Nuclear structure calculations in a large domain of energy excitations

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Topic I

I. Description of the used mean-field model

i. ***Skyrme-Hartree-Fock approximation***

ii. ***Some additional information for the model calculations***

iii. ***Experimental clues for evolution of the shell structure close to the drip lines***

iv. ***Applications of the model:***
   i. *For the isotopic chain Z=14*
   ii. *For the isotonic chain N=28*
   iii. *For the isotopic chain Z=50*
   iv. *For the isotonic chain N=82*

Topic II

II. Description of the excitations in tin nuclei

i. **Giant dipole resonance**

ii. **Pygmy dipole resonance**

iii. **QRPA – description of the microscopic model**

iv. **Results:**

- *i. Neutron skin*
- *ii. Giant dipole resonance*
- *iii. Pygmy dipole resonance*
- *iv. Transitional densities*
- ***v. Description of the first 2^+ states in Sn chain***

Topic III

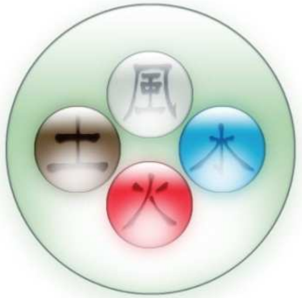
III. Mixed Symmetry States in some N=80

i. What is a mixed symmetry state

ii. Quasiparticle-phonon model

iii. What makes ^{138}Ce different from ^{136}Ba and ^{134}Xe.

IV. Outlook



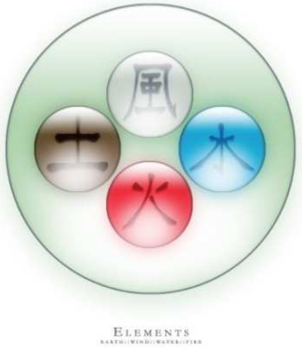
Assumptions of the SHF model

$$H = \sum_{k=1}^{A} h(k)$$

$$h(i)\varphi_k(i) = \varepsilon_k \varphi_k(i)$$

$$i = \{\vec{r}_i, \sigma_i\}, \quad k = [q, n, l, j]$$

$$\varphi_k(i) = \frac{u_k(r)}{r} \mathcal{Y}_{lj}^{m}(\vec{r}_i, \sigma)\chi_q(\tau)$$



Skyrme force potential

$$
\begin{aligned}
V(r_1, r_2) = \;& t_0\,(1 + x_0, \mathbf{P}_\sigma)\,\delta(r) \\
& + \tfrac{1}{2} t_1\,(1 + x_1 \mathbf{P}_\sigma)\,[\mathbf{P}'^2 \delta(r) + \delta(r)\mathbf{P}^2] \\
& + t_2\,(1 + x_2 \mathbf{P}_\sigma)\,\mathbf{P}' \cdot \delta(r)\mathbf{P} \\
& + \tfrac{1}{6} t_3\,(1 + x_3 \mathbf{P}_\sigma)\left[\frac{\rho(r)}{\rho_0}\right]^\sigma \\
& + i W_0 \sigma \cdot [\mathbf{P}' \times \delta(r)\mathbf{P}]
\end{aligned}
$$

Central term | Non-local term | Density dependent term | Spin-orbit term

Skyrme-Hartree-Fock model

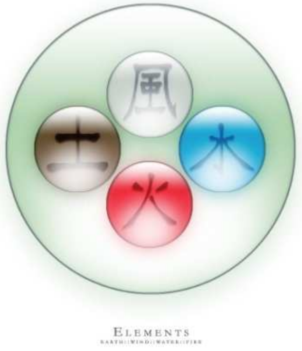
Nucleon density
Kinetic density
Spin-orbit density

$$\langle \varphi | H | \varphi \rangle \equiv \int \mathcal{H}\left(\vec{r}\right) d^3 r$$

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{eff} + \mathcal{H}_{fin} + \mathcal{H}_{so} + \mathcal{H}_{sg} + \mathcal{H}_{Coul}$$

$$\rho_q(r) = \sum_{l,s} |\varphi_i(r)|^2 n_i^q \qquad \tau_q(r) = \sum_{l,s} |\varphi_i(r)|^2 n_i^q$$

$$J_q(r) = \frac{1}{4\pi r^3} \sum_{nlj} \left[j_i(j_i + 1) - l_i(l_i + 1) - \frac{3}{4} \right] n_i^q u_i^2(r)$$



Pairing correlations (BCS)

Two nucleons coupled to a total angular momentum of 0

$$V^{(\mathrm{n\ or\ p})} = V_0^{(\mathrm{n\ or\ p})}\left(1-\frac{\rho(\boldsymbol{R})}{\rho_0}\right)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)$$

$$\langle aa|V|cc\rangle_{00} = (-1)^{l_a+l_c}\frac{1}{2}\frac{\hat{j}_a\hat{j}_c}{4\pi}I_{aacc}$$

$$I_{aacc} = V_0\int_0^\infty \frac{dr}{r^2}\left(1-\frac{\rho}{\rho_0}\right)u_a(r)u_a(r)u_c(r)u_c(r)$$

Pairing correlations II (BCS)

The Gap equation

$$\Delta_a = -\frac{1}{2}\sum_c (-1)^{l_a+l_b}\, \hat{j}_a^{-1}\, \hat{j}_b \langle aa | V | cc \rangle \frac{\Delta_c}{\sqrt{(\varepsilon_c - \lambda)^2 + \Delta_c^2}}$$

$$v_k^2 = \frac{1}{2}\left(1 - \frac{\tilde{\epsilon}_k}{\sqrt{\tilde{\epsilon}_k^2 + \Delta_k^2}}\right)$$

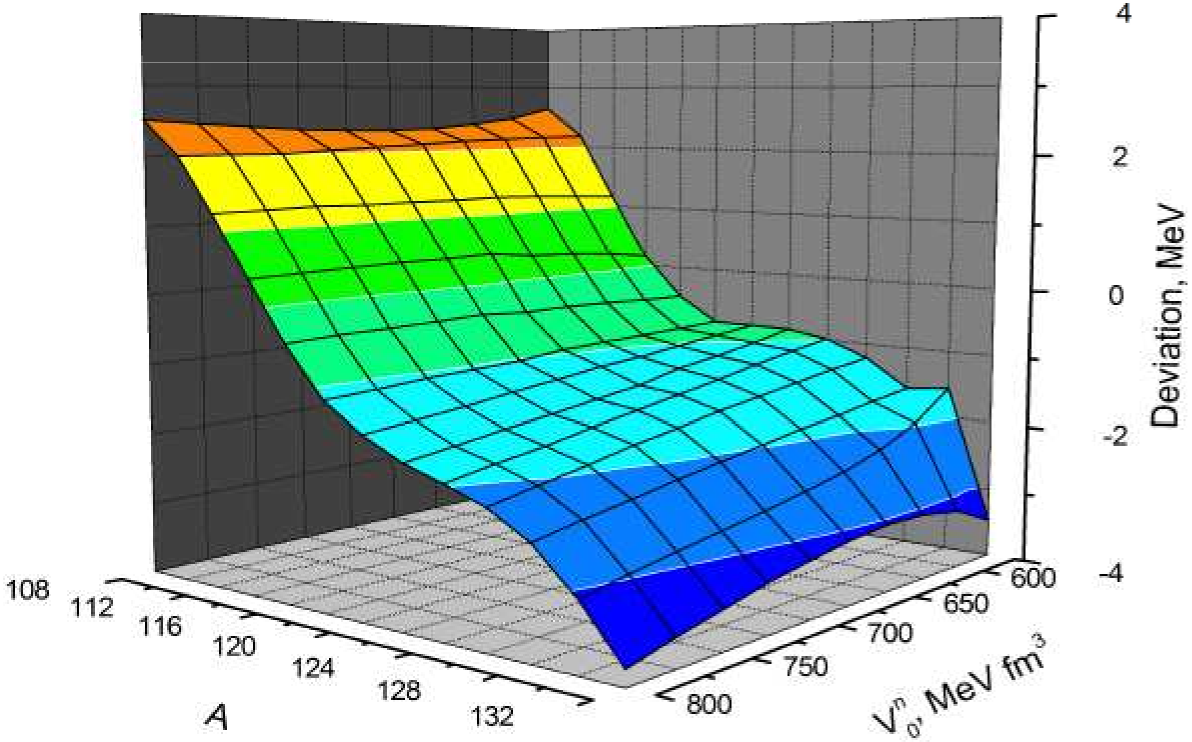
$$u_k^2 = \frac{1}{2}\left(1 + \frac{\tilde{\epsilon}_k}{\sqrt{\tilde{\epsilon}_k^2 + \Delta_k^2}}\right)$$

Pairing correlations II (BCS)

$$S_{2n}^{A,Z} = B(A,Z) - B(A-2,Z)$$

$$S_{2p}^{A,Z} = B(A,Z) - B(A-2,Z-2)$$

$$S_{2n}^{exp} - S_{2n}^{BCS}$$



Taking into account the two-body tensor interaction

$$V_T(r) = v_T(r)\,\tau\cdot\tau'\left[\frac{1}{r^2}(r\cdot\sigma)(r\cdot\sigma') - \frac{1}{3}\sigma\cdot\sigma'\right]$$

$$\begin{aligned} v_T = \tfrac{1}{2} T \{ & [(\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}')(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}') - \tfrac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) k'^2] \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \\ & + \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) [(\boldsymbol{\sigma}_1 \cdot \boldsymbol{k})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{k}) - \tfrac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) k^2] \} \\ & + U \{ (\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}') \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{k}) \\ & - \tfrac{1}{3}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)[\boldsymbol{k}' \cdot \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \boldsymbol{k}] \} , \qquad (1) \end{aligned}$$

Stancu, Brink and Flocard, Phys. Lett. 68B, 108 (1977)

For S waves the contribution of the tensor term is 0

Taking into account the tensor term into the Skrme-Hartree-Fock model

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{eff} + \mathcal{H}_{fin} + \mathcal{H}_{so} + \mathcal{H}_{sg} + \mathcal{H}_{Coul}$$

$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{eff} + \mathcal{H}_{fin} + \mathcal{H}_{so} + \mathcal{H}_{sg} + \mathcal{H}_T + \mathcal{H}_{Coul}$$

$$\mathrm{H}_{sg} = -\frac{1}{16}\left(t_1\left(x_1-1\right)+t_2\left(x_2+1\right)\right)\left[J_p^2+J_n^2\right]-\frac{1}{8}\left(t_1x_1+t_2x_2\right)J_nJ_p$$

$$= \frac{1}{2}\alpha_c\left[J_p^2+J_n^2\right]+\beta_cJ_nJ_p$$

$$\mathcal{H}_T = \frac{1}{2}\alpha_T\left(J_n^2+J_p^2\right)+\beta_TJ_nJ_p$$

Hartree-Fock equations

$$\langle \varphi | H | \varphi \rangle \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{\hbar^2}{2m}\left[-\frac{d^2}{dr^2}\psi(r) + \frac{l(l+1)}{r^2}\psi(r)\right] + V_{eq}^{lj}(r,\epsilon)\psi(r) = \epsilon\psi(r)$$

$$\textbf{where, } \psi(\mathbf{r}) = \sqrt{\frac{m^*(r)}{m}}\frac{\mathbf{u}(\mathbf{r})}{\mathbf{r}}$$

$$V_{\rm eq}^{lj}(r,\epsilon) = \frac{m^*(r)}{m}U_0(r) + \frac{m^*(r)}{m}U_{\rm so}^{lj}(r) + V_{\rm eq}^{\rm m*}\,,$$

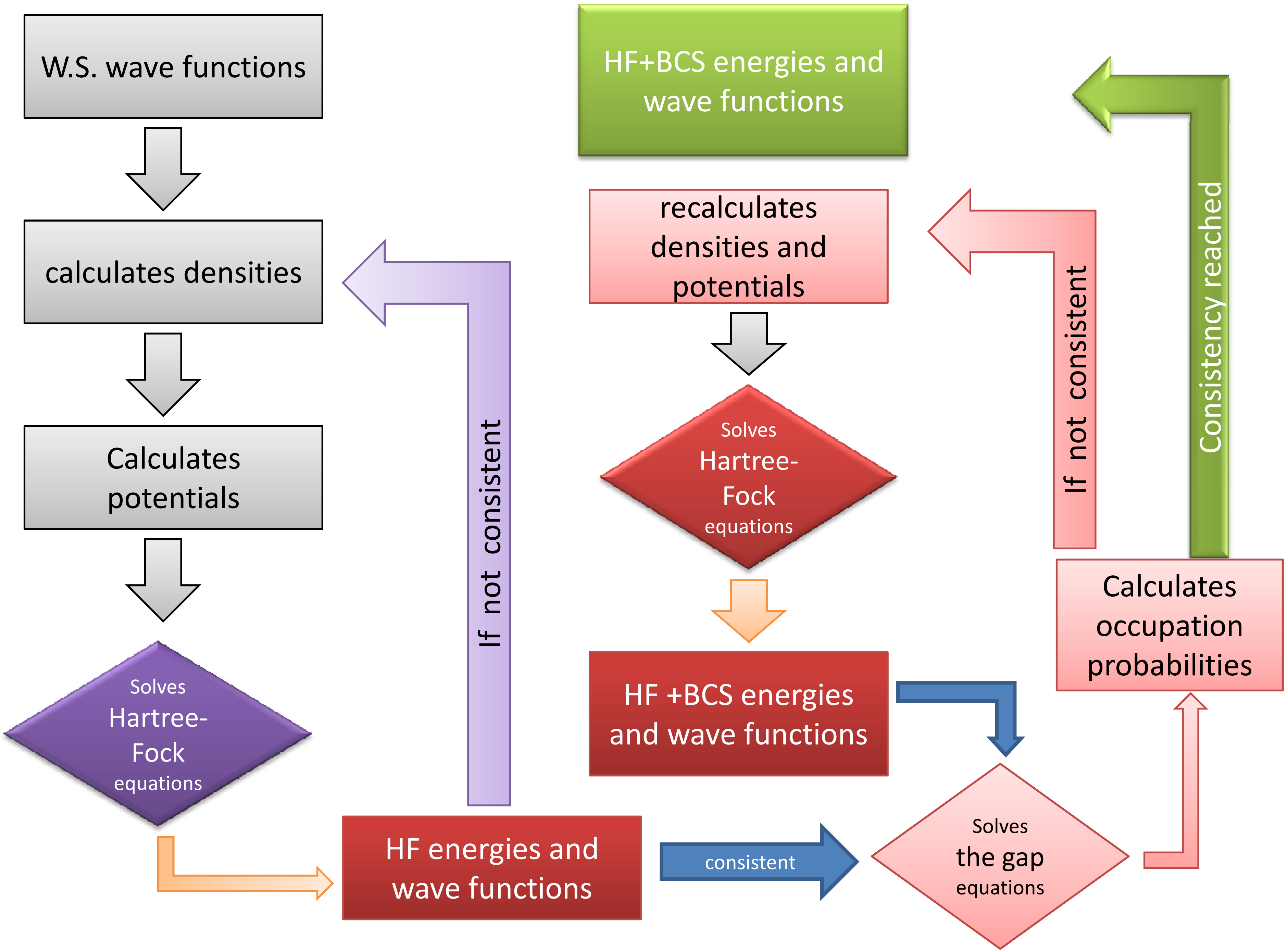
Taking into account the tensor term into the Hartree-Fock model II

$$U^{q}_{s.o.}(r) = \frac{W_0}{2r}\left(2\frac{d\rho_q}{dr} + \frac{d\rho_{q'}}{dr}\right) + \left(\alpha\frac{J_q}{r} + \beta\frac{J_{q'}}{r}\right)$$

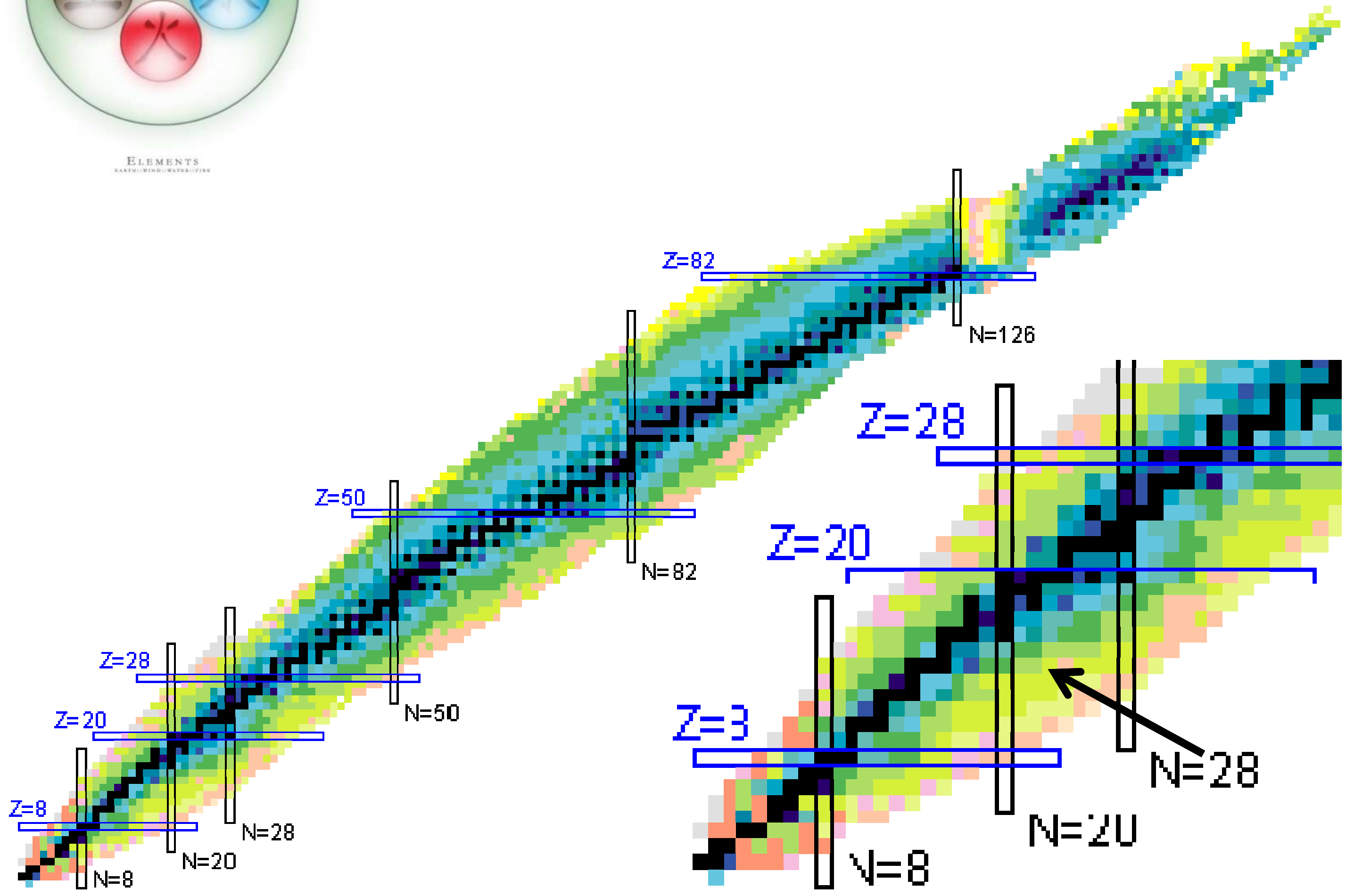
$$\alpha = \alpha_T + \alpha_C \qquad \beta = \beta_T + \beta_C$$

$$\epsilon = \epsilon_{\mathrm{kin}} + \epsilon_{\mathrm{cen.}} + \epsilon_{\mathrm{s.o.}} + \epsilon_{m*}$$

$$\epsilon_{\mathrm{s.o.}} = \epsilon_{s.o.} + \epsilon_C + \epsilon_T$$


W.S. wave functions
calculates densities
Calculates potentials
Solves Hartree-Fock equations
HF energies and wave functions
If not consistent
consistent
HF+BCS energies and wave functions
recalculates densities and potentials
Solves Hartree-Fock equations
HF +BCS energies and wave functions
Solves the gap equations
Calculates occupation probabilities
If not consistent
Consistency reached

Experimental clues for the shell structure evolution close to the drip lines



T. Otsuka, R. Fujimoto, Y. Utsuno, B. A. Brown, M. Honma, and T. Mizusaki, Physical Review Letters **87 (2001), 082502.**

T. Otsuka, T. Suzuki, R. Fujimoto, H. Grawe, and Y. Akaishi, Physical Review Letters **95 (2005), 232502.**

Experimental clues for the shell structure evolution close to the drip lines

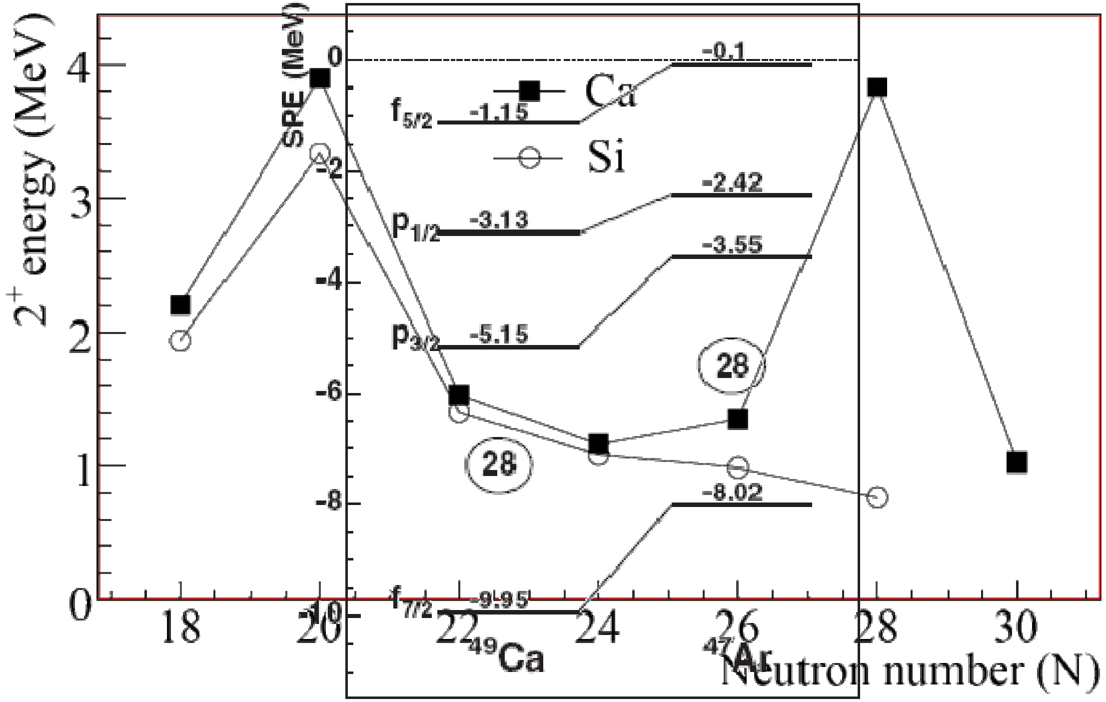


FIG. 3: Energies of the 2^+ states measured in the Ca and Si isotopes. Present result for ^{42}Si –770(19) keV– brings evidence for the the collapse of the N=28 shell closure at Z=14.

- Shrinking of 1 MeV of the N=28 gap for ^{42}Si (starting value of about 4.8 MeV for ^{48}Ca).
- Shrinking of the proton *sd* orbital of 1.94 MeV for ^{42}Si in comparison to ^{34}Si

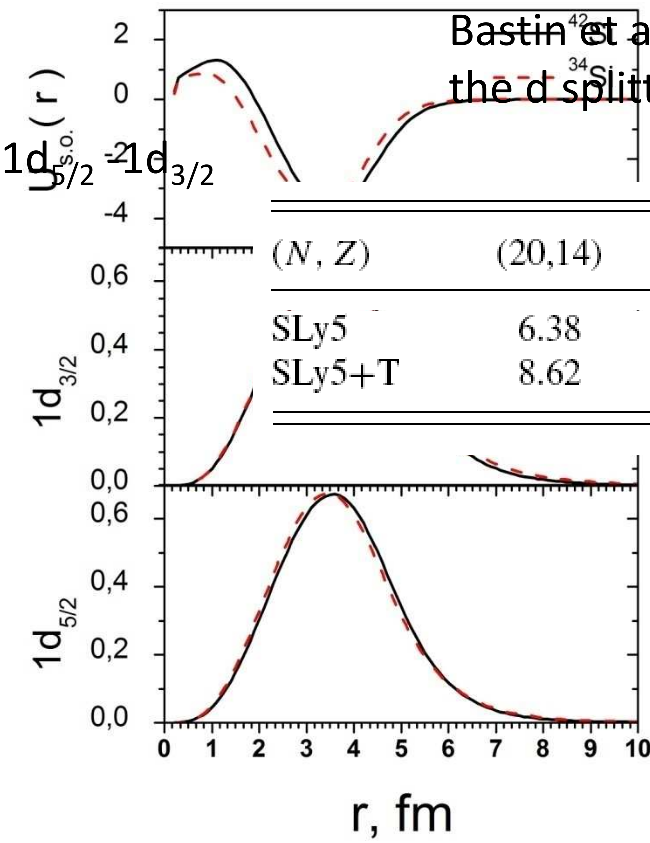
L. Gaudefroy et al., Physical Review Letters **97 (2006) 092501**
B. Bastin et al., Physical Review Letters **99 (2007) 022503**

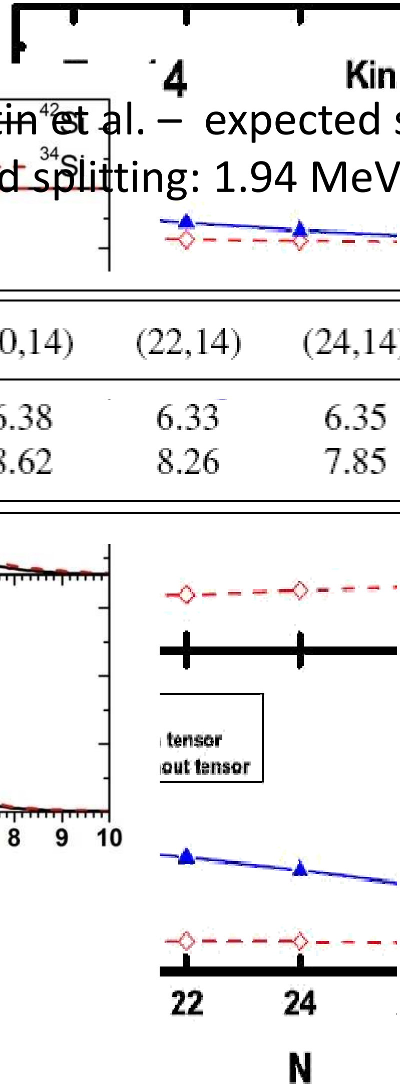
Results:

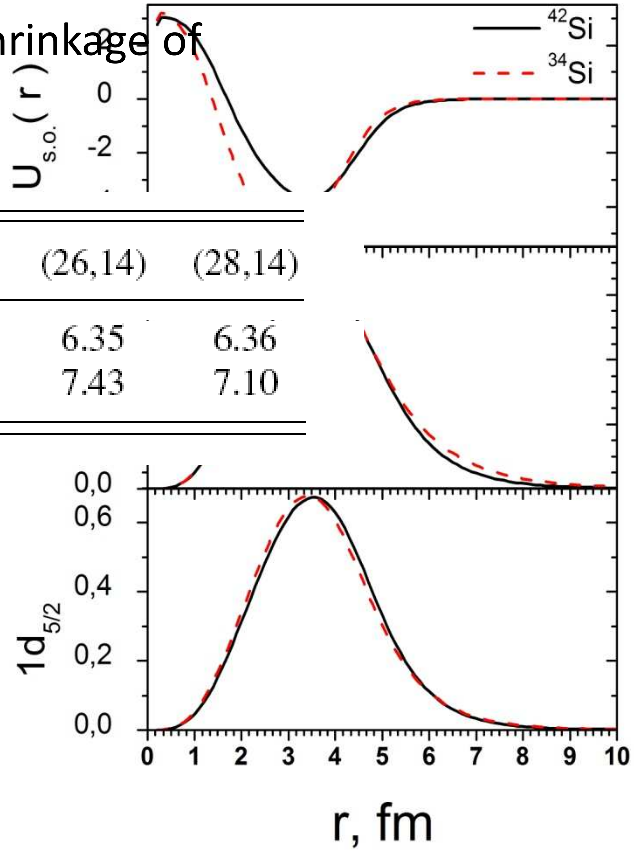
Z=14

Bastin et al. – expected shrinkage of the d splitting: 1.94 MeV

$1d_{5/2} - 1d_{3/2}$







(N, Z)	(20,14)	(22,14)	(24,14)	(26,14)	(28,14)
SLy5	6.38	6.33	6.35	6.35	6.36
SLy5+T	8.62	8.26	7.85	7.43	7.10

Without Tensor term

With tensor term

DT, H. Liang, N. V. Giai, and C. Stoyanov
Phys. Rev. C 77, 054316 (2008)

Results:

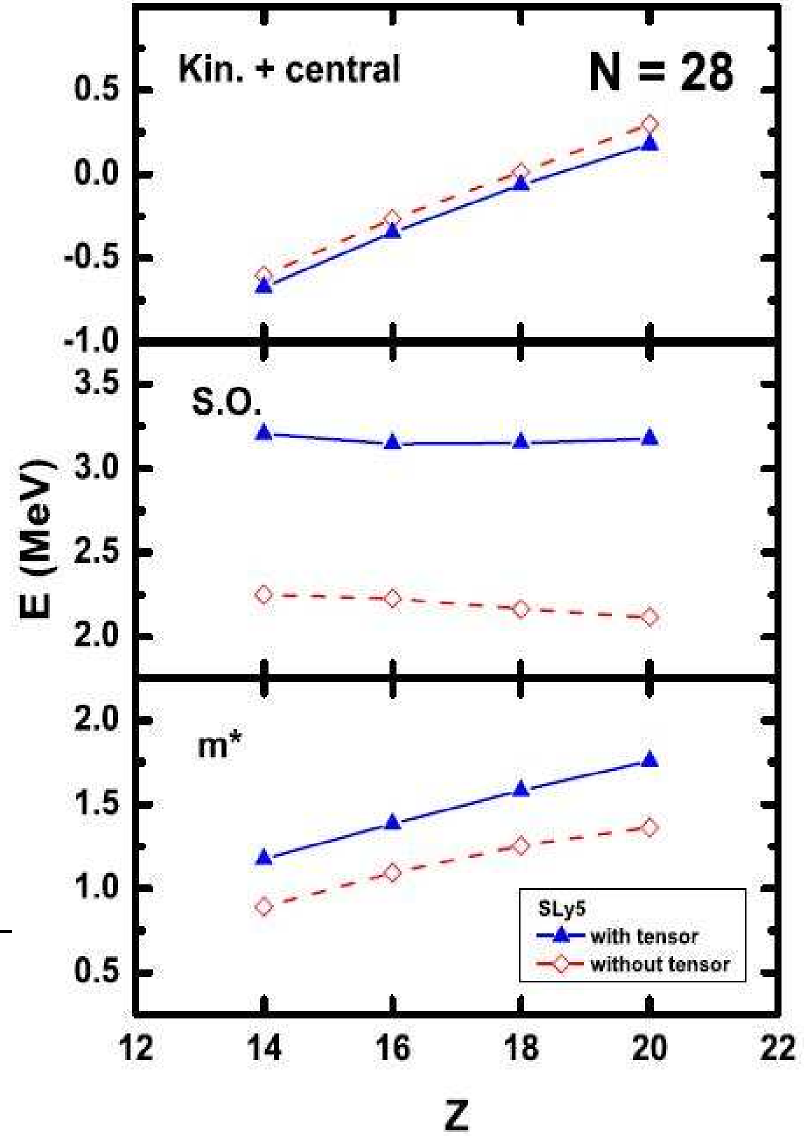
N=28

$1f_{7/2} – 2p_{3/2}$

Bastin et al. expected reduction of the gap in Si: 1.2MeV

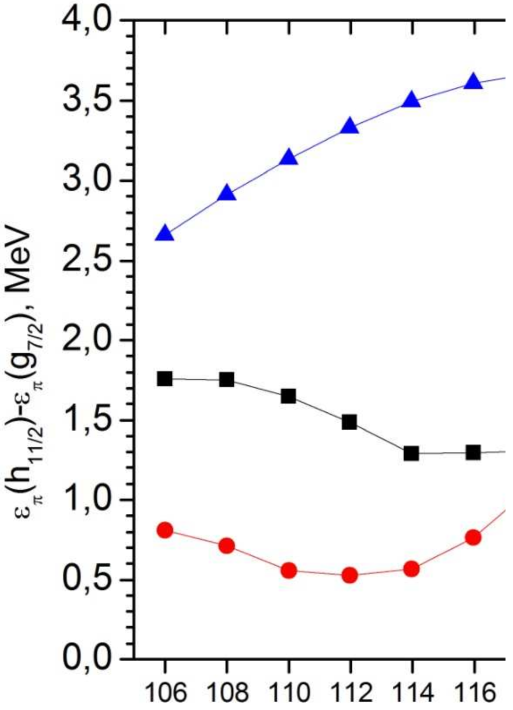
Gaudefroy et al. – observed reduction of the gap in Ar 330keV

(N,Z)	(28,14)	(28,16)	(28,18)	(28,20)
SLy5	2.55	3.09	3.49	3.75
SLy5+T	3.79	4.08	4.66	5.15

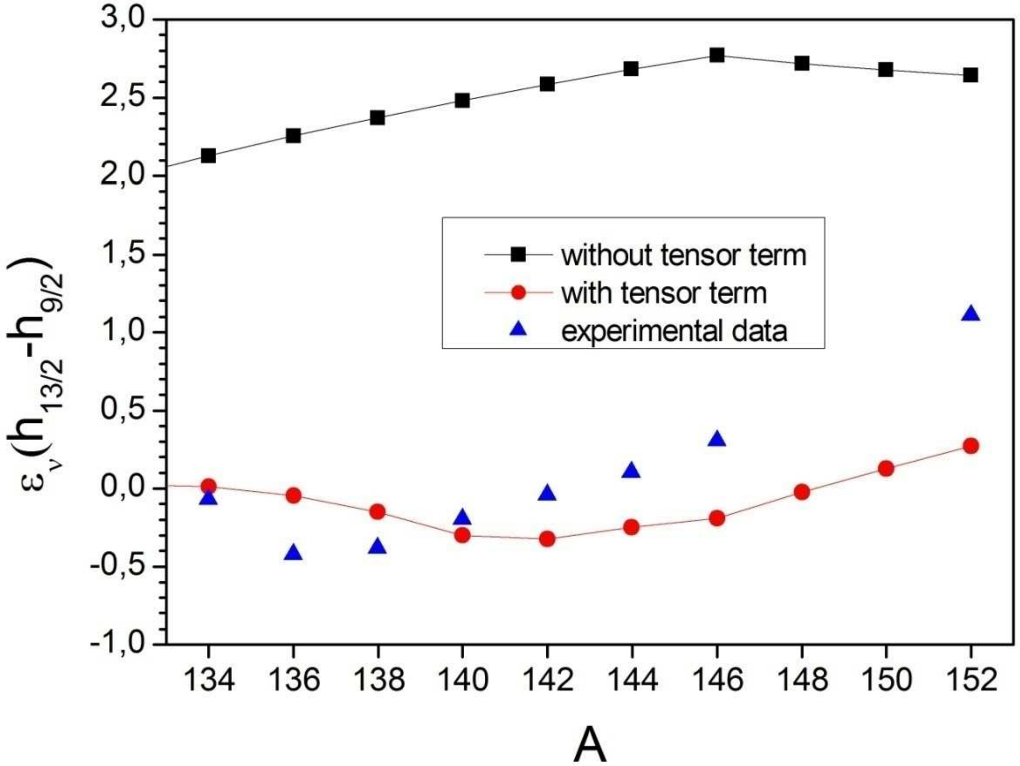


Results:
Z=50, N=82

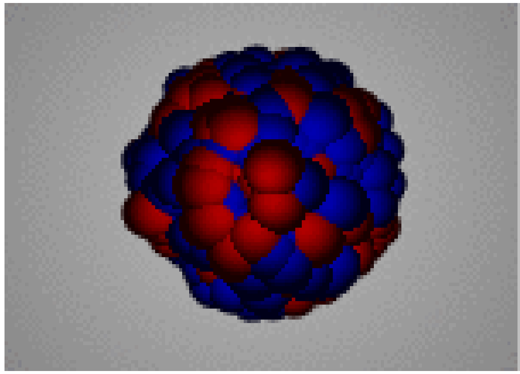
Z=50



N=82



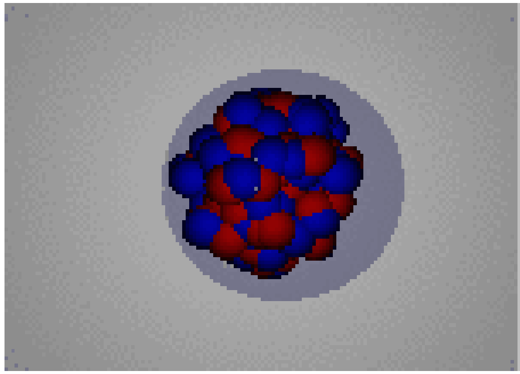
J.P. Schiffer, et al., Phys. Rev. Lett. 92 (2004) 162501



Giant Dipole Resonance

- ✓ High frequency, collective excitation of the nuclei
- ✓ Their basic features depend on the bulk structure
- ✓ Their characteristics change slowly with the mass number
- ✓ Their energy is situated well above the one particle separation energy (10-20 MeV)
- ✓ Their width is about 2.5 – 6 MeV
- ✓ The effective cross section may be described well by the following Lorentz type distribution formula.

$$E \quad \sigma_\gamma(E) = \frac{\sigma_{Max}}{1+\left[\frac{(E^2-E_{GDR}^2)^2}{E^2\Gamma^2}\right]} \quad A^{-1/6}$$



Pygmy Dipole Resonance

- ❑ It is an effect from structural changes when one goes further towards the drip lines
- ❑ Appearance of a diffused surface – the skin
- ❑The energy of the Pygmy Dipole Resonance is a function of the collectivity and the energy of the Giant Dipole Resonance

$$E_{PDR} = \sqrt{\frac{Z}{A-Z}\,\frac{N_s}{A-N_s}}\,E_{GDR}$$

Quasiparticle Random Phase Approximation (QRPA)

Landau-Migdal representation of the residual force

$$V_{ph} = N_0^{-1} \sum_l [F_l + G_l \sigma_1 \cdot \sigma_2 + (F_l + G_l \sigma_1 \cdot \sigma_2)\tau_1 \cdot \tau_2]\, \delta(\mathbf{r_1} - \mathbf{r_2}),$$

Linking to the Skyrme parameters

$$\begin{aligned}
F_0 &= N_0 \left[\frac{3}{4} t_0 + \frac{1}{16} t_3 \rho^\alpha (\alpha + 1)(\alpha + 2) + \frac{1}{8} k_F^2 \left[3t_1 + (5 + 4x_2) t_2 \right] \right] \\
F_0' &= N_0 \left[\frac{1}{4} t_0 (1 + 2x_0) + \frac{1}{24} t_3 \rho^\alpha (1 + 2x_3) + \frac{1}{8} k_F^2 \left[t_1 (1 + 2x_1) - t_2 (1 + 2x_2) \right] \right] \\
G_0 &= -N_0 \left[\frac{1}{4} t_0 (1 - 2x_0) + \frac{1}{24} t_3 \rho^\alpha (1 - 2x_3) + \frac{1}{8} k_F^2 \left[t_1 (1 - 2x_1) - t_2 (1 + 2x_2) \right] \right] \\
G_0' &= -N_0 \left[\frac{1}{4} t_0 + \frac{1}{24} t_3 \rho^\alpha + \frac{1}{8} k_F^2 (t_1 - t_2) \right]
\end{aligned}$$

QRPA Hamiltonian

$$H = \sum_\tau \sum_{jm}^{\tau} (E_f - \lambda_\tau) a^+_{jm} a_{jm}$$

$$- \frac{1}{4} \sum_\tau V_\tau^{(0)} P_0^+(\tau) P_0(\tau)$$

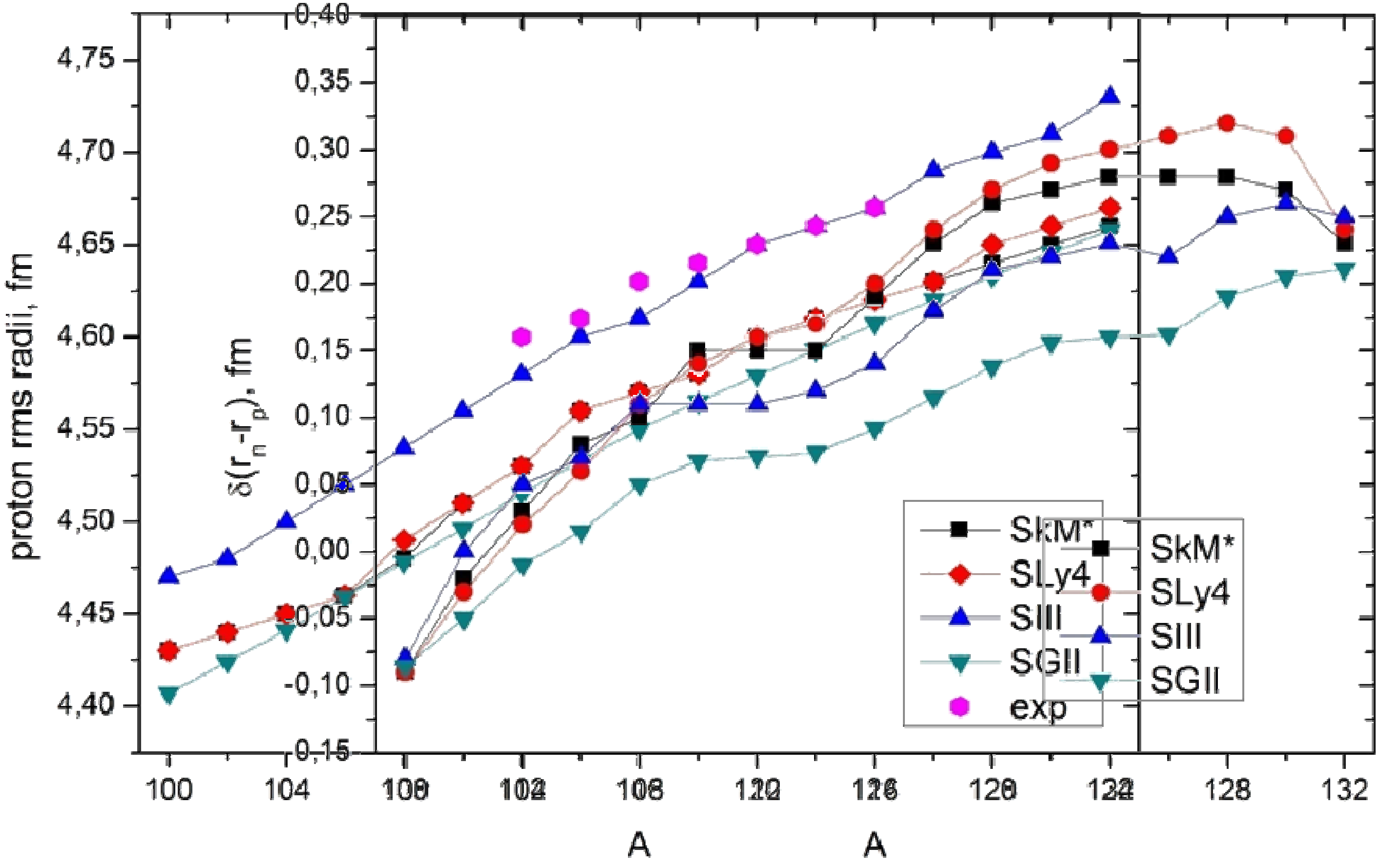
$$- \frac{1}{2} \sum_\tau \sum_{k=1}^{N} \sum_{q=\pm1} \sum_{\lambda\mu} \kappa_0^{M,k} + q\kappa_1^{(M,k)} : M^{(k)+}_{\lambda\mu}(\tau) M^{(k)}_{\lambda\mu}(\mathbf{q}\tau) :$$

$$- \frac{1}{2} \sum_\tau \sum_{k=1}^{N} \sum_{q=\pm1} \sum_{\lambda\mu} \sum_{L=\lambda,\lambda\pm1} \kappa_0^{S,k} + q\kappa_1^{(S,k)} : S^{(k)+}_{\lambda L\mu}(\tau) S^{(k)}_{\lambda L\mu}(\mathbf{q}\tau) :$$

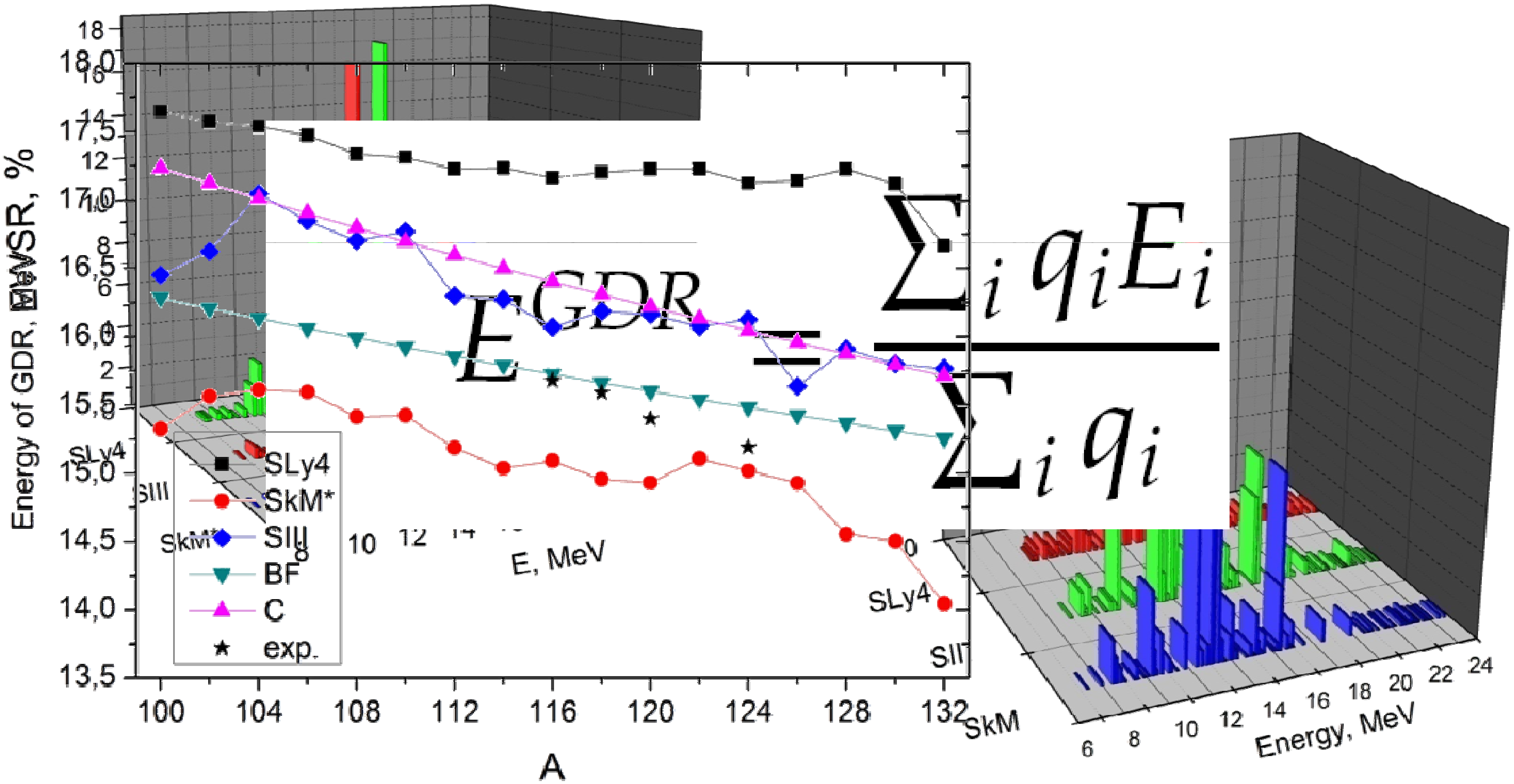
- Hartree-Fock term
- Pairing term
- Isoscalar term
- Isovector term

$$\begin{pmatrix} k_0^{M,k} \\ k_1^{M,k} \\ k_0^{S,k} \\ k_1^{S,k} \end{pmatrix} = -N_0^{-1} \frac{R\omega_k}{2r_k^2} \begin{pmatrix} F_0(r_k) \\ F_0'(r_k) \\ G_0(r_k) \\ G_0'(r_k) \end{pmatrix}$$

Results for the isotopic chain Z=50: RootMeanSquare radii

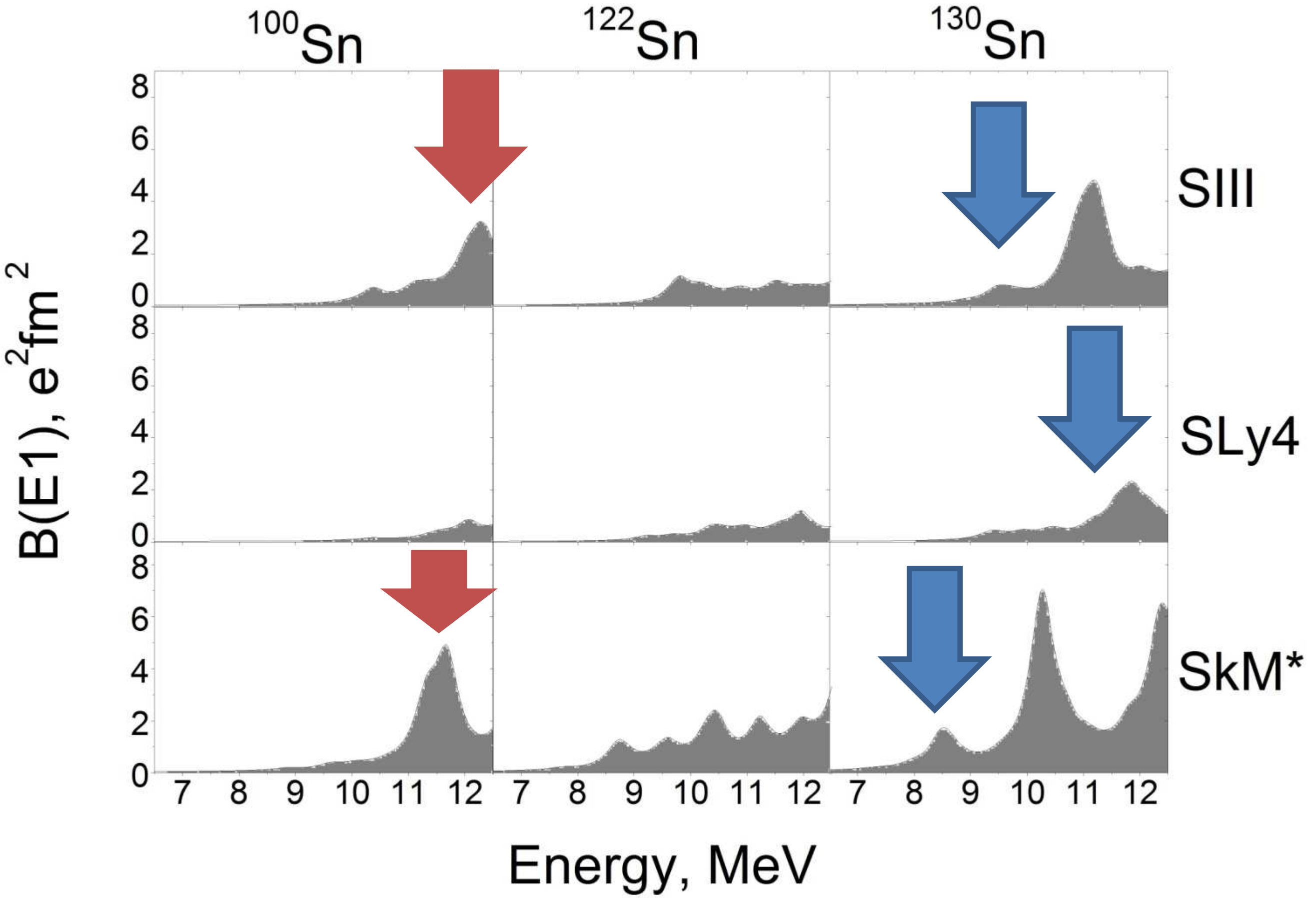


Results for the isotopic chain Z=50: Energy of the Giant Dipole Resonance



$$E^{GDR} = \frac{\sum_i q_i E_i}{\sum_i q_i}$$

Results for the isotopic chain Z=50: Evolution of the dipole excitations with the mass number



Results for the isotopic chain Z=50: Transitional densities

$$\delta\rho^T_{if} = \sum_{j_1 j_2;\lambda\mu} \left[i^\lambda Y_{\lambda\mu}(r)\right]^\dagger \rho^{\lambda T}_{j_1 j_2}(r)\langle\Psi(f)|\Gamma^+_{\lambda\mu}(j_1 j_2)|\Psi(i)\rangle$$

$$\rho^{\lambda T}_{j_1 j_2}(r) = u_{j_1}(r)u_{j_2}(r)\frac{1}{\hat{\lambda}}\langle j_1\|i^\lambda Y_\lambda\|j_2\rangle\langle q|\tau^T_3|q\rangle$$

$$|\Psi(i)\rangle = |0\rangle$$

$$|\Psi(f)\rangle = Q^+_{\lambda\mu i}|0\rangle$$

I

$$\delta\rho^T_{if} = \sum_{j_1 j_2;\lambda\mu} \left[i^\lambda Y_{\lambda\mu}(r)\right]^\dagger \rho^{\lambda T}_{j_1 j_2}(r)\langle 0|Q_{\lambda\mu i}\Gamma^+_{\lambda\mu}(j_1 j_2)|0\rangle$$

$$= \sum_{j_1 j_2 \ldots} \ldots (j_1 j_2)]|0\rangle$$

$$\delta\rho^T = \sum \rho^{\lambda T}(r) g^{\lambda i}$$

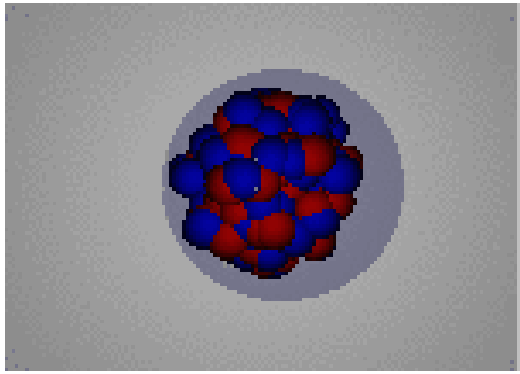
$$\rho^{1p}_{j_1 j_2}(r) = \frac{1}{2}\rho^{10}_{j_1 j_2}(r) - \rho^{11}_{j_1 j_2}(r)$$

$$\rho^{1n}_{j_1 j_2}(r) = \frac{1}{2}\rho^{10}_{j_1 j_2}(r) + \rho^{11}_{j_1 j_2}(r)$$

Results for the isotopic chain Z=50: Transitional densities

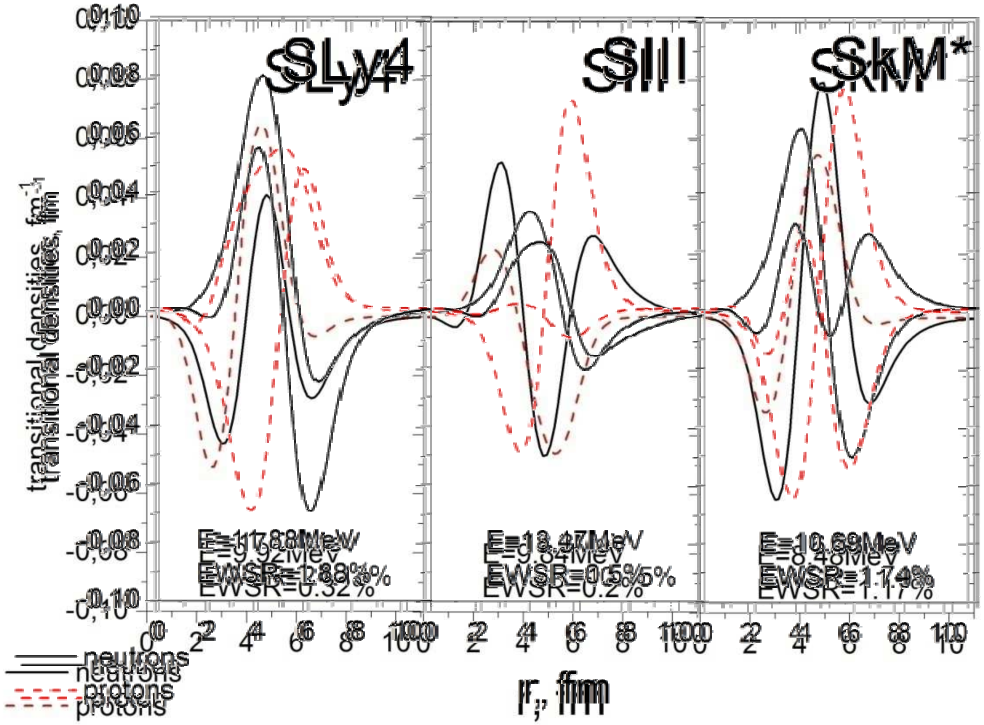
$$\rho^{1p}_{j_1 j_2}(r) = \frac{1}{2}\rho^{10}_{j_1 j_2}(r) - \rho^{11}_{j_1 j_2}(r)$$

$$\rho^{1n}_{j_1 j_2}(r) = \frac{1}{2}\rho^{10}_{j_1 j_2}(r) + \rho^{11}_{j_1 j_2}(r)$$



Results for the isotopic chain Z=50: Transition densities

1. Transitional densities for excitations below 10 MeV;
2. Transitional densities for excitations belonging to the Giant Dipole Resonance;
3. Excitations of a mixed type.



Results for the isotopic chain Z=50: First 2^+ excited state

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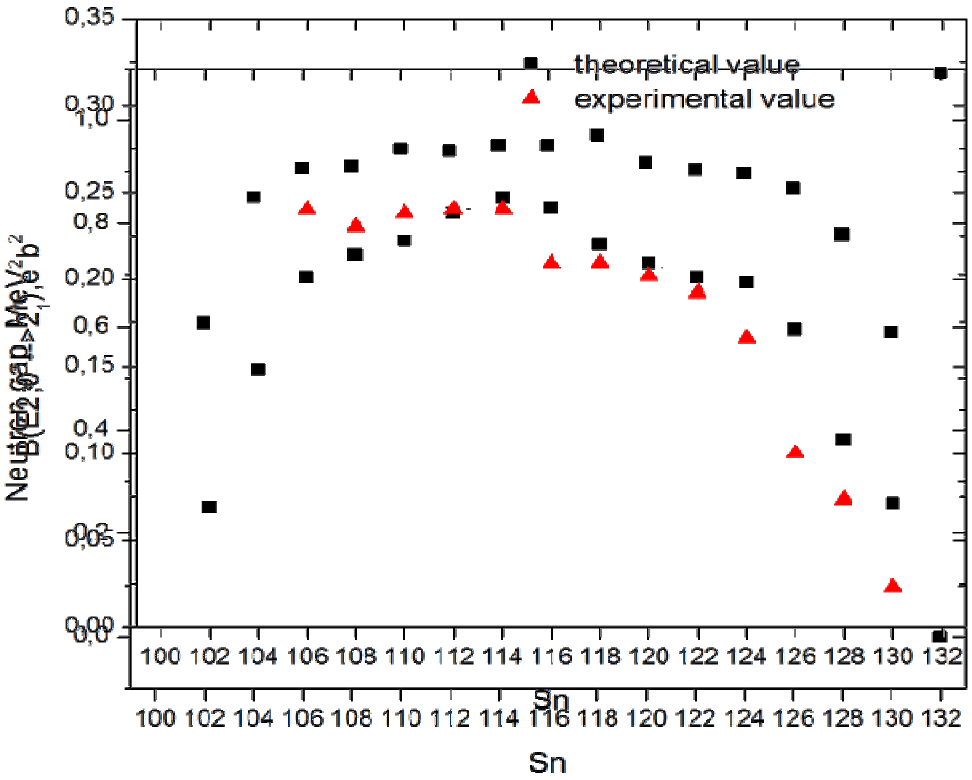
[2] J. Cederkall, A. Ekstrom, C. Fahlander, A. M. Hurst, M. Hjorth-Jensen, F. Ames, A. Banu, P. A. Butler, T. Davinson, U. Datta Pramanik, et al., Phys. Rev. Lett. 98, 172501 (2007)
[3] C. Vaman, C. Andreoiu, D. Bazin, A. Becerril, B. A. Brown, C. M. Campbell, A. Chester, J. M. Cook, D. C. Dinca, A. Gade, et al. Phys. Rev. Lett. 99, 162501 (2007).
[4] A. Ekstrom, J. Cederkall, C. Fahlander, M. Hjorth-Jensen, F. Ames, P. A. Butler, T. Davinson, J. Eberth, F. Fincke, A.Gorgen et al. Phys. Rev. Lett, 101, 012502 (2008)[5] V. Zelevinsky. and A. Volya, Nucl. Phys., A752, 325c (2005)
[6] A. Ansari, Phys. Lett., B623, 37 (2005)
[7] I. Talmi, Nucl. Phys. A172, 1 (1971).
[8] J.R. Beene et al, Nucl. Phys. A746, 417c (2004).
[9] Nguyen Van Giai, Ch. Stoyanov, V.V.Voronov, Phys Rev C57, 1204 (1998)
[10] V. G. Soloviev, *Theory of atomic nuclei : Quasiparticles and Phonons (Institute of Physics Publishing, Bristol, 1992).*

Motivation:

In recent experiments [1-4] the E2 strengths have been measured in the neutron deficient $^{106-112}$Sn isotopes. Using different models [5-8] a lot of theoretical effort was devoted in order to understand the effect of different aspects of the nuclear structure on the B(E2,g.s.→2_1^+) strength in the tin chain. We try to describe the effect of the pairing on the E2 strength in these nuclei, and compare to the experimental results.



Mixed symmetry states (MSS)

Mixed symmetry states in the framework of a microscopic model:

2_1^+ oscillations of the proton and neutron systems **in phase**;
(***isoscalar*** vibrations)

(Fully Symmetric State)

2_2^+ oscillations **out-of-phase**;
(***isovector***) vbrations of protons and neutrons

(Mixed Symmetry State)

A. A.Faessler, R. Nojarov, Phys. Lett., **B166**, 367 (1986)
B. R. Nojarov, A. Faessler, J. Phys. G, **13**, 337 (1987)

Quasiparticle Phonon Model (QPM)

$$H_{QPM} = H_{sp} + V_{pair} + V_M^{ph} + H_{SM}^{ph} + H_M^{pp}$$

$$H_{pair} = -\sum_{j,j',m,m'} G(jm, j-m; j'm', j', -m')\, a^{\dagger}_{jm} a^{\dagger}_{j-m} a_{j'-m'} a_{j'm'}$$

$$H_{s.p.} = \sum_{j,m} \left(E_0(j) - \lambda\right) a^{\dagger}_{jm} a_{jm}$$

$$V(r) = -\frac{V_0^{N,Z}}{1 + \exp\left[\frac{1}{a}(r - R_0)\right]} \quad \text{central term}$$

$$V_{ls}(r) = -\xi \frac{1}{r}\frac{dV(r)}{dr}(\mathbf{l} \cdot \mathbf{s}) \quad \text{spin-orbit term}$$

parameter	neutrons	protons
V_0, MeV	45.95	53.44
R_0, fm	1.27	1.31
a, fm	1.61	1.538
ξ, fm^2	0.413	0.349

Quasiparticle Phonon Model (QPM)

1. V.G. Soloviev, *Theory of atomic nuclei : Quasiparticles and Phonons* (Institute of Physics Publishing, Bristol and Philadelphia, 1992).

$$H_{QPM} = \sum_{i\mu} \omega_{i\lambda} Q^{\dagger}_{i\lambda\mu} Q_{i\lambda\mu} + H_{vq}$$

$$Q^{\dagger}_{i\lambda\mu} = \frac{1}{2} \sum_{jj'} \left\{ \psi^{i\lambda}_{jj'} [\alpha^{\dagger}_{j} \alpha^{\dagger}_{j'}]_{\lambda\mu} - (-1)^{\lambda-\mu} \varphi^{i\lambda}_{jj'} [\alpha_{j'} \alpha_{j}]_{\lambda-\mu} \right\}_{\tau}$$

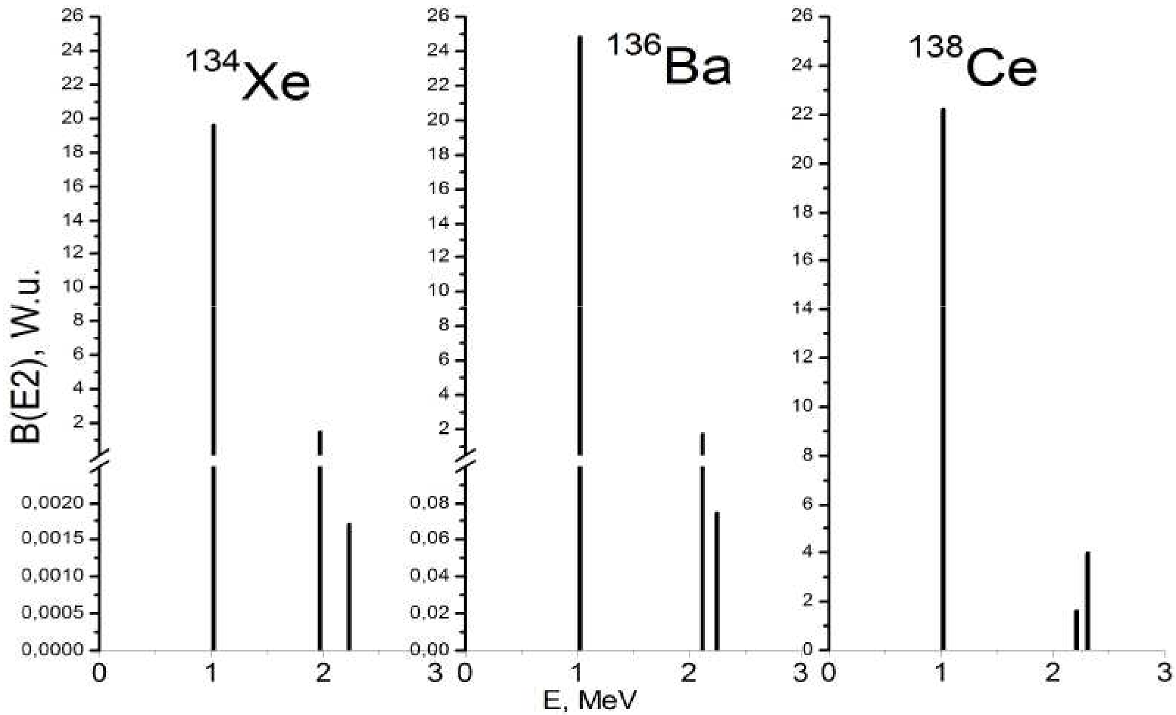
$$\Psi_{\nu}(\lambda\mu) = \sum_{i} R_i(\nu J) Q^{+}_{i\lambda\mu}|0\rangle + \sum_{\ldots} P^{j_1 m_1}_{j_2 m_2}(\nu\lambda)[Q^{+}_{j_1 m_1} \otimes Q^{+}_{j_2 m_2}]_{\lambda\mu}|0\rangle$$

$$[Q_{\lambda\mu i}, Q^{+}_{\lambda'\mu' i'}] = \frac{\delta_{\lambda\lambda'}\delta_{\mu\mu'}\delta_{ii'}}{2} \sum_{jj'} \left[X^{\lambda i}_{jj'} X^{\lambda i'}_{jj'} - Y^{\lambda i}_{jj'} Y^{\lambda i'}_{jj'} \right] - \sum_{jj' j_2 m m' m_2} \alpha^{+}_{jm} \alpha_{j'm'}$$

$$\left[X^{\lambda i}_{j' j_2} X^{\lambda' i'}_{j j_2} \langle j'm' j_2 m_2 | \lambda\mu \rangle \langle jm j_2 m_2 | \lambda'\mu' \rangle \right.$$

$$\left. -(-1)^{\lambda+\lambda'+\mu+\mu'} Y^{\lambda i}_{j j_2} Y^{\lambda' i'}_{j' j_2} \langle jm j_2 m_2 | \lambda - \mu \rangle \langle j'm' j_2 m_2 | \lambda' - \mu' \rangle \right]$$

Results

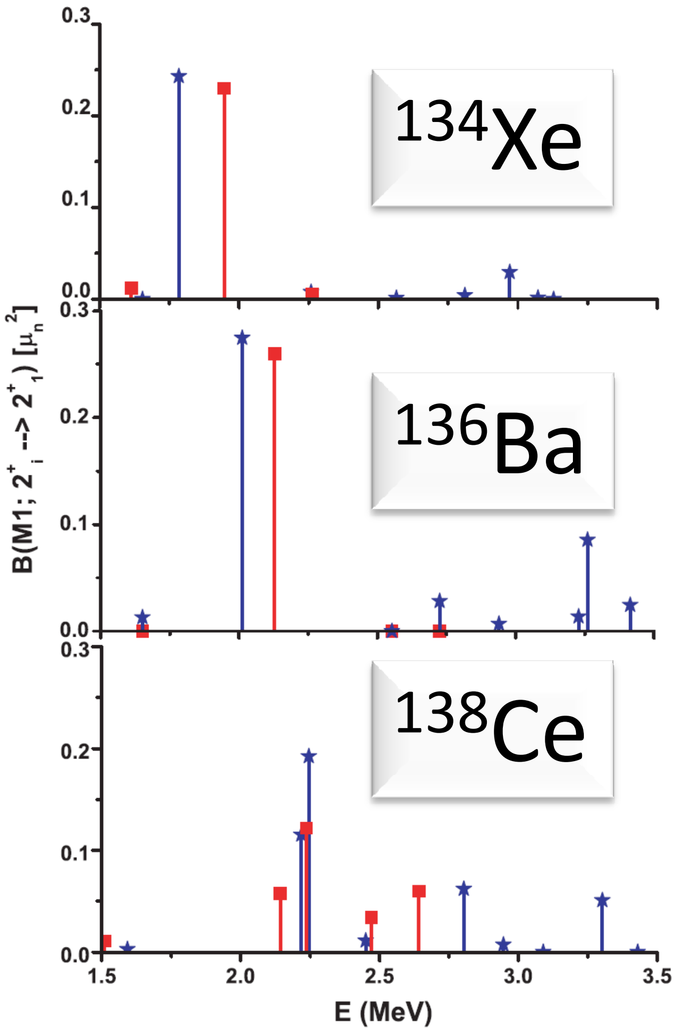


Nucleus	$q_1 q_2$	$E_{q_1+q_2}$, MeV
^{134}Xe	$(1_{g_{7/2}} \otimes 1_{g_{7/2}})_\pi$	2.124
	$(1_{g_{7/2}} \otimes 2_{d_{5/2}})_\pi$	2.943
	$(2_{d_{5/2}} \otimes 2_{d_{5/2}})_\pi$	3.762
^{136}Ba	$(1_{g_{7/2}} \otimes 1_{g_{7/2}})_\pi$	2.333
	$(1_{2_{3/2}} \otimes 3_{s_{1/2}})_\pi$	2.886
	$(2_{d_{5/2}} \otimes 2_{d_{5/2}})_\pi$	3.440
^{138}Ce	$(1_{g_{7/2}} \otimes 1_{g_{7/2}})_\pi$	2.627
	$(1_{g_{7/2}} \otimes 2_{d_{5/2}})_\pi$	2.892
	$(2_{d_{5/2}} \otimes 2_{d_{5/2}})_\pi$	3.158

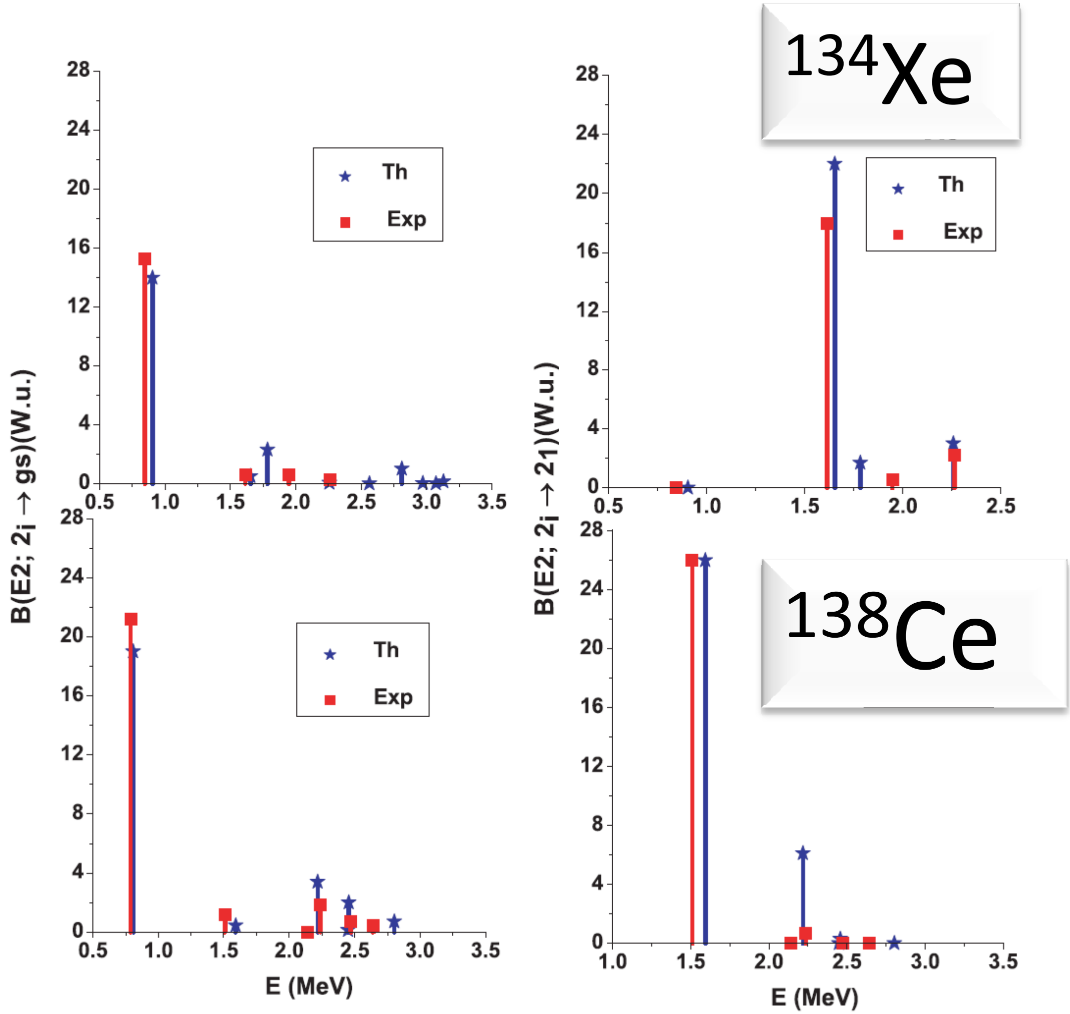
Energies of the two-quasi-particle states for the first 2^+ excitation

First excitation – np collective, isoscalar
Second excitation – np collective, isovector

Results

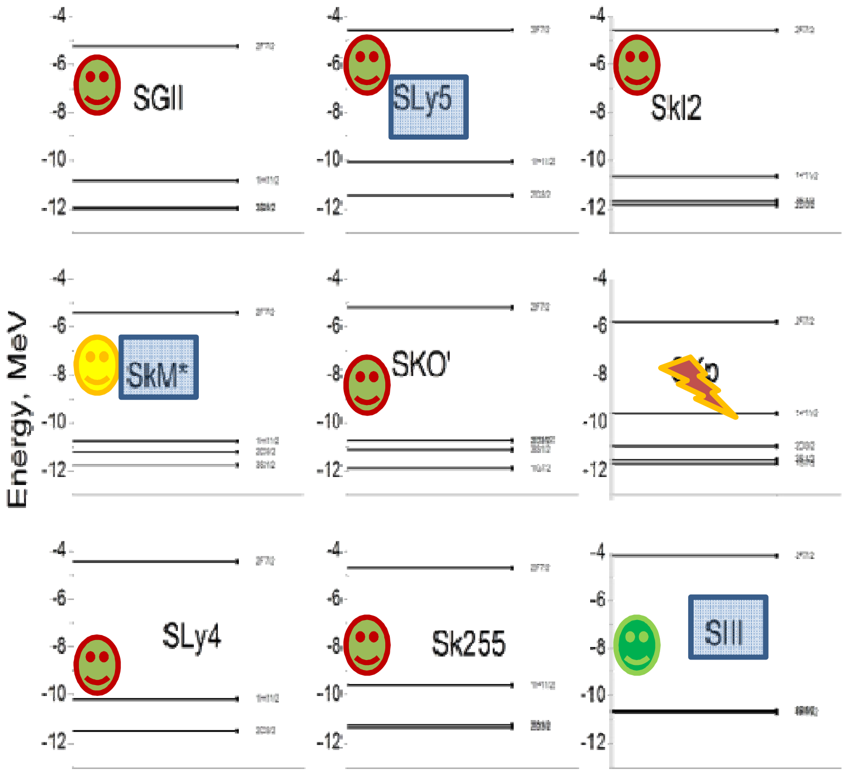


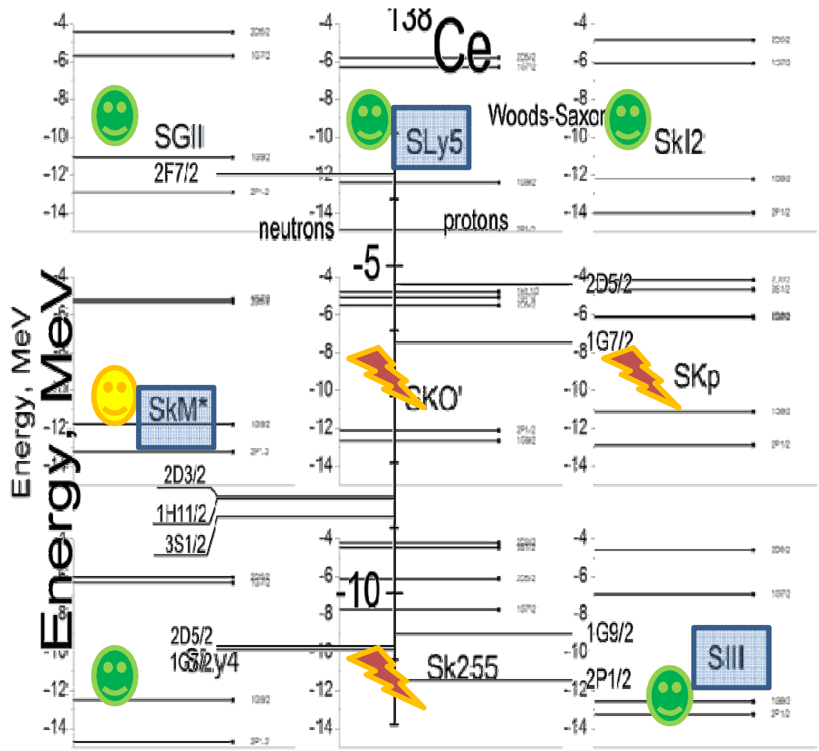
134Xe
136Ba
138Ce
B(M1; 2+i --> 2+1) [μn2]
E (MeV)
0.0
0.1
0.2
0.3
1.5
2.0
2.5
3.0
3.5



134Xe
138Ce
Th
Exp
B(E2; 2i → gs)(W.u.)
B(E2; 2i → 21)(W.u.)
E (MeV)
0
4
8
12
16
20
24
28
0.5
1.0
1.5
2.0
2.5
3.0
3.5

The effect of the different Skyrme forces on the single particle scheme (^{138}Ce)





Results for ^{138}Ce

This ratio probes:

1. **The** **isoscalar** **(B(2⁺)<1)** and
2. **The** **isovector** **(B(2+)>1)**

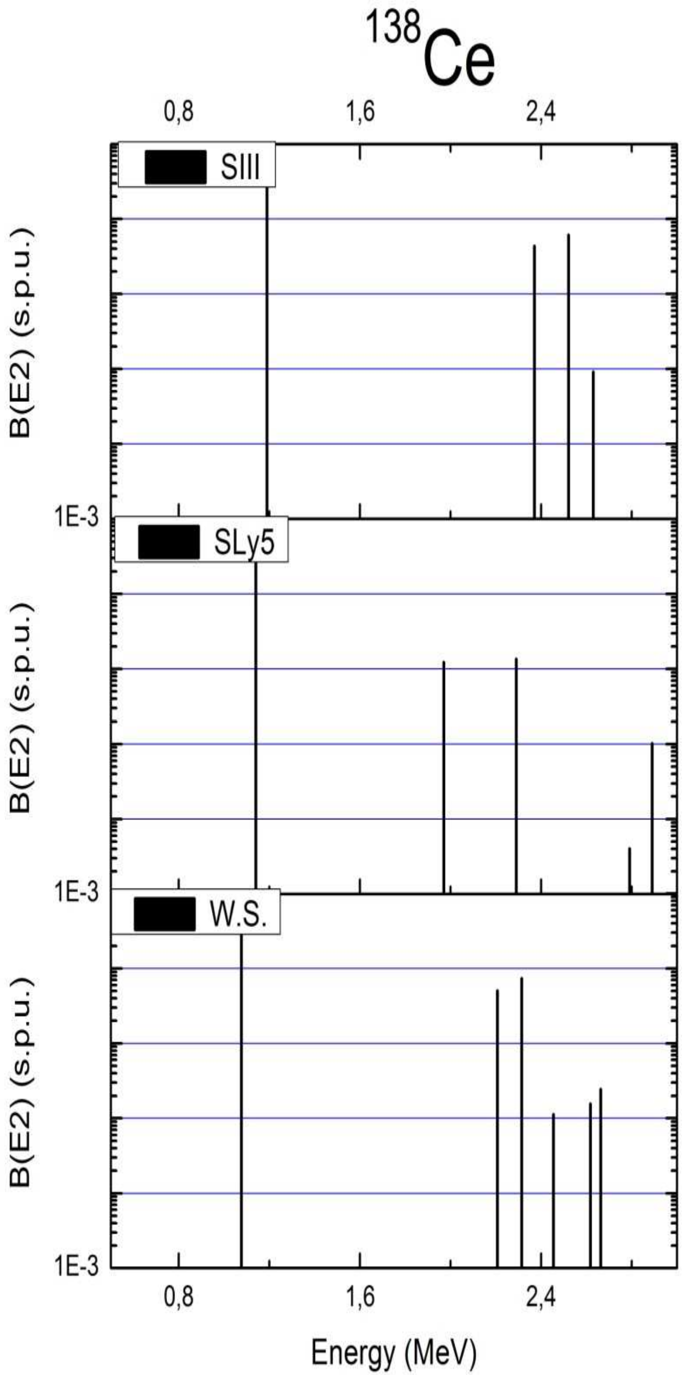
properties of the 2^+ state under consideration

$$B\left(2^+\right)=\frac{\left|\left\langle 2^+\right|\left|\sum_k^p r_k^2 Y_{2\mu}\left(\Omega_k\right)-\sum_k^n r_k^2 Y_{2\mu}\left(\Omega_k\right)\right|\left|g.s\right\rangle\right|^2}{\left|\left\langle 2^+\right|\left|\sum_k^p r_k^2 Y_{2\mu}\left(\Omega_k\right)+\sum_k^n r_k^2 Y_{2\mu}\left(\Omega_k\right)\right|\left|g.s\right\rangle\right|^2}$$

$B(2^+_1)=0.0012$

$B(2^+_2)= 2.609$

$B(2^+_3)= 1.235$



QPM Results for ^{136}Be and^{138}Ce

Nucleus	$J_i \rightarrow J_f$	B(E2)		B(M1)	
		EXP	QPM (SIII)	EXP	QPM (SIII)
^{136}Ba	$0^+_{gs} \rightarrow 2^+_1$	0.400(5)	0.24		
	$0^+_{gs} \rightarrow 2^+_2$	0.016(4)	0.09		
	$0^+_{gs} \rightarrow 2^+_3$	0.045(5)	0.03		
	$2^+_2 \rightarrow 2^+_1$	0.09(4)	0.12		
	$2^+_2 \rightarrow 2^+_1$				0.007
	$2^+_3 \rightarrow 2^+_1$			0.26(3)	0.21
^{138}Ce	$2^+_1 \rightarrow 0^+_{gs}$	21.2(14)	11		
	$2^+_2 \rightarrow 0^+_{gs}$	1.16(8)	4.5		
	$2^+_3 \rightarrow 0^+_{gs}$		3.9		
	$2^+_4 \rightarrow 0^+_{gs}$	1.86(16)	0.35		
	$2^+_2 \rightarrow 2^+_1$	28(2)	26	0.011(2)	0.003
	$2^+_3 \rightarrow 2^+_1$		6.1	0.058(6)	0.23
	$2^+_4 \rightarrow 2^+_1$	0.65(10)	0.28	0.122(10)	0.13

QPM versus experimental strengths of E2 and M1 transitions. The E2 strengths are given in W.u. for ^{138}Ce, and in e^2b^2 for ^{136}Ba. The M1 strengths are in μ_N^2.

Outlook

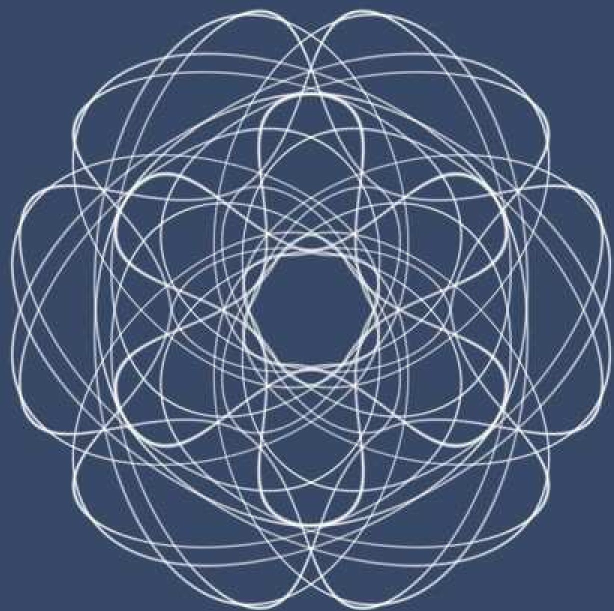
1) Expanding our QPM calculations towards other N=80 isotones; looking into N=78 and N=84 isotones.

2) A self consistent description (starting from Skyrme type force) in the QPM model.

3) Including particle-vibration coupling.

4) Constructing a new QRPA code, and extending the one we have.

Thank You for Your attention



analog*

digital

* an organ or structure that is similar in function to one in another kind of organism but is of dissimilar evolutionary origin