Stany jąder ciężkich jako laboratorium badania procesów fundamentalnych

Jacek Dobaczewski
Uniwersytet Warszawski & Uniwersytet w Jyväskylä

Seminarium „Struktura jądra atomowego”
Uniwersytet Warszawski
7 kwietnia 2011

Publications and talks

• **J. Dobaczewski and J. Engel:**
Nuclear Time-Reversal Violation and the Schiff Moment of ^{225}Ra, Phys. Rev. Lett. 94 (2005) 232502

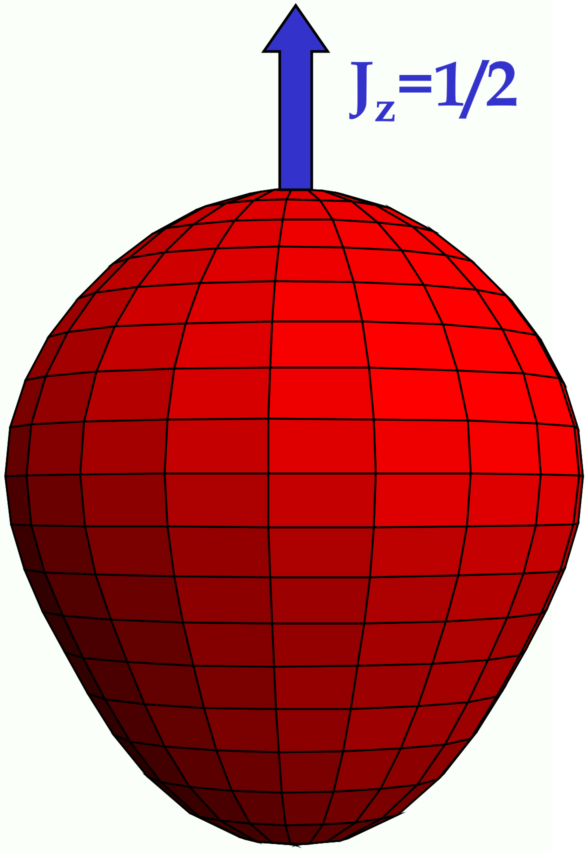
• **J. Engel:**
INT Program 05-3, *Nuclear Structure Near the Limits of Stability*, http://www.int.washington.edu/talks/WorkShops/int_05_3/People/Engel_J/dipole_seattle_trans.pdf

• **S. Ban, J. Engel, J. Dobaczewski, and A. Shukla:**
Fully self-consistent calculations of nuclear Schiff moments, Phys. Rev. C 82 (2010) 015501

• **E. Litvinova, H. Feldmeier, J. Dobaczewski, and V. Flambaum:**
Nuclear structure of lowest ^{229}Th states and time-dependent fundamental constants, Phys. Rev. C 79, 064303 (2009)

Skyrme-Hartree-Fock
J. Dobaczewski, J. Engel,
Phys. Rev. Lett. 94, 232502 (2005)

Experiment
R.G. Helmer *et al.*, Nucl. Phys. A474 (1987) 77



$\beta_{10} = \ 0.023$

$\beta_{20} = \ 0.161$

$\beta_{30} = -0.128$

$\beta_{40} = \ 0.091$

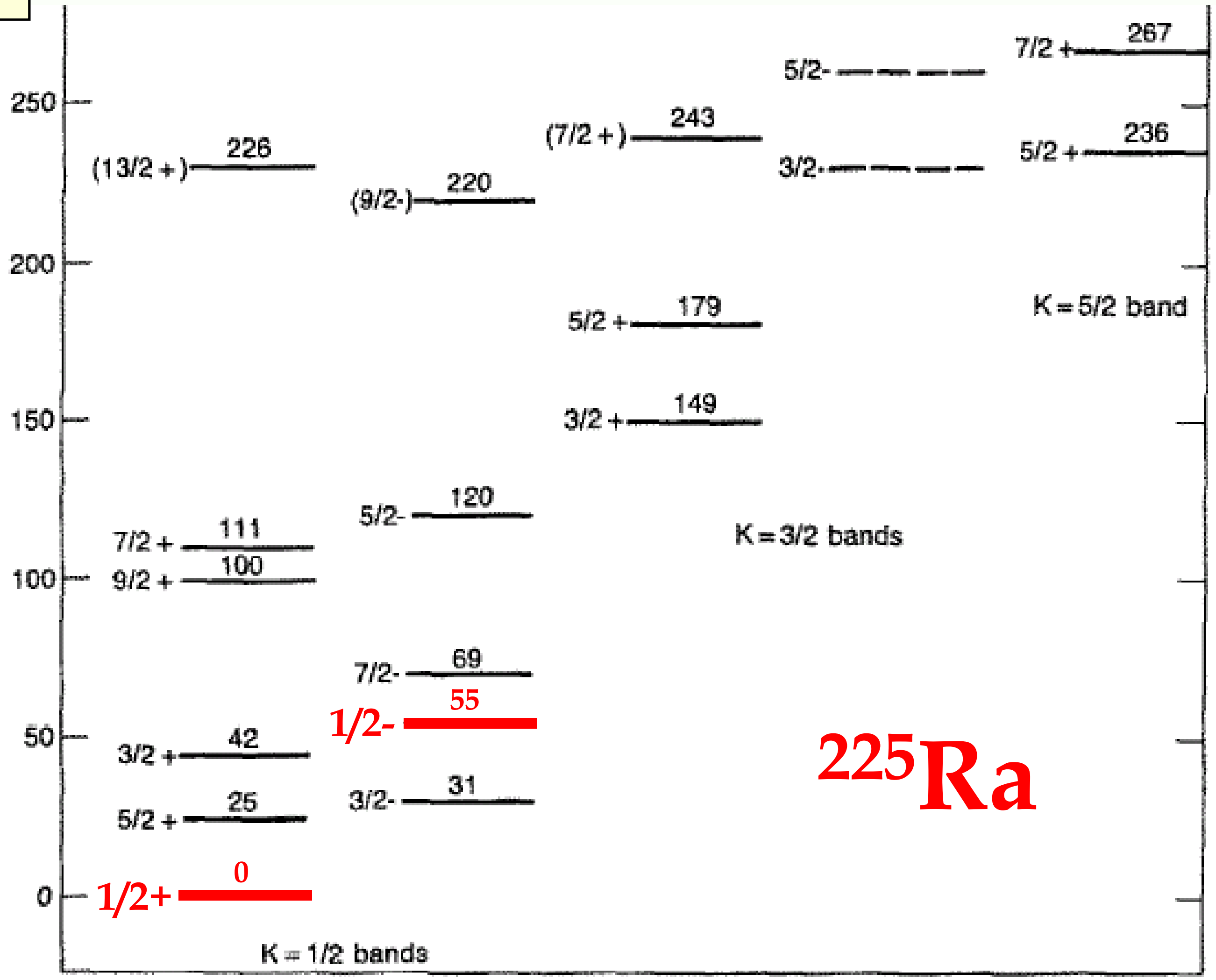
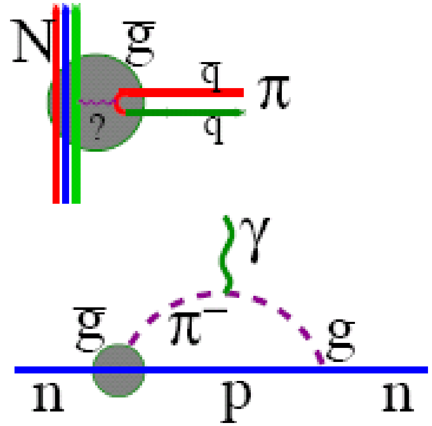


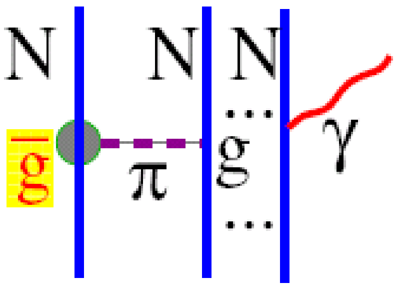
Fig. 5. Proposed grouping of the low-lying states of ^{225}Ra into rotational bands. The two members of the $K^{\pi} = \frac{3}{2}^{-}$ band have been reported in a study of the ^{225}Fr decay [20]; they are not observed in the present study.

How Do Things Get EDM's?

- Underlying theory generates T-violating πNN vertex:
- A neutron gets a EDM from a diagram like this:



- A nucleus can get one from a nucleon EDM or through a T-violating nucleon-nucleon interaction, e.g.



$$W \propto \left\{ \left[\bar{g}_0 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - \frac{\bar{g}_1}{2} \left(\tau_1^z + \tau_1^z \right) + \bar{g}_2 \left(3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right) \right] \left(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2 \right) - \frac{\bar{g}_1}{2} \left(\tau_1^z - \tau_2^z \right) \left(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 \right) \right\} \cdot \left(\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2 \right) \frac{\exp\left(-m_\pi |\mathbf{r}_1 - \mathbf{r}_2| \right)}{m_\pi |\mathbf{r}_1 - \mathbf{r}_2|}$$

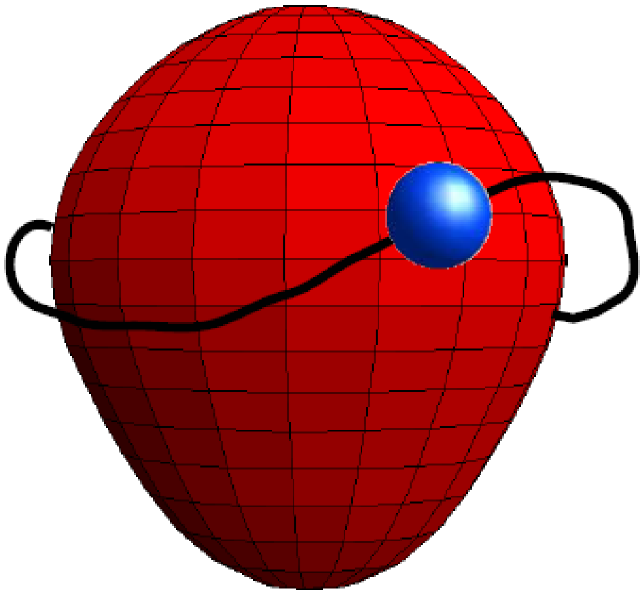
- Finally, nuclear EDM induces atomic EDM.

The goal of the atomic experiments discussed here is to constrain (or determine) the three $\bar{g}$'s.

Now What About Schiff Moments?

Need T-violating nuclear interaction W to get one. Treating W as perturbation:

$$\langle \vec{S} \rangle = \sum_m \frac{\langle 0 | \vec{S} | m \rangle \langle m | W | 0 \rangle}{E_0 - E_m} + c.c.$$



where $|0\rangle$ is the unperturbed nuclear ground state.

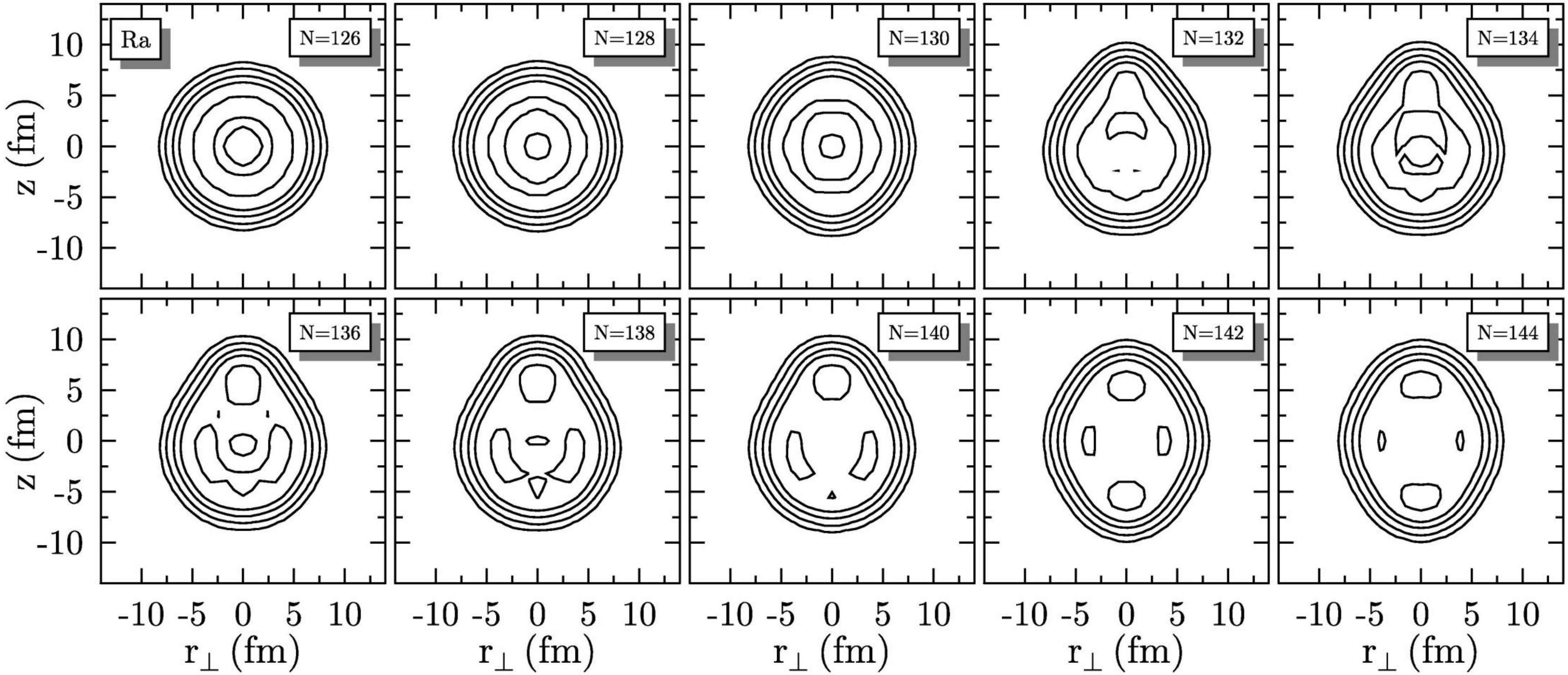
$\langle \vec{S} \rangle$ will not be enhanced if nucleus is only quadrupole deformed. Need octupole deformation too.

Then, two collective effects help you out:

1. Parity doubling
2. Large and robust intrinsic Schiff moments

Shapes of Radium isotopes

Skyrme HF+BCS calculations with the SkO′ interaction and density-independent zero-range pairing force, axial symmetry

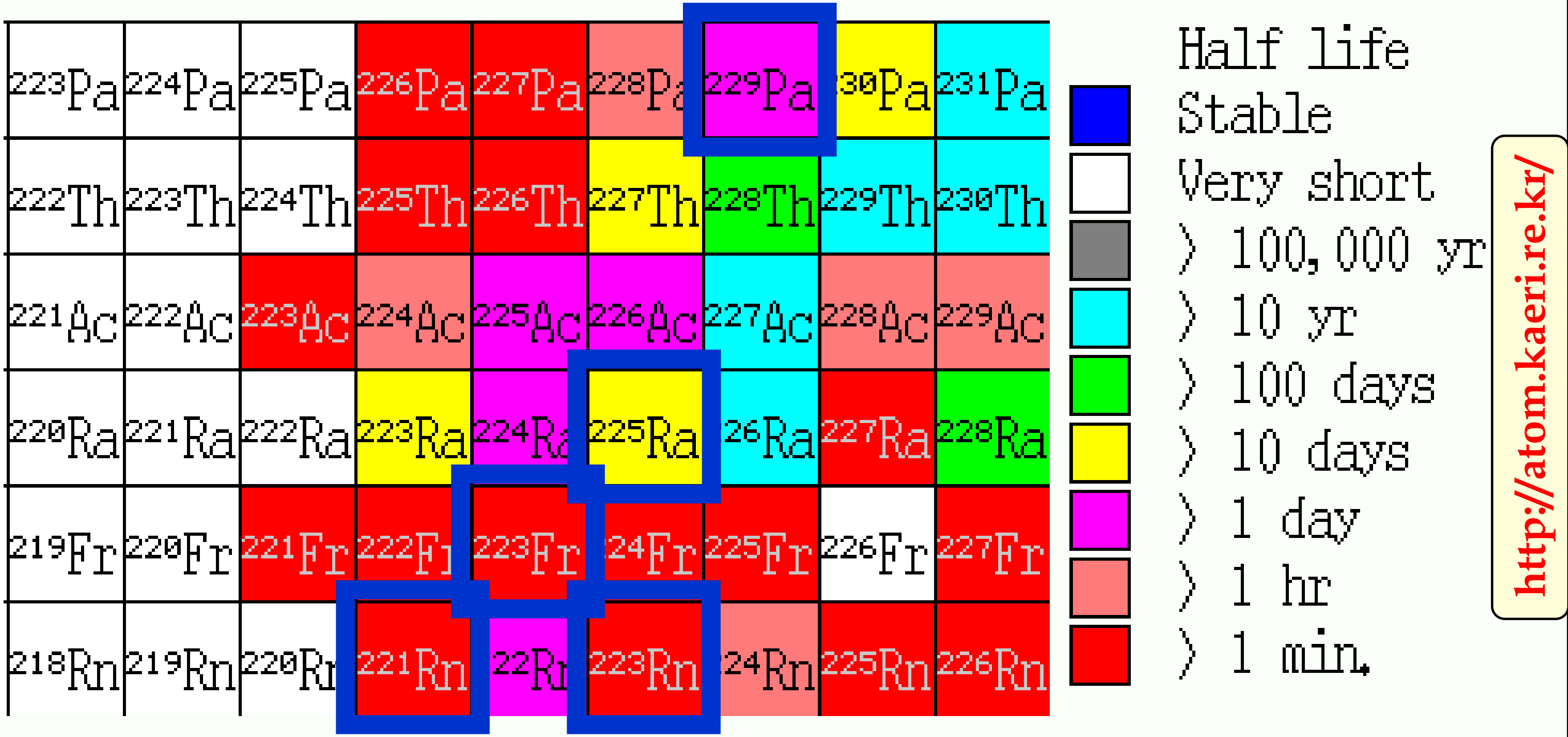


J. Engel *et al.*, Phys. Rev. C68, 025501 (2003)

Heavy nuclei with octupole deformation

Z=91

Z=86



N=132

N=140

Nuclear Energy Density Functional

We consider the EDF in the form,

$$\mathcal{E} = \int d^3r \mathcal{H}(r),$$

where the energy density $\mathcal{H}(r)$ can be represented as a sum of the kinetic and iteraction energy densities,

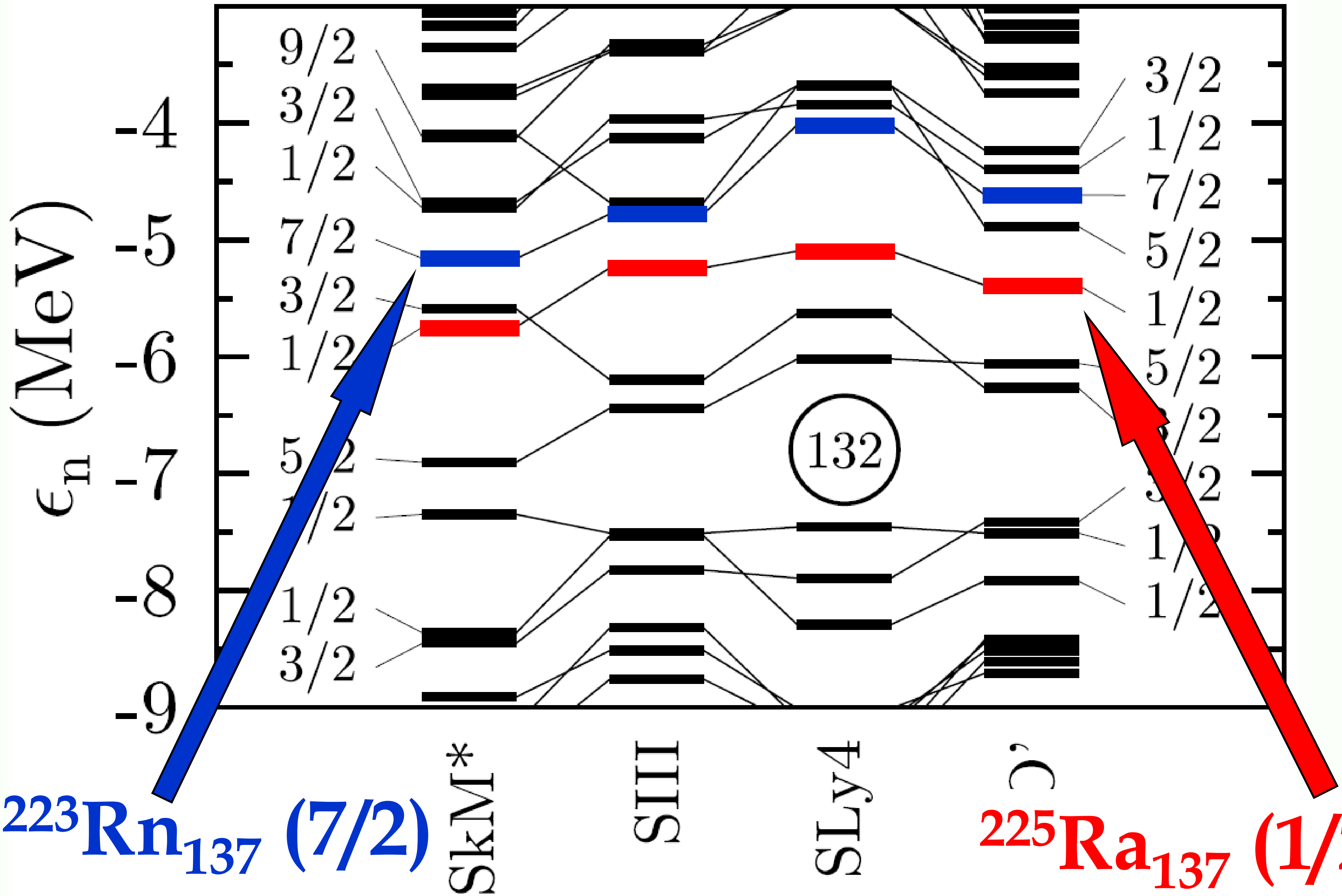
$$\mathcal{H}(r) = \frac{\hbar^2}{2m}\tau_0(r) + \sum_{t=0,1}\mathcal{E}_t^{\rm int}(r),$$

where

$$\begin{aligned}\mathcal{E}_t^{\rm int} = \big[& C_t^\rho \rho_k^2 + C_t^{\Delta\rho}\rho_k\Delta\rho_k + C_t^\tau(\rho_k\tau_k - \vec{j}_k^{\,2}) \\ & + C_t^s \vec{s}_k^{\,2} + C_t^{\Delta s}\vec{s}_k\cdot\Delta\vec{s}_k + C_t^T(\vec{s}_k\cdot\vec{T}_k - \mathsf{J}_{abk}\mathsf{J}_{abk}) \\ & + C_t^F(\vec{s}_k\cdot\vec{F}_k - \tfrac{1}{2}\mathsf{J}_{aak}\mathsf{J}_{bbk} - \tfrac{1}{2}\mathsf{J}_{abk}\mathsf{J}_{bak}) + C_t^{\nabla s}(\vec{\nabla}\cdot\vec{s}_k)^2 \\ & + C_t^{\nabla J}(\rho_k\vec{\nabla}\cdot\vec{J}_k + \vec{s}_k\cdot(\vec{\nabla}\times\vec{j}_k))\big],\end{aligned}$$

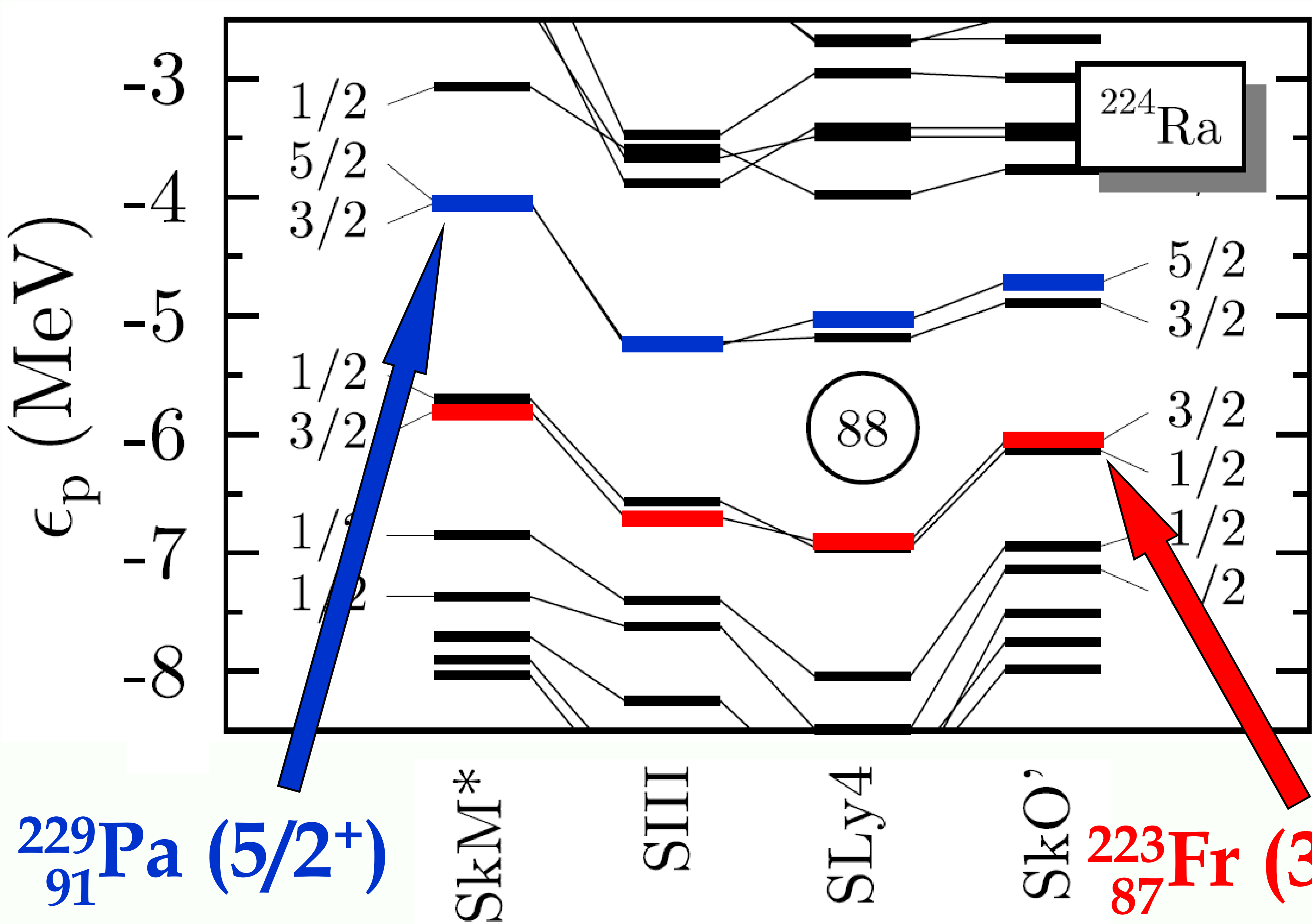
Following the parametrization used for the Skyrme forces, we assume the dependence of the coupling parameters C_t^ρ on the isoscalar density ρ_0 as: $C_t^\rho = C_{t0}^\rho + C_{t\mathrm{D}}^\rho\rho_0^\alpha$.

Neutron single-particle spectra

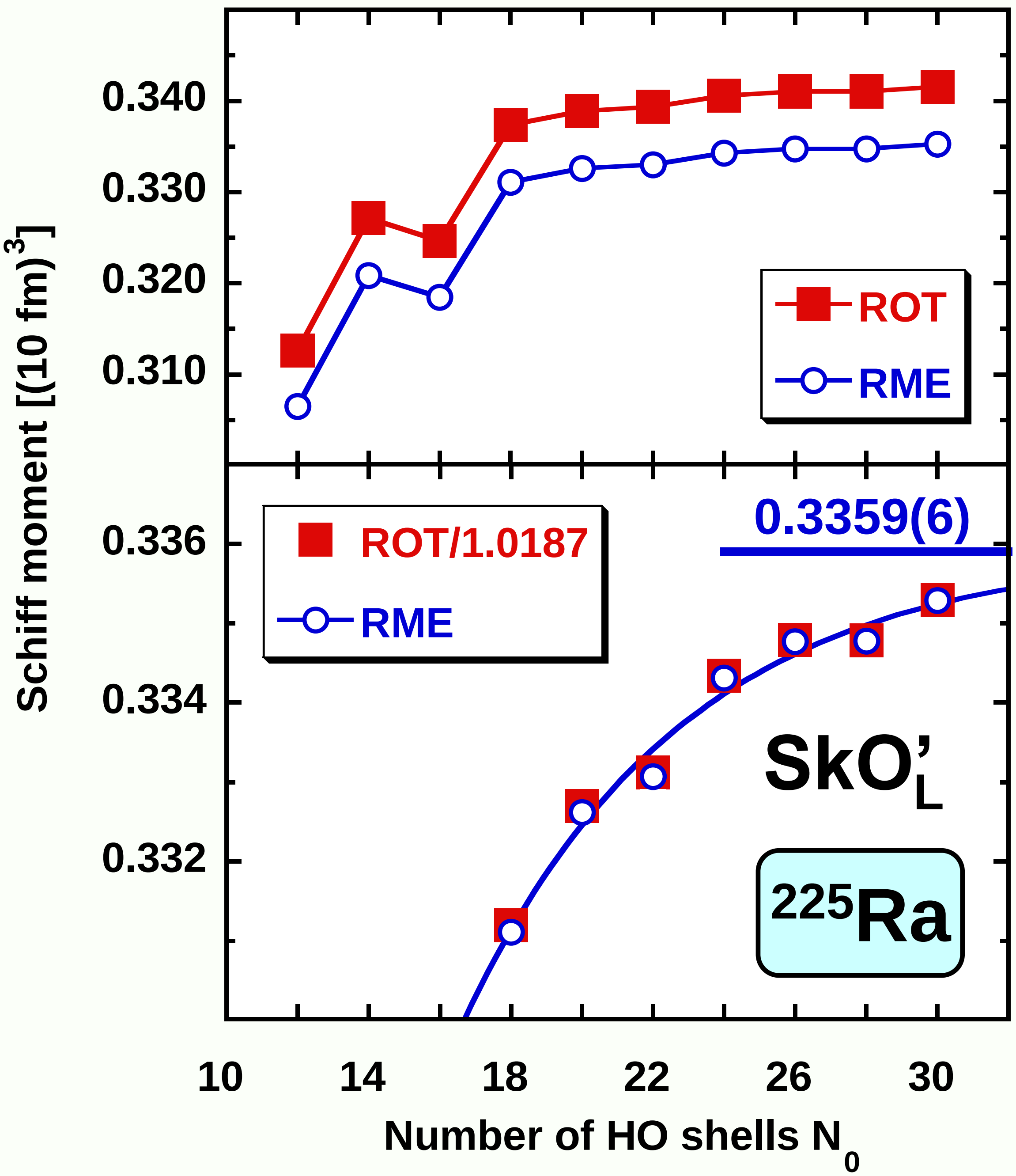


J. Engel *et al.*, Phys. Rev. C68, 025501 (2003)

Proton single-particle spectra



J. Engel *et al.*, Phys. Rev. C68, 025501 (2003)



The reduced matrix element (RME) <I=1/2||S||I=1/2> calculated by

(i) projecting the I=1/2 state from the intrinsic state

(ii) using the rotational approximation (ROT) $\frac{2}{\sqrt{6}} S_{intr}$

To evaluate $\langle \hat{V}_{\rm PT} \rangle$ we constructed a new version of the code HFODD (v2.14 e) [18,19]. The code uses a triaxial harmonic-oscillator basis and Gaussian integration to solve self-consistent mean-field equations for zero-range Skyrme interactions. Evaluating matrix elements of the finite-range interaction (3) is much harder numerically, but efficient techniques have already been developed [20] for Gaussian interactions, which are separable in three Cartesian directions. The spatial dependence in Eq. (3) is different, the derivative of a Yukawa function, and we also include short-range correlations between nucleons (which the mean field does not capture) by multiplying the interaction by the square of a correlation function [21] that cuts off the two-nucleon wave functions below a relative distance of about a Fermi:

$$f(r) = 1 - e^{-1.1r^2}(1 - 0.68r^2), \qquad (6)$$

with $r \equiv |\mathbf{r}_1 - \mathbf{r}_2|$ in Fermis and the coefficients of r^2 in fm^{-2}. The resulting product looks very different from a Gaussian, but we were able to reproduce it quite accurately (see Fig. 2) with the sum of four Gaussians:

$$g(r) \equiv f(r)^2 \frac{e^{-a_\pi r}}{r^2}\left(1 + \frac{1}{a_\pi r}\right) \approx 1.75e^{-1.1r^2} + 0.53e^{-0.68r^2} + 0.11e^{-0.21r^2} + 0.004e^{-0.06r^2}, \qquad (7)$$

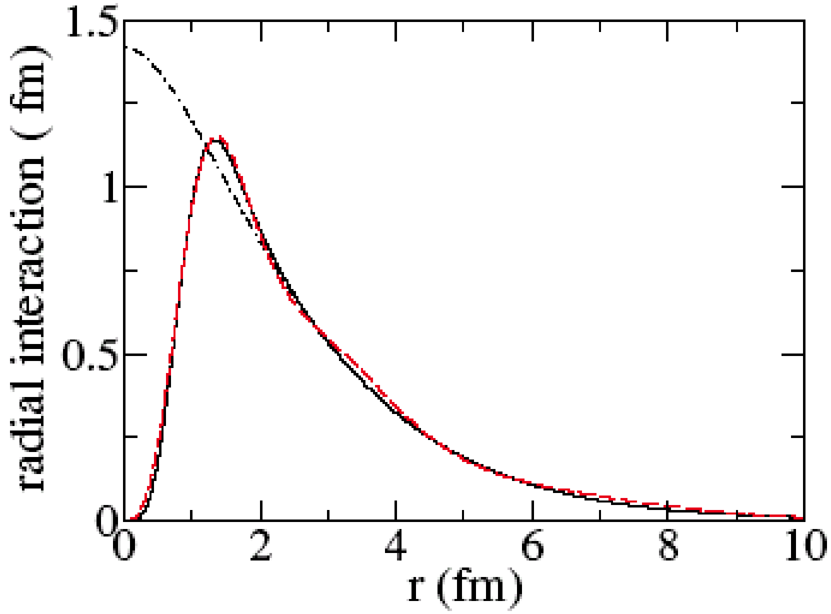
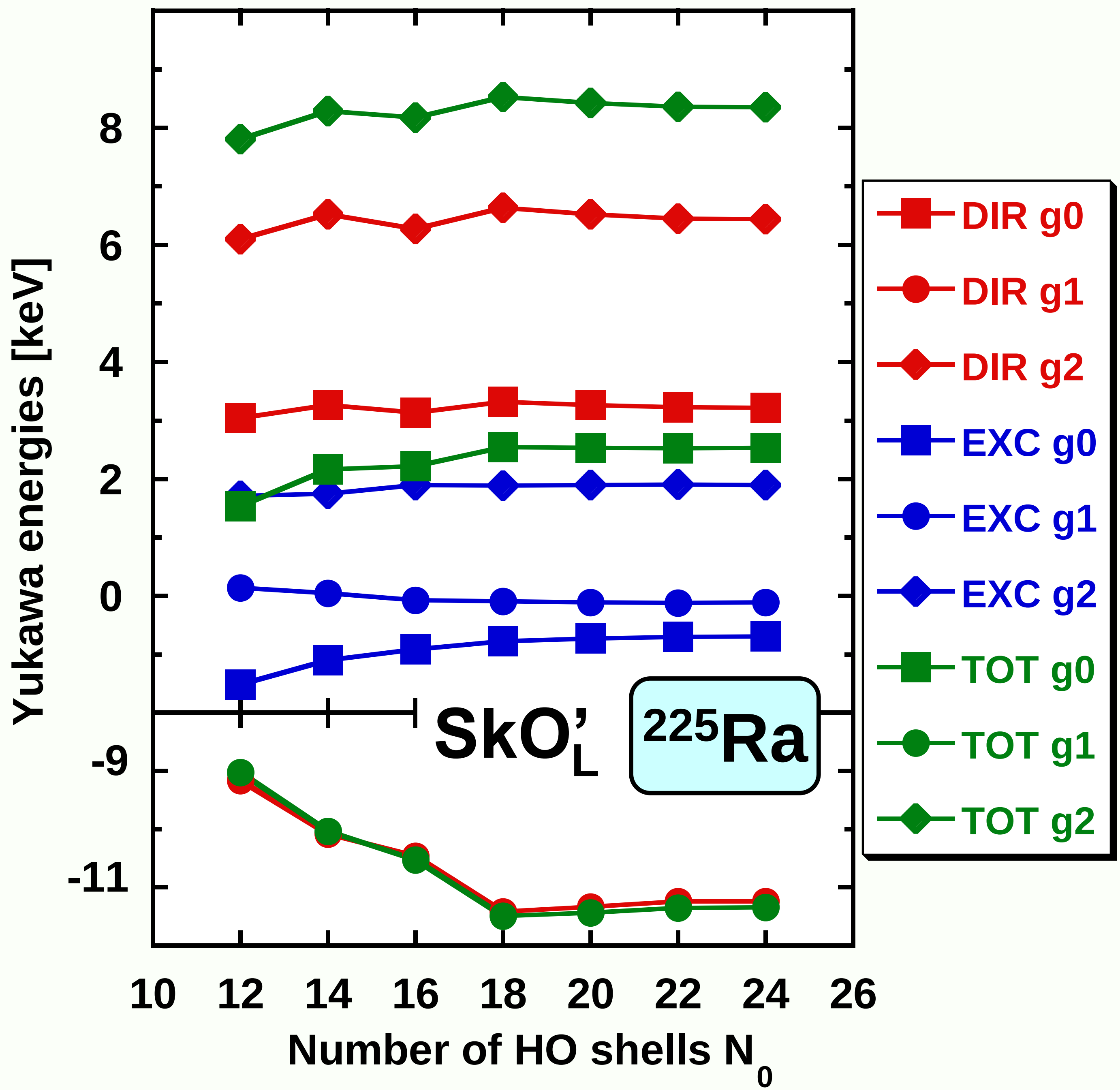
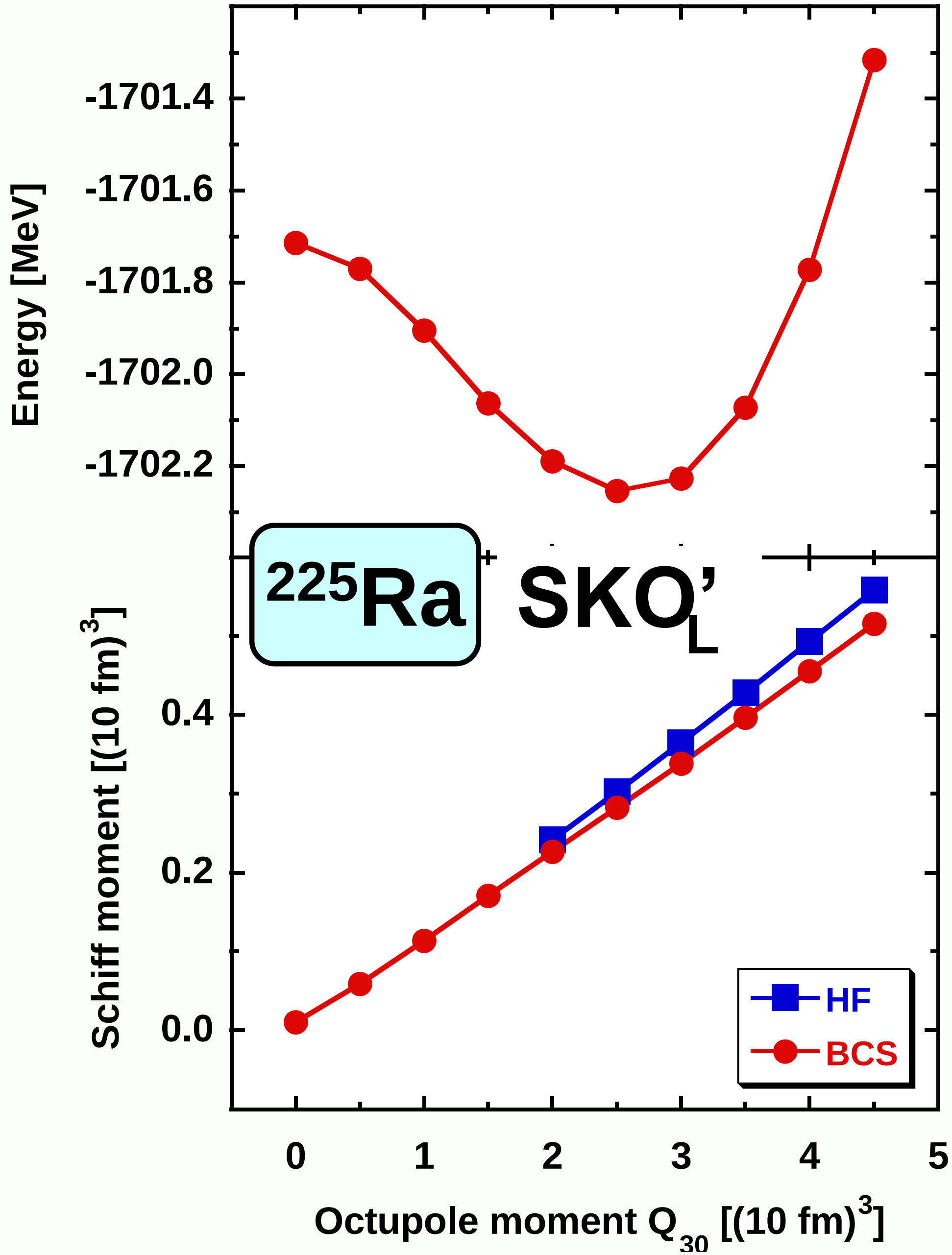


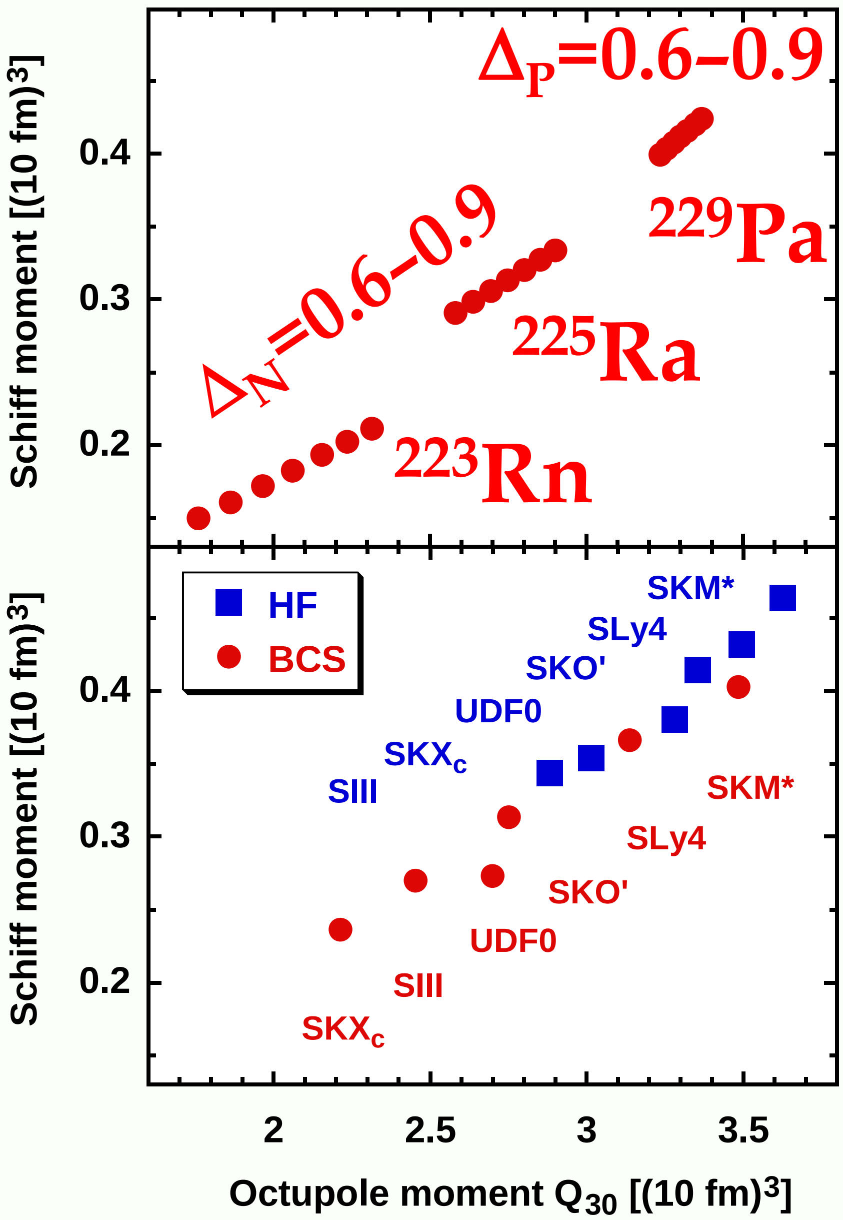
FIG. 2 (color online). The function $g(r)$ in Eq. (7) multiplied by r^3 (solid line), the Gaussian fit multiplied by r^3 (dashed line), and $r^3 g(r)/f(r)^2$, the radial T-odd interaction without short-range correlations (dot-dashed line). The factor r^3 is to account for the volume element and the additional factor of $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$ in Eq. (3).



Average values of the Yukawa P- and T-breaking finite-range interaction calculated with the short-range corrections included

Schiff moments vs. octupole deformation





Nuclear Energy Density Functional

We consider the EDF in the form,

$$\mathcal{E} = \int d^3r \mathcal{H}(r),$$

where the energy density $\mathcal{H}(r)$ can be represented as a sum of the kinetic and iteraction energy densities,

$$\mathcal{H}(r) = \frac{\hbar^2}{2m}\tau_0(r) + \sum_{t=0,1}\mathcal{E}_t^{\rm int}(r),$$

where

$$\begin{aligned}\mathcal{E}_t^{\rm int} = \big[& C_t^\rho \rho_k^2 + C_t^{\Delta\rho}\rho_k\Delta\rho_k + C_t^\tau(\rho_k\tau_k - \vec{j}_k^{\,2}) \\ & + C_t^s \vec{s}_k^{\,2} + C_t^{\Delta s}\vec{s}_k\cdot\Delta\vec{s}_k + C_t^T(\vec{s}_k\cdot\vec{T}_k - \mathsf{J}_{abk}\mathsf{J}_{abk}) \\ & + C_t^F(\vec{s}_k\cdot\vec{F}_k - \tfrac{1}{2}\mathsf{J}_{aak}\mathsf{J}_{bbk} - \tfrac{1}{2}\mathsf{J}_{abk}\mathsf{J}_{bak}) + C_t^{\nabla s}(\vec{\nabla}\cdot\vec{s}_k)^2 \\ & + C_t^{\nabla J}(\rho_k\vec{\nabla}\cdot\vec{J}_k + \vec{s}_k\cdot(\vec{\nabla}\times\vec{j}_k))\big],\end{aligned}$$

Following the parametrization used for the Skyrme forces, we assume the dependence of the coupling parameters C_t^ρ on the isoscalar density ρ_0 as: $C_t^\rho = C_{t0}^\rho + C_{t\mathrm{D}}^\rho\rho_0^\alpha$.

Time-odd mean fields and Landau parameters

The Landau-Migdal interaction is defined as

$$\begin{aligned}
&\tilde{F}(\vec{k}_1\sigma_1\tau_1\sigma_1'\tau_1';\vec{k}_2\sigma_2\tau_2\sigma_2'\tau_2') \\
&= \frac{\delta^2\mathcal{E}}{\delta\tilde{\rho}(\vec{k}_1\sigma_1\tau_1\sigma_1'\tau_1')\delta\tilde{\rho}(\vec{k}_2\sigma_2\tau_2\sigma_2'\tau_2')} \\
&= \tilde{f}(\vec{k}_1,\vec{k}_2) + \tilde{f}'(\vec{k}_1,\vec{k}_2)\;\vec{\tau}_1\cdot\vec{\tau}_2 \\
&\quad +\tilde{g}(\vec{k}_1,\vec{k}_2)\;\vec{\sigma}_1\cdot\vec{\sigma}_2 \\
&\quad +\tilde{g}'(\vec{k}_1,\vec{k}_2)\;(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2).
\end{aligned}$$

The isoscalar-scalar, isovector-scalar, isoscalar-vector, and isovector-vector channels of the residual interaction are given by

$$\begin{aligned}
\tilde{f}(\vec{k}_1,\vec{k}_2) &= \frac{\delta^2\mathcal{E}}{\delta\tilde{\rho}_{00}(\vec{k}_1)\delta\tilde{\rho}_{00}(\vec{k}_2)} \\
\tilde{f}'(\vec{k}_1,\vec{k}_2) &= \frac{\delta^2\mathcal{E}}{\delta\tilde{\rho}_{1t_3}(\vec{k}_1)\delta\tilde{\rho}_{1t_3}(\vec{k}_2)} \\
\tilde{g}(\vec{k}_1,\vec{k}_2) &= \frac{\delta^2\mathcal{E}}{\delta\tilde{\vec{s}}_{00}(\vec{k}_1)\delta\tilde{\vec{s}}_{00}(\vec{k}_2)} \\
\tilde{g}'(\vec{k}_1,\vec{k}_2) &= \frac{\delta^2\mathcal{E}}{\delta\tilde{\vec{s}}_{1t_3}(\vec{k}_1)\delta\tilde{\vec{s}}_{1t_3}(\vec{k}_2)}
\end{aligned}$$

Assuming that only states at the Fermi surface contribute, i.e., $|\vec{k}_1| = |\vec{k}_2| = k_{\rm F}$, $\tilde{f}$, $\tilde{f}'$, $\tilde{g}$, and $\tilde{g}'$ depend on the angle θ between $\vec{k}_1$ and $\vec{k}_2$ only, and can be expanded into Legendre polynomials, e.g.

$$\tilde{f}(\vec{k}_1,\vec{k}_2) = \frac{1}{N_0}\sum_{\ell=0}^{\infty} f_\ell\, P_\ell(\theta).$$

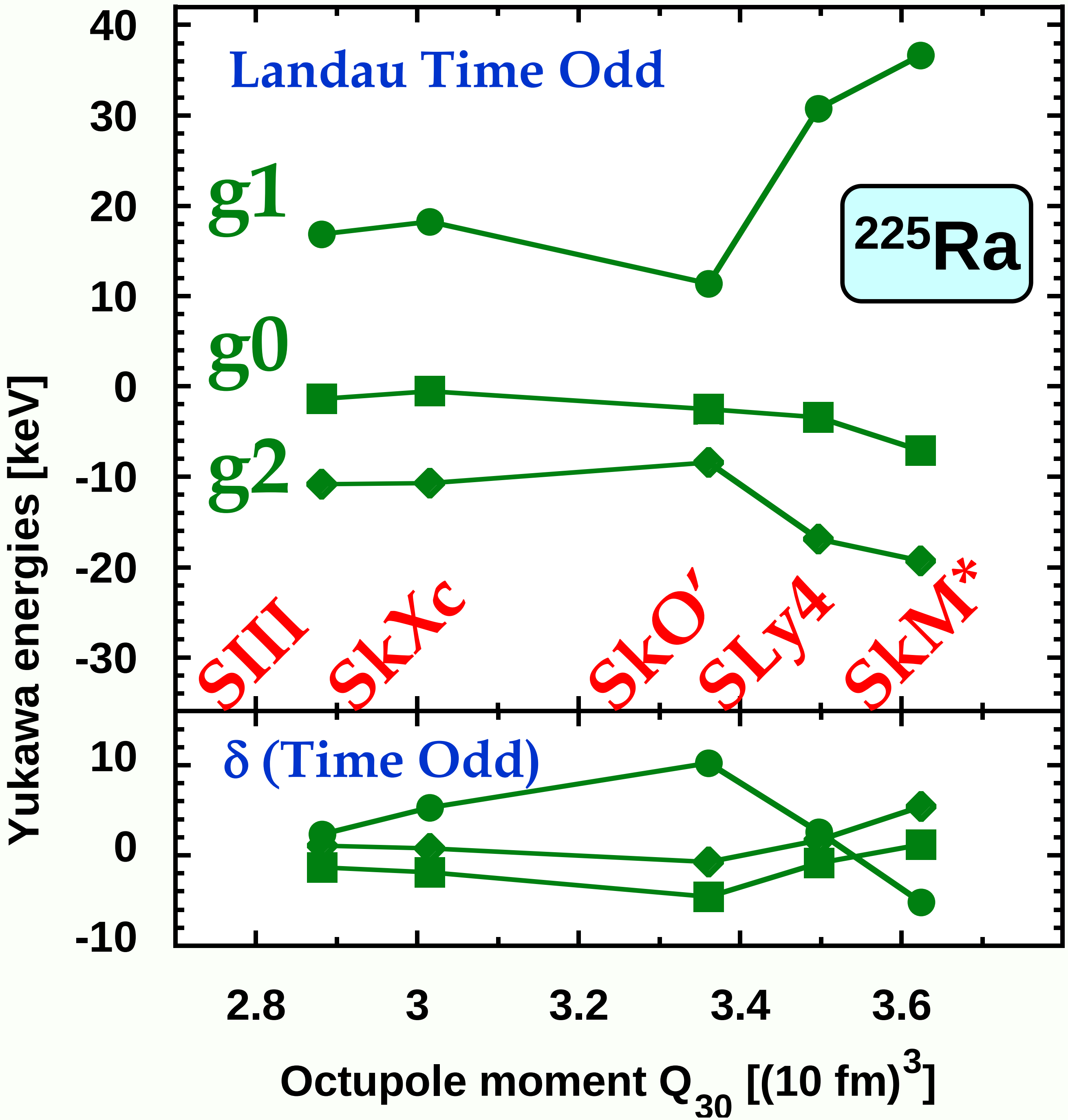
The normalization factor N_0 is the level density at the Fermi surface

$$\frac{1}{N_0} = \frac{\pi^2\hbar^2}{2m^*k_{\rm F}} \quad .$$

The Landau parameters g_0 and g_0' corresponding to the general energy functional are

$$\begin{aligned}
g_0 &= N_0(2C_0^s + 2C_0^T\,\beta\,\rho_{00}^{2/3}), \\
g_0' &= N_0(2C_1^s + 2C_1^T\,\beta\,\rho_{00}^{2/3}),
\end{aligned}$$

g_0=0.4 g_0'=1.2



The time-odd terms of the energy density functional included by conserving the gauge invariance and reproducing the Landau parameters

The Bottom Line

^{225}Ra

TABLE I. Coefficients of $g\bar{g}_i$, in units of $e\,\mathrm{fm}^3$, in the expression Eq. (5) for the Schiff moment of ^{225}Ra, calculated with the SkO′ Skyrme interaction. The abbreviation "src" stands for "-short-range correlations."

	a_0	a_1	a_2
Zero range (direct only)	−5.1	10.4	−10.1
Finite range (direct only)	−1.9	6.3	−3.8
Finite range + src (direct only)	−1.7	6.0	−3.5
Finite range + src (direct+exchange)	−1.5	6.0	−4.0

SIII	−1.0	7.0	−3.9
SkM*	−4.7	21.5	−11.0
SLy4	−3.0	16.9	−8.8

J.D. & J. Engel, Phys. Rev. Lett. 94, 232502 (2005)

Schiff moment in symmetric nuclei

^{199}Hg

Three equivalent methods can be used:

a) Solving self-consistent field equations with $H \equiv H_{\rm Skyrme} + \lambda V_{PT}$, and then evaluating the expectation value of $\vec{S}/\lambda$,

b) Solving the mean-field equations with $H \equiv H_{\rm Skyrme} + \lambda\vec{S}$, and then evaluating the expectation value of V_{PT}/λ.

c) Solving the mean-field equations with $H \equiv H_{\rm Skyrme}$, and then evaluating $\sum_i \frac{\langle 0|\vec{S}|i\rangle_{\rm RPA}\langle i|V_{PT}|0\rangle_{\rm RPA}}{(E_0 - E_i)} + c.c\,,$

The Bottom Line

^{199}Hg

TABLE IV. Results for coefficients a_i and b, in $e\,\text{fm}^3$, in ^{199}Hg. The third column gives ground-state energy in mega-electron-volts, the fourth the deformation, and the fifth the excitation energy (also in mega-electron-volts) of the lowest configuration with the same value of Ω^π as the experimental ground state. The first three lines are in the HF approximation, and the next two are in the HFB approximation. The last two lines report results of previous work, with the numbers for Ref. [8] representing the average over several interactions.

	E_{gs}	β	$E_{\text{exc.}}$	a_0	a_1	a_2	b
SLy4	−1561.42	−0.13	0.97	0.013	−0.006	0.022	0.003
SIII	−1562.63	−0.11	0	0.012	0.005	0.016	0.004
SV	−1556.43	−0.11	0.68	0.009	−0.0001	0.016	0.002
SLy4	−1560.21	−0.10	0.83	0.013	−0.006	0.024	0.007
SkM*	−1564.03	0	0.82	0.041	−0.027	0.069	0.013

S. Ban *et al.*, Phys. Rev. C82, 015501 (2010)

Error estimates of Schiff moments in ^{225}Ra

Parity-symmetry restoration

Insignificant

Rotational-symmetry restoration

Error ~2% – rotational approximation excellent.

Configurations

Single-particle states not in the correct order, necessity to consider excited configurations

Phase space

Error ~1‰ for calculations with N=20 HO shells

Short-range correlations

Error unknown

Pairing

Error 20-30%, for the Schiff moment correlated with the octupole deformation

EDF parameterisation, time even

Error ~50%, for the Schiff moment correlated with the octupole deformation

EDF parameterisation, time odd

Error ~50%

Time-dependent fine-structure constant α

Accelerating Universe may imply the time-dependent fine-structure constant. Some evidence is available in astronomical observations.

With varying α, all eigen-energies of charged quantum systems vary in time:

$$\frac{dE}{dt} = \dot{\alpha}\frac{\partial E}{\partial \alpha} \equiv \frac{\dot{\alpha}}{\alpha}V_C.$$

Therefore, transition energies vary with rates given by differences of Coulomb energies V_C:

$$\frac{d\Delta E}{dt} \equiv \frac{\dot{\alpha}}{\alpha}\Delta V_C.$$

Structure of ^{229}Th

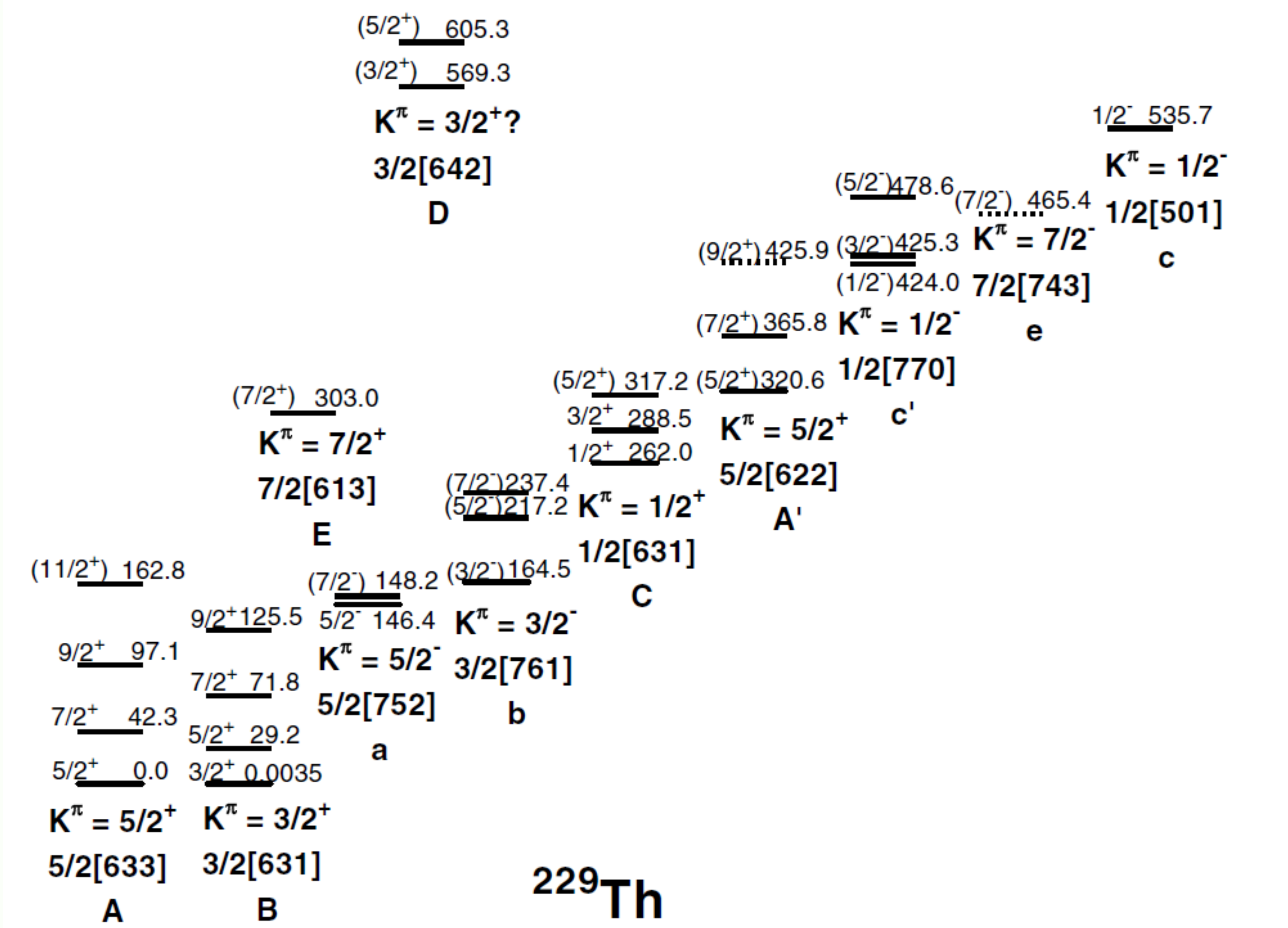
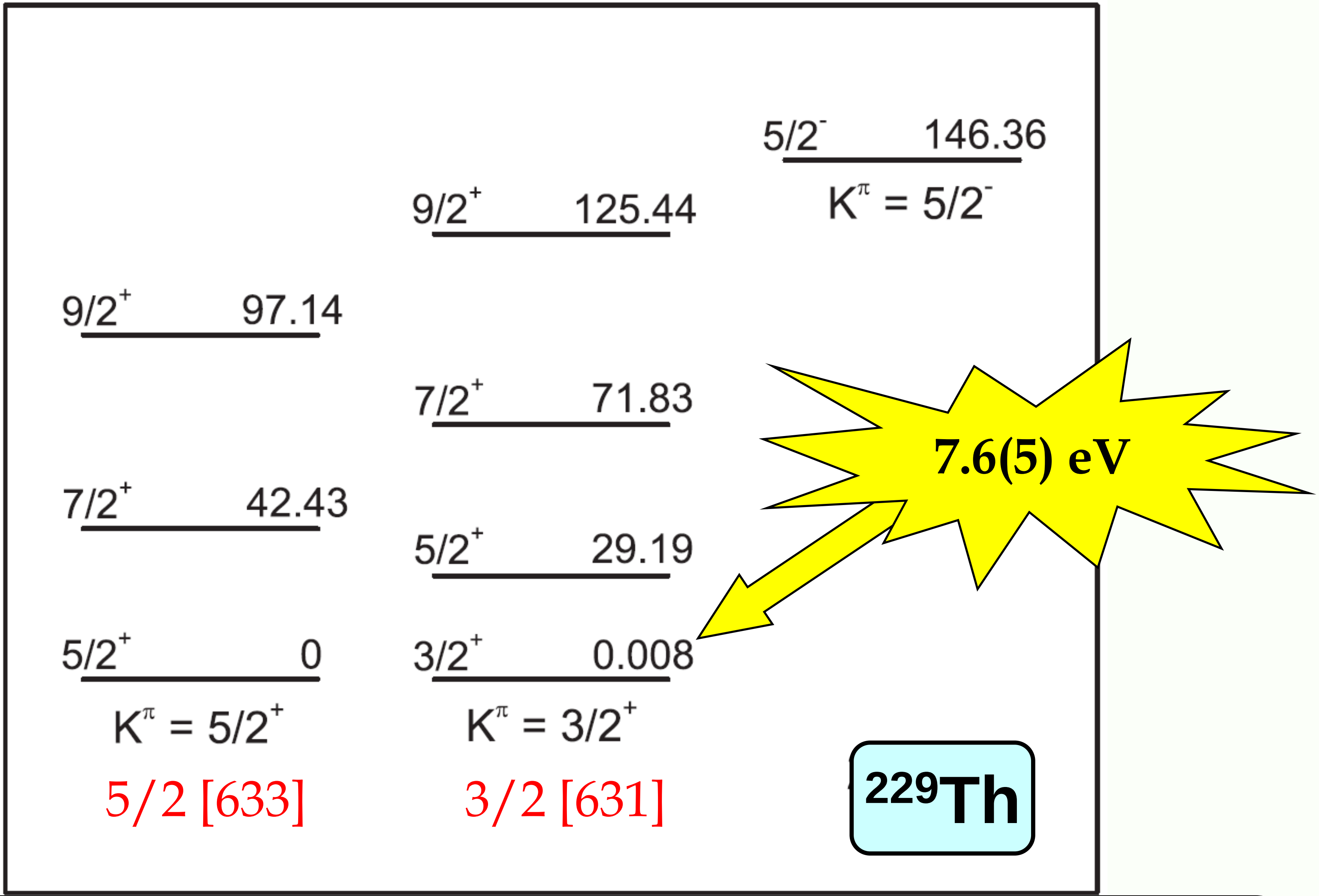


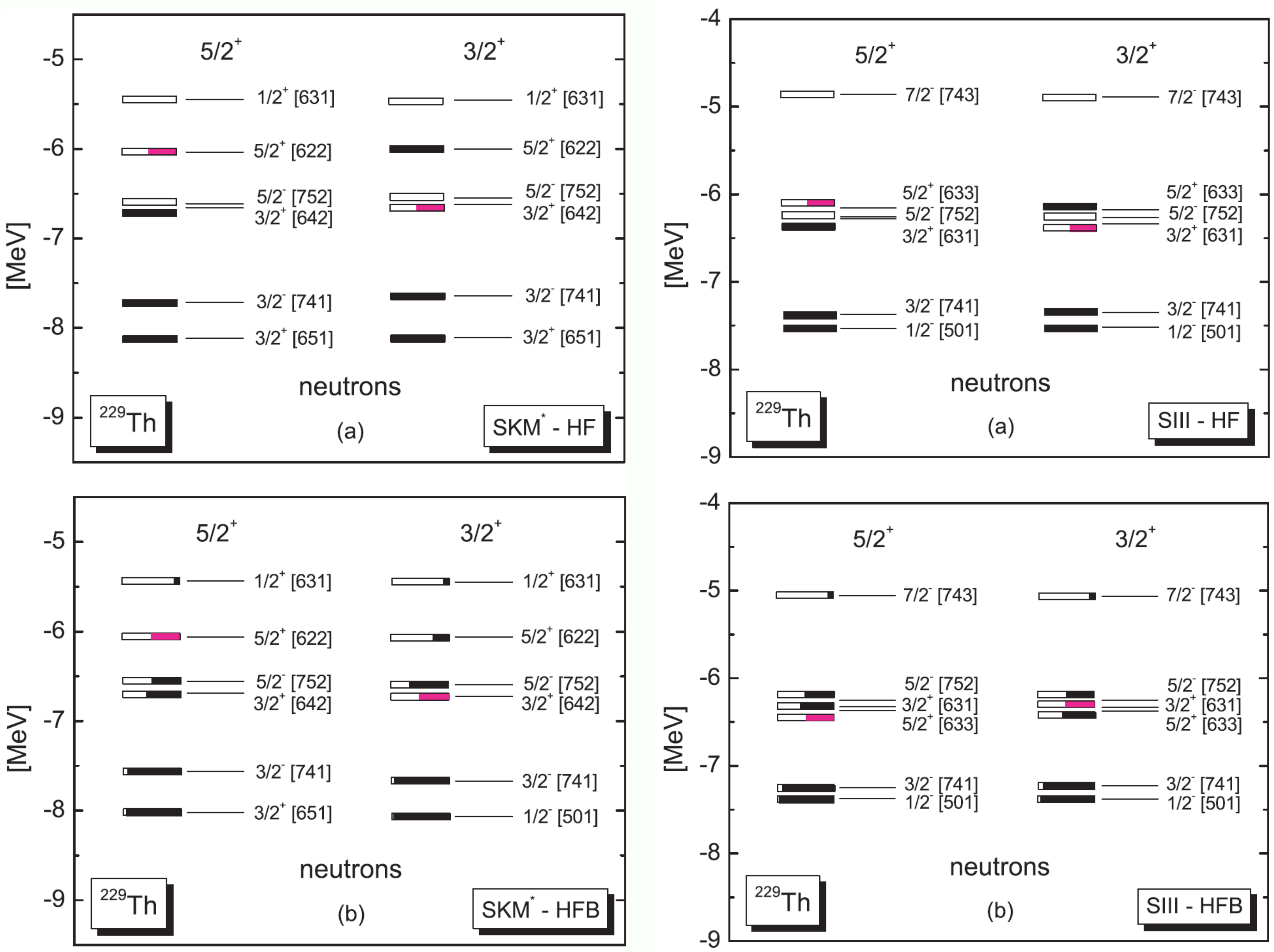
FIG. 7. Experimental levels in ^{229}Th grouped into rotational bands. Dashed lines represent levels populated in the α decay [8,9,16]. Each band is labeled by the dominant single-quasiparticle component of the wave function as obtained in the calculations in Sec. IV C. The structure of the intrinsic states forming the base for each rotational band is given in Table III. The pairs of bands labeled by the same small and capital letters are interpreted as parity partner bands.

E. Ruchowska, W.A. Płóciennik, J. Żylicz, *et al.*, Phys. Rev. C73, 044326 (2006)



B.R. Beck *et al.*, Phys. Rev. Lett. 98, 142501 (2007)

NASA/electron beam ion trap x-ray microcalorimeter spectrometer with an experimental energy resolution of 26 eV (FWHM) used. A difference technique was applied to the gamma ray decay of the 71.83 keV level that populates both members of the doublet.



E. Litvinova *et al.*, Phys. Rev. C79, 064303 (2009)

Differences of Coulomb energies in ^{229}Th

TABLE I. Total, Coulomb, neutron, and proton kinetic energies of the ^{229}Th $5/2^+$ ground state calculated with different energy functionals. Differences of these energies between $3/2^+$ first excited state and $5/2^+$ ground state.

	Exp.	SkM*		SIII		NL3 RH
		HF	HFB	HF	HFB	
$5/2^+$	Ref. [40]					
E^{tot} (MeV)	−1748.334	−1739.454	−1747.546	−1741.885	−1748.016	−1745.775
V_C (MeV)		923.927	924.854	912.204	912.216	948.203
T_n (MeV)		2785.404	2800.225	2783.593	2794.909	2059.640
T_p (MeV)		1458.103	1512.705	1442.018	1477.485	1106.697
$3/2^+ - 5/2^+$	Ref. [12]					
ΔE^{tot}(MeV)	0.000 008	0.619	−0.046	0.141	−0.074	2.407
ΔV_C (MeV)		0.451	−0.307	−0.098	0.001	1.011
ΔT_n (MeV)		2.570	0.954	−0.728	0.087	−2.181
ΔT_p (MeV)		0.688	0.233	−0.163	−0.022	−1.996

Conclusions

- Calculations of the nuclear Schiff moments bring invaluable information on the link between the hypothetical T-breaking NN interactions and prospective measurements of the atomic electric dipole moments.

- Present-day uncertainties in the nuclear-physics interactions and methods allow for the determination of the Schiff moments up to a factor of about 2.

- Calculation of the charge polarizations in two nuclear configurations of ^{229}Th allow us to evaluate the possibility of measuring the time-variation of the fine-structure constant α.

- Pairing effects are essential in estimating the Coulomb effects in configurations close to the Fermi surface – they dramatically decrease the differences of the corresponding Coulomb energies.

- Applications of nuclear-structure methods and expertise to fundamental problems of physics are important, easy, and rewarding

Thank you

The T Operator in QM is Different

.

- Not linear:

$$T[x,p]T^{-1} = -[x,p]$$

  so i is odd under T.

- Has no eigenstates in the conventional sense:

$$T|a\rangle = |a\rangle \quad \longrightarrow \quad T(\alpha|a\rangle) = \alpha^* T|a\rangle = \alpha^*|a\rangle \neq \alpha|a\rangle$$

  for α complex

- Typical physical states $|J,M\rangle$ not even close to eigenstates of T

As a result, T violation doesn't show up as “mixing of states with opposite T”

What Do EDM's Have to Do With T

Consider nondegenerate ground state $|g:J,M\rangle$. Symmetry under rotations $R_y(\pi) \Rightarrow$ for a vector operator like $\vec{d} \equiv \sum_i e_i \vec{r}_i$,

$$\langle g:J,M|\vec{d}|g:J,M\rangle = -\langle g:J,-M|\vec{d}|g:J,-M\rangle \ .$$

T takes M to $-M$, like $R_y(\pi)$. But $\vec{d}$ is *odd* under $R_y(\pi)$ and *even* under T, so for T conserved

$$\langle g:J,M|\vec{d}|g:J,M\rangle = +\langle g:J,-M|\vec{d}|g:J,-M\rangle \ .$$

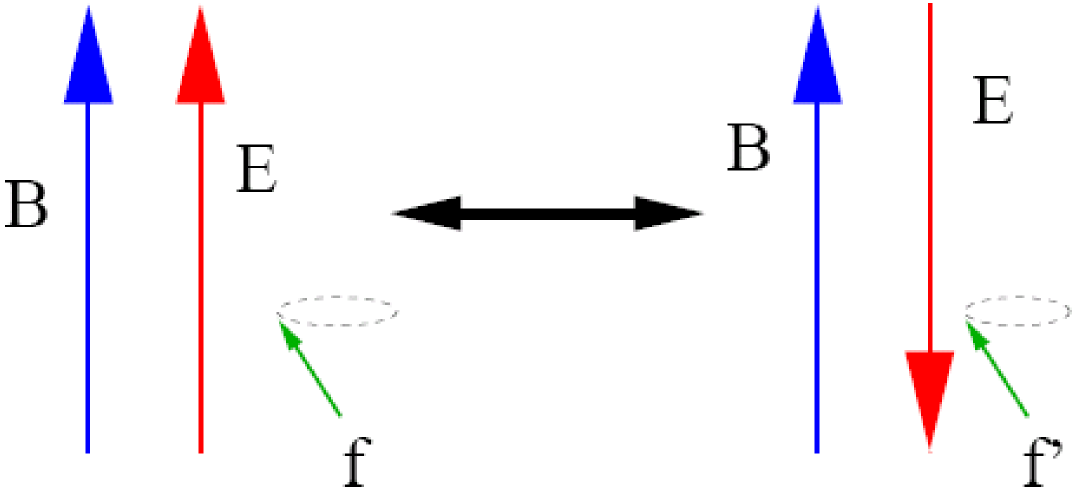
Together with the first equation, this implies

$$\langle \vec{d} \rangle = 0 \ .$$

If T is violated, argument fails because T can take $|g:JM\rangle$ to a *different* state with $J,-M$.

There are EDM Experiments on Neutrons, Atoms . . .

Basic principle:



$$H = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

and there is a change in precession frequency (linear in E) when $\vec{E}$ is flipped.

Shielding by Electrons

Unfortunately for atomic experiments

Theorem (Schiff)

The nuclear dipole moment causes the atomic electrons to rearrange themselves so that they develop a dipole moment opposite that of the nucleus. In the limit of nonrelativistic electrons and a point nucleus the electrons' dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes!

Skip proof

Proof

Consider atom with nonrelativistic constituents (with dipole moments $\vec{d}_k$) held together by electrostatic forces. The atom has a "bare" edm $\boxed{\vec{d} \equiv \sum_k \vec{d}_k}$ and a Hamiltonian

$$\begin{aligned} H &= \sum_k \frac{p_k^2}{2m_k} + \sum_k V(\vec{r}_k) \quad - \sum_k \vec{d}_k \cdot \vec{E}_k \\ &= \qquad H_0 \qquad + \sum_k (1/e_k)\vec{d}_k \cdot \vec{\nabla} V(\vec{r}_k) \\ &= \qquad H_0 \qquad + i \sum_k (1/e_k) \left[\vec{d}_k \cdot \vec{p}_k, H_0\right] \\ & \qquad\quad \Uparrow \qquad\qquad\qquad\qquad \Uparrow \\ & \text{K.E. } + \text{ Coulomb} \qquad \text{dipole perturbation} \end{aligned}$$

Shielding by Electrons

The perturbing Hamiltonian

$$H_d = i \sum_k (1/e_k) \left[\vec{d}_k \cdot \vec{p}_k, H_0 \right]$$

shifts the ground state $|0\rangle$ to

$$\begin{aligned}
|\tilde{0}\rangle &= |0\rangle + \sum_m \frac{|m\rangle\langle m|H_d|0\rangle}{E_0 - E_m} \\
&= |0\rangle + \sum_m \frac{|m\rangle\langle m| i \sum_k (1/e_k) \vec{d}_k \cdot \vec{p}_k |0\rangle (E_0 - E_m)}{E_0 - E_m} \\
&= \left(1 + i \sum_k (1/e_k) \vec{d}_k \cdot \vec{p}_k \right) |0\rangle
\end{aligned}$$

Shielding by Electrons

The induced dipole moment $\vec{d}'$ is

$$\vec{d}' = \langle\tilde{0}|\sum_j e_j \vec{r}_j|\tilde{0}\rangle$$

$$= \langle 0|\left(1 - i\textstyle\sum_k (1/e_k)\vec{d}_k\cdot\vec{p}_k\right)\left(\textstyle\sum_j e_j\vec{r}_j\right) \times \left(1 + i\textstyle\sum_k (1/e_k)\vec{d}_k\cdot\vec{p}_k\right)|0\rangle$$

$$= i\langle 0|\left[\textstyle\sum_j e_j\vec{r}_j, \textstyle\sum_k (1/e_k)\vec{d}_k\cdot\vec{p}_k\right]|0\rangle$$

$$= -\langle 0|\sum_k \vec{d}_k|0\rangle \quad = \quad -\sum_k \vec{d}_k$$

$$= -\vec{d}$$

So the net EDM is zero!

All is Not Lost, Though. . .

Th nucleus has finite size. Shielding is not complete, and nuclear T violation can still induce atomic EDM $\vec{d}$.

Post-screening nucleus-electron interaction doesn't explicitly involve the nuclear EDM $\vec{D}$, but rather a related quantity:

The nuclear "Schiff moment"

$$\vec{S} \equiv \sum_p e_p \left(r_p^2 - \frac{5}{3} \langle R_{\rm ch}^2 \rangle \right) \vec{r}_p \ .$$

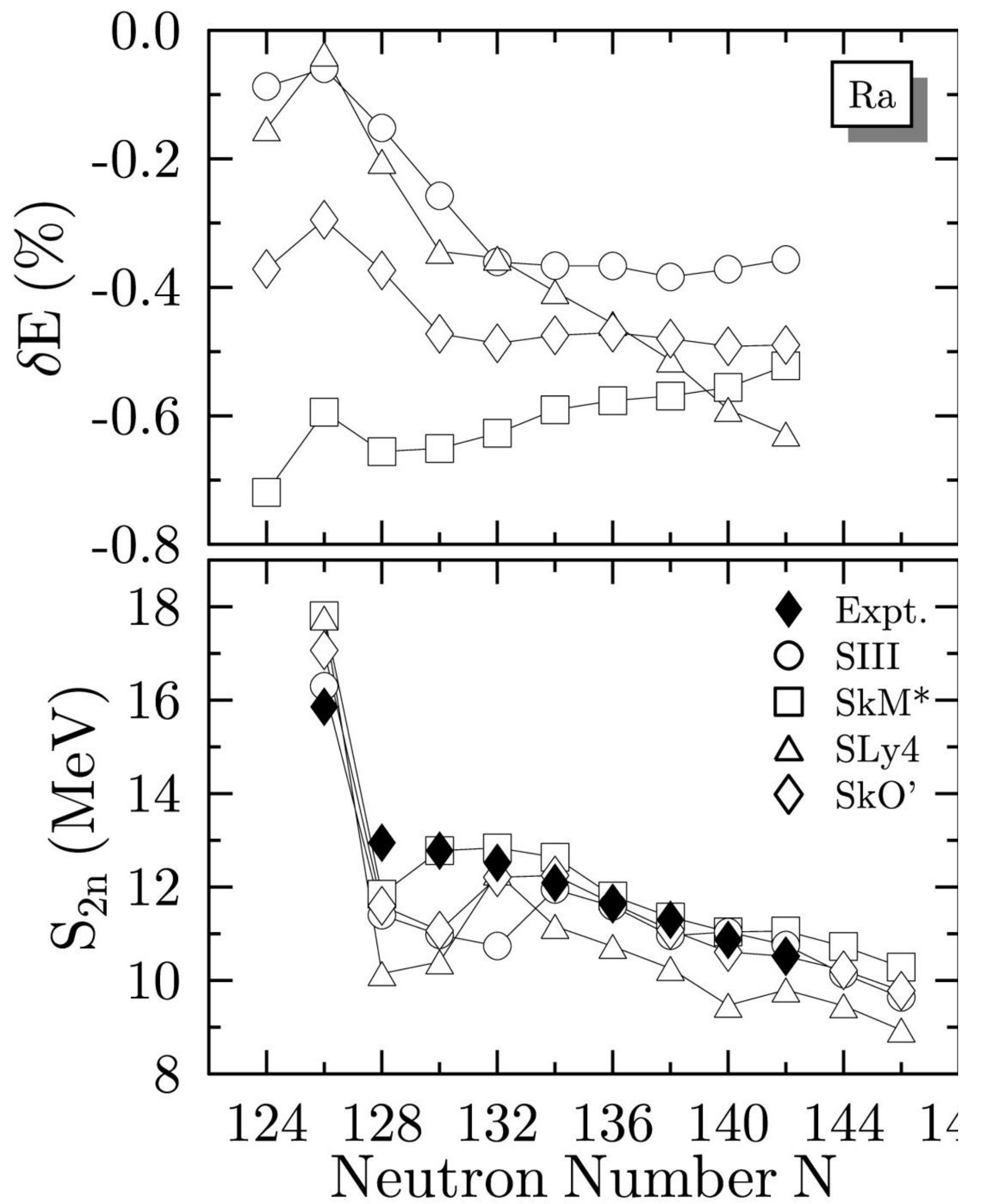
If, as you'd expect, $\langle \vec{S} \rangle \approx R_N^2 \langle \vec{D} \rangle$, then $\vec{d}$ is down from $\langle \vec{D} \rangle$ by

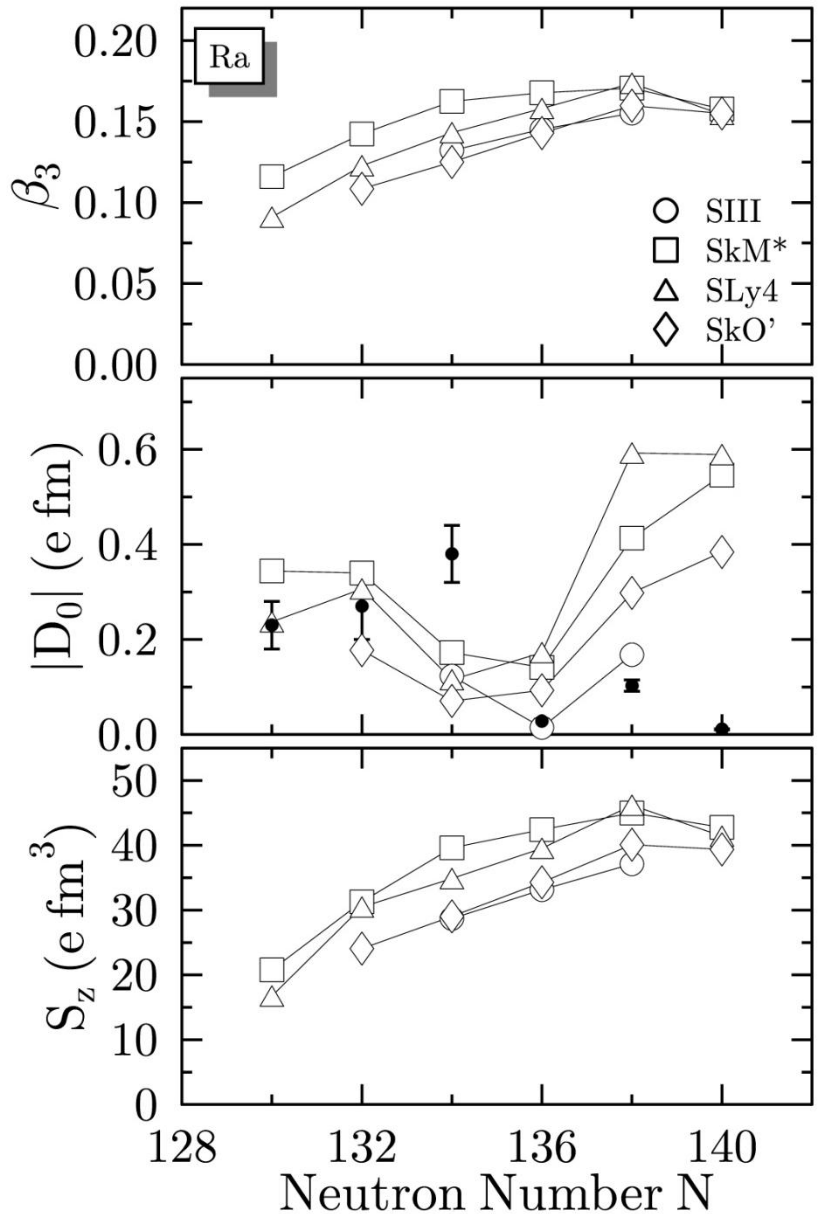
$$O\left(R_N^2 / R_A^2\right) \approx 10^{-8} \ .$$

Ughh! Fortunately the large nuclear charge and relativistic wave functions offset this factor by $10Z^2 \approx 10^5$.

Overall suppression of $\langle \vec{D} \rangle$ is only about 10^{-3}.

Ground-state properties





J. Engel *et al.*, Phys. Rev. C68, 025501 (2003)

Nuclear densities as composite fields

Density matrix:

$$\rho(\vec{r}\sigma,\vec{r}'\sigma') = \langle\Phi|a^+(\vec{r}'\sigma')a(\vec{r}\sigma)|\Phi\rangle$$

Scalar and vector part:

$$\rho(\vec{r},\vec{r}') = \sum_\sigma \rho(\vec{r}\sigma,\vec{r}'\sigma)$$

$$\vec{s}(\vec{r},\vec{r}') = \sum_{\sigma\sigma'} \rho(\vec{r}\sigma,\vec{r}'\sigma')\langle\sigma'|\vec{\sigma}|\sigma\rangle$$

Symmetries:

$$\rho^T(\vec{r},\vec{r}') = \rho^*(\vec{r},\vec{r}') = \rho(\vec{r}',\vec{r})$$

$$\vec{s}^T(\vec{r},\vec{r}') = -\vec{s}^*(\vec{r},\vec{r}') = -\vec{s}(\vec{r}',\vec{r})$$

Local densities:

Matter:	$\rho(\vec{r}) = \rho(\vec{r},\vec{r})$
Momentum:	$\vec{j}(\vec{r}) = (1/2i)[(\vec{\nabla}-\vec{\nabla}')\rho(\vec{r},\vec{r}')]_{r=r'}$
Kinetic:	$\tau(\vec{r}) = [\vec{\nabla}\cdot\vec{\nabla}'\rho(\vec{r},\vec{r}')]_{r=r'}$
Spin:	$\vec{s}(\vec{r}) = \vec{s}(\vec{r},\vec{r})$
Spin momentum:	$J_{\mu\nu}(\vec{r}) = (1/2i)[(\nabla_\mu-\nabla'_\mu)s_\nu(\vec{r},\vec{r}')]_{r=r'}$
Spin kinetic:	$\vec{T}(\vec{r}) = [\vec{\nabla}\cdot\vec{\nabla}'\vec{s}(\vec{r},\vec{r}')]_{r=r'}$
Tensor kinetic:	$\vec{F}(\vec{r}) = \frac{1}{2}[(\vec{\nabla}\otimes\vec{\nabla}'+\vec{\nabla}'\otimes\vec{\nabla})\cdot\vec{s}(\vec{r},\vec{r}')]_{r=r'}$

Complete local energy density

E. Perlińska, *et al.*, Phys. Rev. C69 (2004) 014316

The energy density can be written in the following form:

$$\mathcal{H}(\vec{r}) = \frac{\hbar^2}{2m}\tau_0(\vec{r}) + \sum_{t=0,1}\left(\chi_t(\vec{r}) + \breve{\chi}_t(\vec{r})\right),$$

The p-h and p-p interaction energy densities, $\chi_t(\vec{r})$ and $\breve{\chi}_t$, for t=0 depend quadratically on the isoscalar densities, and for t=1 – on the isovector ones. Based on general rules of constructing the energy density, one obtains

Mean field

$$\begin{aligned}
\chi_0(\vec{r}) &= C_0^\rho \rho_0^2 + C_0^{\Delta\rho}\rho_0\Delta\rho_0 + C_0^\tau \rho_0\tau_0 \\
&+ C_0^{J0} J_0^2 + C_0^{J1}\vec{J}_0^2 + C_0^{J2}\underline{\mathsf{J}}_0^2 + C_0^{\nabla J}\rho_0\vec{\nabla}\cdot\vec{J}_0 \\
&+ C_0^s \vec{s}_0^2 + C_0^{\Delta s}\vec{s}_0\cdot\Delta\vec{s}_0 + C_0^T\vec{s}_0\cdot\vec{T}_0 \\
&+ C_0^j \vec{j}_0^2 + C_0^{\nabla j}\vec{s}_0\cdot(\vec{\nabla}\times\vec{j}_0) \\
&+ C_0^{\nabla s}(\vec{\nabla}\cdot\vec{s}_0)^2 + C_0^F\vec{s}_0\cdot\vec{F}_0, \\
\chi_1(\vec{r}) &= C_1^\rho \vec{\rho}^{\,2} + C_1^{\Delta\rho}\vec{\rho}\circ\Delta\vec{\rho} + C_1^\tau\vec{\rho}\circ\vec{\tau} \\
&+ C_1^{J0}\vec{J}^2 + C_1^{J1}\vec{\vec{J}}^{\,2} + C_1^{J2}\vec{\underline{\mathsf{J}}}^{\,2} + C_1^{\nabla J}\vec{\rho}\circ\vec{\nabla}\cdot\vec{\vec{J}} \\
&+ C_1^s\vec{\vec{s}}^{\,2} + C_1^{\Delta s}\vec{\vec{s}}\cdot\circ\Delta\vec{\vec{s}} + C_1^T\vec{\vec{s}}\cdot\circ\vec{\vec{T}} \\
&+ C_1^j\vec{\vec{j}}^{\,2} + C_1^{\nabla j}\vec{\vec{s}}\cdot\circ(\vec{\nabla}\times\vec{\vec{j}}) \\
&+ C_1^{\nabla s}(\vec{\nabla}\cdot\vec{\vec{s}})^2 + C_1^F\vec{\vec{s}}\cdot\circ\vec{\vec{F}},
\end{aligned}$$

where $\times$ stands for the vector product

Pairing

$$\begin{aligned}
\breve{\chi}_0(\vec{r}) &= \breve{C}_0^s|\breve{\vec{s}}_0|^2 + \breve{C}_0^{\Delta s}\Re(\breve{\vec{s}}_0^{\,*}\cdot\Delta\breve{\vec{s}}_0) \\
&+ \breve{C}_0^T\Re(\breve{\vec{s}}_0^{\,*}\cdot\breve{\vec{T}}_0) + \breve{C}_0^j|\breve{\vec{j}}_0|^2 \\
&+ \breve{C}_0^{\nabla j}\Re(\breve{\vec{s}}_0^{\,*}\cdot(\vec{\nabla}\times\breve{\vec{j}}_0)) \\
&+ \breve{C}_0^{\nabla s}|\vec{\nabla}\cdot\breve{\vec{s}}_0|^2 \\
&+ \breve{C}_0^F\Re(\breve{\vec{s}}_0^{\,*}\cdot\breve{\vec{F}}_0), \\
\breve{\chi}_1(\vec{r}) &= \breve{C}_1^\rho|\breve{\vec{\rho}}|^2 + \breve{C}_1^{\Delta\rho}\Re(\breve{\vec{\rho}}^{\,*}\circ\Delta\breve{\vec{\rho}}) \\
&+ \breve{C}_1^\tau\Re(\breve{\vec{\rho}}^{\,*}\circ\breve{\vec{\tau}}) \\
&+ \breve{C}_1^{J0}|\breve{\vec{J}}|^2 + \breve{C}_1^{J1}|\breve{\vec{\vec{J}}}|^2 \\
&+ \breve{C}_1^{J2}|\breve{\vec{\underline{\mathsf{J}}}}|^2 \\
&+ \breve{C}_1^{\nabla J}\Re(\breve{\vec{\rho}}^{\,*}\circ\vec{\nabla}\cdot\breve{\vec{\vec{J}}}).
\end{aligned}$$

We briefly review some definitions and ideas. The Schiff moment is given accurately by the first-order expression

$$S \equiv \langle \Psi_0 | \hat{S}_z | \Psi_0 \rangle = \sum_{i \neq 0} \frac{\langle \Psi_0 | \hat{S}_z | \Psi_i \rangle \langle \Psi_i | \hat{V}_{\rm PT} | \Psi_0 \rangle}{E_0 - E_i} + {\rm c.c.}, \quad (1)$$

where $|\Psi_0\rangle$ is the member of the ground-state multiplet with $J_z = J = 1/2$ (positive parity), the sum is over excited states, and $\hat{S}_z$ is the operator

$$\hat{S}_z = \frac{e}{10} \sum_p \left(r_p^2 - \frac{5}{3} \bar{r}_{\rm ch}^2 \right) z_p, \quad (2)$$

with the sum here over protons, and $\bar{r}_{\rm ch}^2$ the mean-square charge radius. The operator $\hat{V}_{\rm PT}$ in Eq. (1) is the T- (and parity-) violating nucleon-nucleon interaction mediated by the pion [7,15]:

$$\hat{V}_{\rm PT}(\boldsymbol{r}_1 - \boldsymbol{r}_2) = -\frac{g m_\pi^2}{8\pi m_N} \left\{ (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2) \left[\bar{g}_0 \vec{\tau}_1 \cdot \vec{\tau}_2 - \frac{\bar{g}_1}{2} (\tau_{1z} + \tau_{2z}) + \bar{g}_2 (3\tau_{1z}\tau_{2z} - \vec{\tau}_1 \cdot \vec{\tau}_2) \right] - \frac{\bar{g}_1}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)(\tau_{1z} - \tau_{2z}) \right\} \frac{\exp(-m_\pi |\boldsymbol{r}_1 - \boldsymbol{r}_2|)}{m_\pi |\boldsymbol{r}_1 - \boldsymbol{r}_2|^2} \left[1 + \frac{1}{m_\pi |\boldsymbol{r}_1 - \boldsymbol{r}_2|} \right], \quad (3)$$

where arrows denote isovector operators, τ_z is $+1$ for neutrons, m_N is the nucleon mass, and (in this equation only) we use the convention $\hbar = c = 1$. The $\bar{g}$'s are the unknown isoscalar, isovector, and isotensor T-violating pion-nucleon coupling constants, and g is the usual strong πNN coupling constant.

The asymmetric shape of ^{225}Ra implies parity doubling (see, e.g., Ref. [16]), i.e., the existence of a very low-energy $|1/2^-\rangle$ state, in this case 55 keV [17] above the ground state $|\Psi_0\rangle \equiv |1/2^+\rangle$, that dominates the sum in

$$S \approx -\frac{2}{3} \langle \hat{S}_z \rangle \frac{\langle \hat{V}_{\rm PT} \rangle}{(55~{\rm keV})}, \quad (4)$$

where the brackets indicate expectation values in the intrinsic state. Using Eq. (3) for $\hat{V}_{\rm PT}$, we can express the dependence of the Schiff moment on the undetermined T-violating πNN vertices as

$$S = a_0 g \bar{g}_0 + a_1 g \bar{g}_1 + a_2 g \bar{g}_2. \quad (5)$$

The coefficients a_i, which are the result of the calculation,