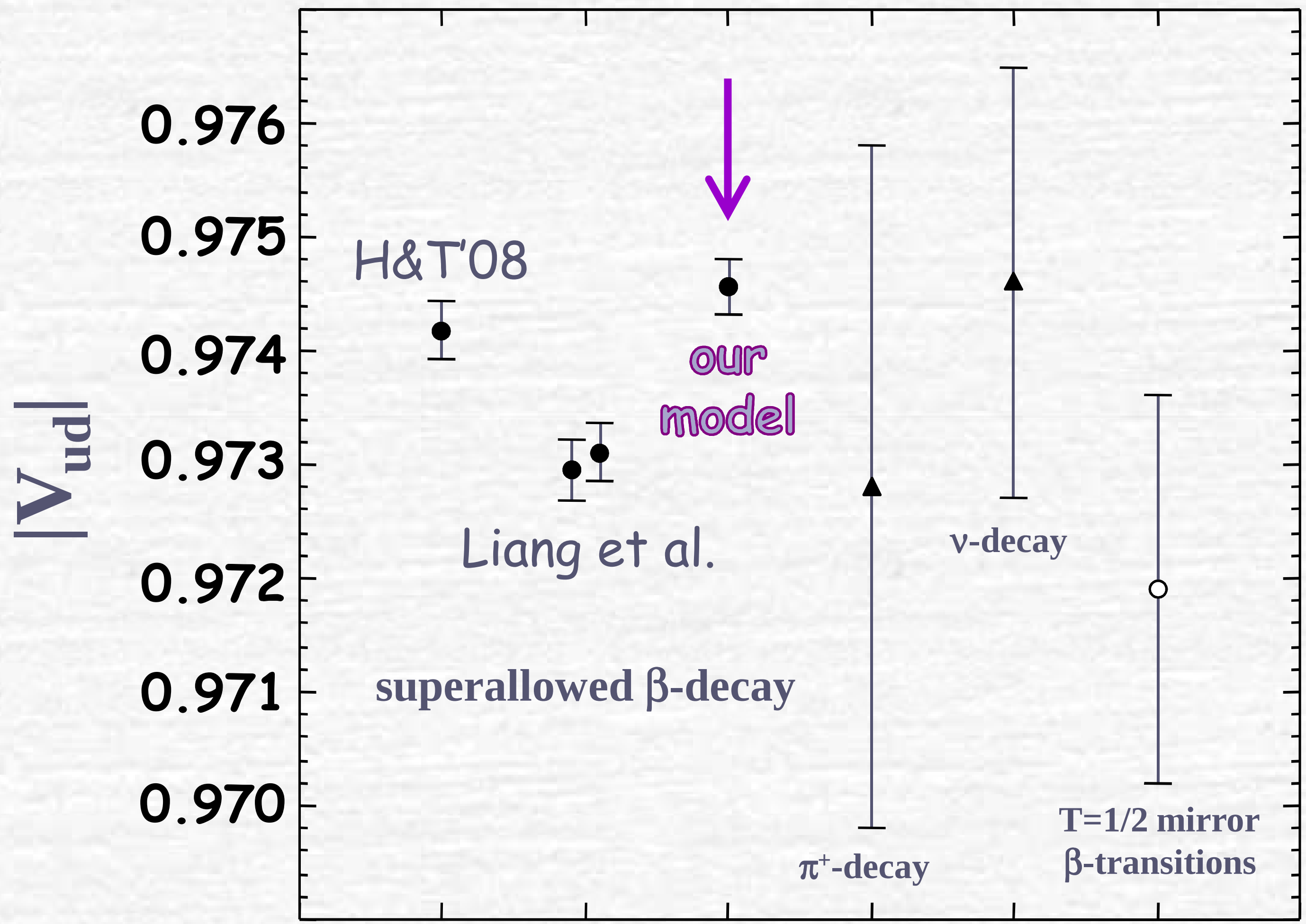
V_{ud}=0.97418(26)

our (no A=38):

Ft=3069.6(10)

V_{ud}=0.97459(24)

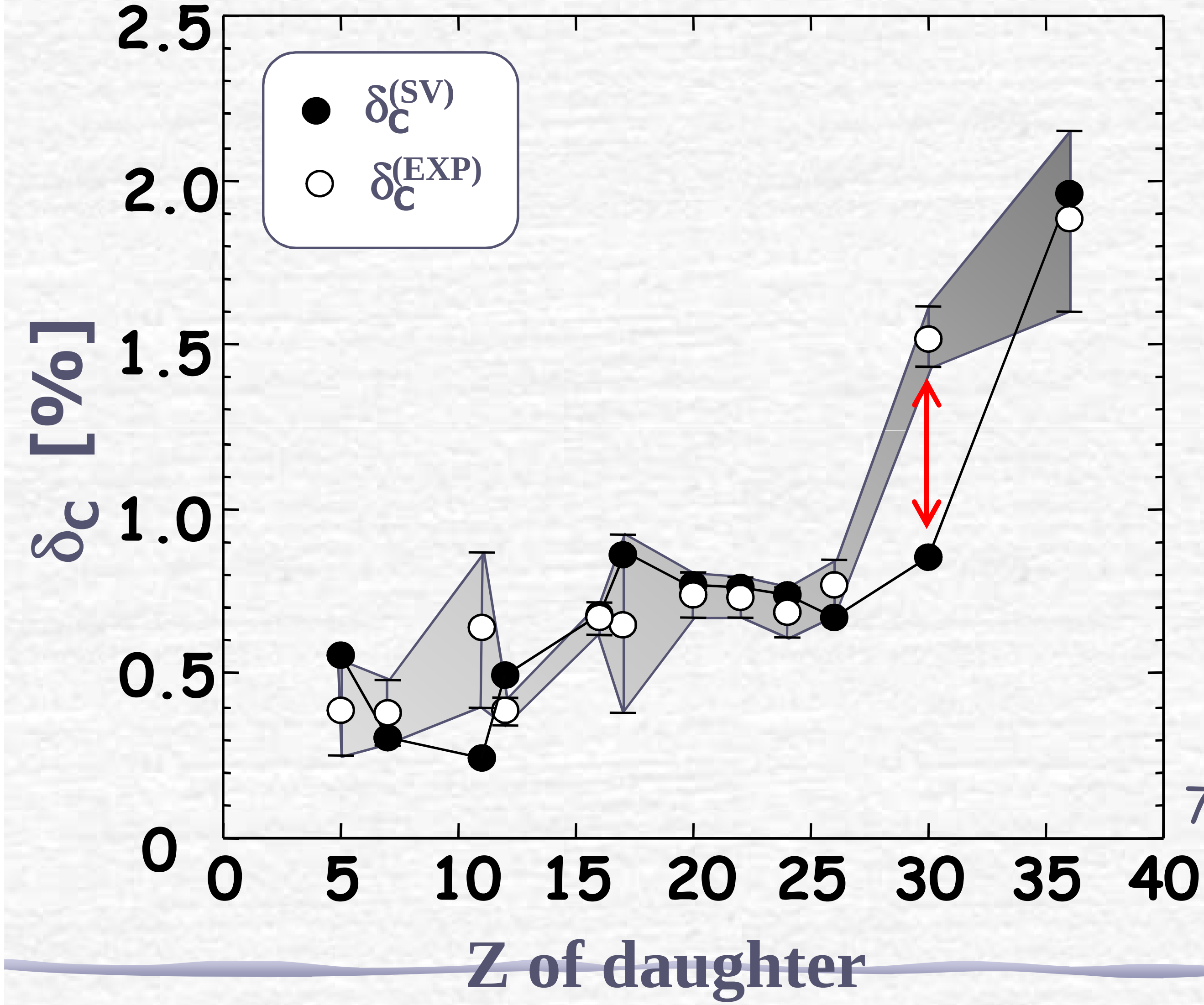
$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2$=
=1.00070(98)



0.976
0.975
0.974
0.973
0.972
0.971
0.970
|Vud|
H&T'08
our
model
Liang et al.
superallowed β-decay
π+-decay
ν-decay
T=1/2 mirror
β-transitions

Confidence level test based on the CVC hypothesis

T&H PRC82, 065501 (2010)

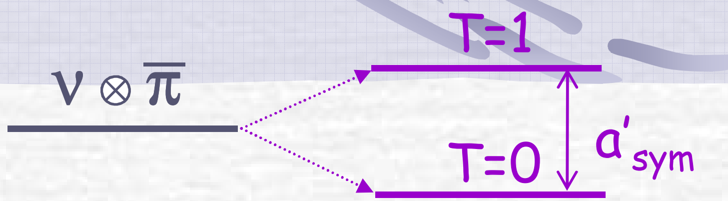


$$\delta_C^{(EXP)} = 1+\delta_{NS} - \frac{\overline{\mathcal{F}t}}{ft(1+\delta_R')}$$

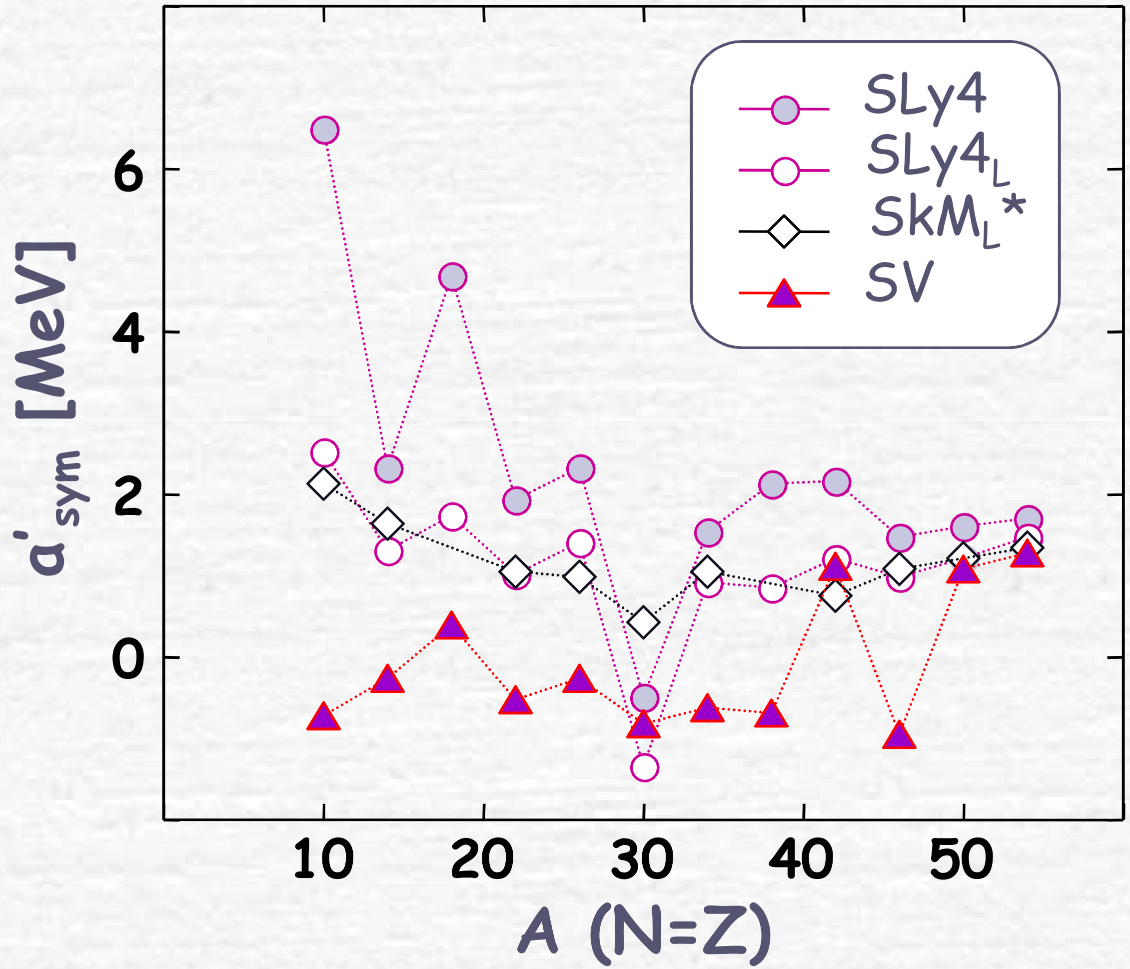
Minimize RMS deviation between the caluclated and experimental δ_C with respect to $\overline{\mathcal{F}t}$

χ^2/n_d=4.6
for Ft = 3069.9s
75% contribution to the χ^2 comes from A=62

„NEW OPPORTUNITIES" IN STUDIES OF THE SYMMETRY ENERGY:



$$E'_{sym} = \frac{1}{2} a'_{sym} T(T+1)$$



In infinite nuclear matter we have:

SLy4:
a_{sym}=32.0MeV

SV:
a_{sym}=32.8MeV

SkM*:
a_{sym}=30.0MeV

$$a_{sym} = \frac{m}{m^*} e_F + a_{int}$$

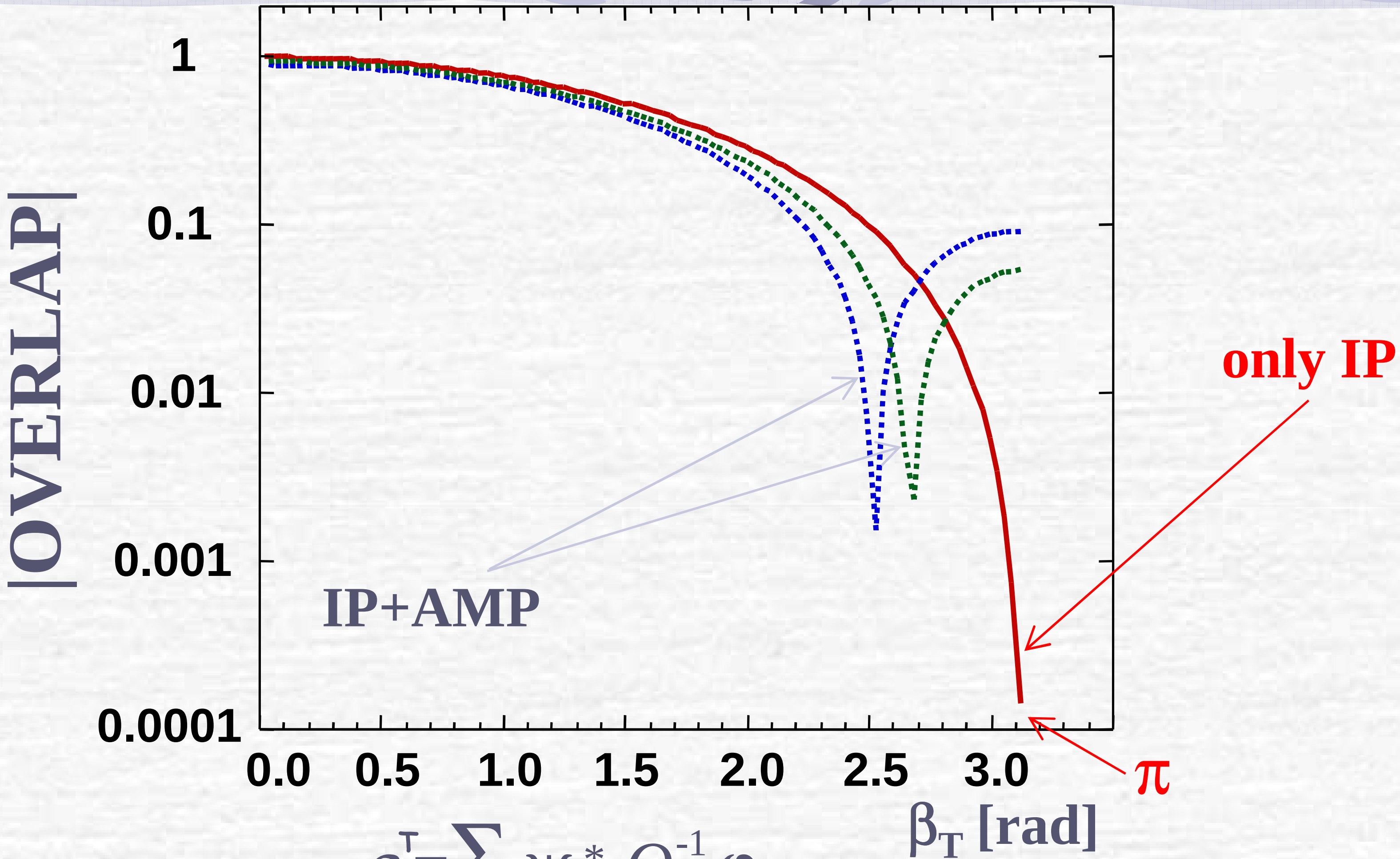
⇓

SLy4: 14.4MeV
SV: 1.4MeV
SkM*: 14.4MeV

Summary and outlook

- Elementary excitations in binary systems may differ from simple particle-hole (quasi-particle) exciatations especially when interaction among particles posseses additional symmetry (like the isospin symmetry in nuclei)
- Projection techniques seem to be necessary to account for those excitations - how to construct non-singular EDFs?
  [Isopin projection, unlike the angular-momentum and particle-number projections, is practically non-singular !!!]
- Superallowed beta decay:
  → encomapsses extremely rich physics: CVC, V_{ud}, unitarity of the CKM matrix, scalar currents… connecting nuclear and particle physics
  → … there is still something to do in δ_c business …
- How to include pairing into the scheme?

Why we have to to use Skyrme-V?



$$\rho^{\tau} = \sum_{ij} \psi_i^* \, O_{ij}^{-1} \, \varphi_j$$

HF sp state

inverse of the overlap matrix

space & isospin rotated sp state