

# Linear response in infinite matter with tensor force

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# Outline

1 Introduction

2 Landau

3 RPA:  $V_{ph}=0$

4 RPA:  $V_{ph} \neq 0$

5 Summary

# Introduction

The tensor part is an essential part of the NN interaction.

- Early studies in mean-field framework:
  - T.H.R. Skyrme included it
  - F. Stancu, H. Flocard, D.M. Brink, PLB (1977)
  - K.F. Liu et al. NPA (1991)
- Recent attempts at adding a tensor term to mean field/density functional model:
  - Gogny: T. Otsuka et al. PRL (2005)
  - RHF: W.H. Long, N.V. Giai and J. Meng, PLB (2006)
  - Skyrme EDF:
    - B.A. Brown et al. PRC (2006)
    - T. Lesinski et al. PRC (2007)
    - M. Zalewski et al. PRC (2008)
    - J. Dobaczewski arxiv (2006)
    - ...

# Introduction: tensor

In the original paper of T.H.R Skyrme reads

$$\begin{aligned} V^T &= \frac{T}{2} \left\{ \left[ (\sigma_1 \cdot \mathbf{k}')(\sigma_2 \cdot \mathbf{k}') - \frac{1}{3}(\sigma_1 \cdot \sigma_2)\mathbf{k}'^2 \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) \right. \\ &+ \delta(\mathbf{r}_1 - \mathbf{r}_2) \left. \left[ (\sigma_1 \cdot \mathbf{k})(\sigma_2 \cdot \mathbf{k}) - \frac{1}{3}(\sigma_1 \cdot \sigma_2)\mathbf{k}^2 \right] \right\} \\ &+ \frac{U}{2} \left\{ (\sigma_1 \cdot \mathbf{k}')\delta(\mathbf{r}_1 - \mathbf{r}_2)(\sigma_2 \cdot \mathbf{k}) + (\sigma_2 \cdot \mathbf{k}')\delta(\mathbf{r}_1 - \mathbf{r}_2)(\sigma_1 \cdot \mathbf{k}) \right\} \\ &- \frac{2}{3} [(\sigma_1 \cdot \sigma_2)\mathbf{k}' \cdot \delta(\mathbf{r} - \mathbf{r}')\mathbf{k}] \end{aligned}$$

or in modern Functional language

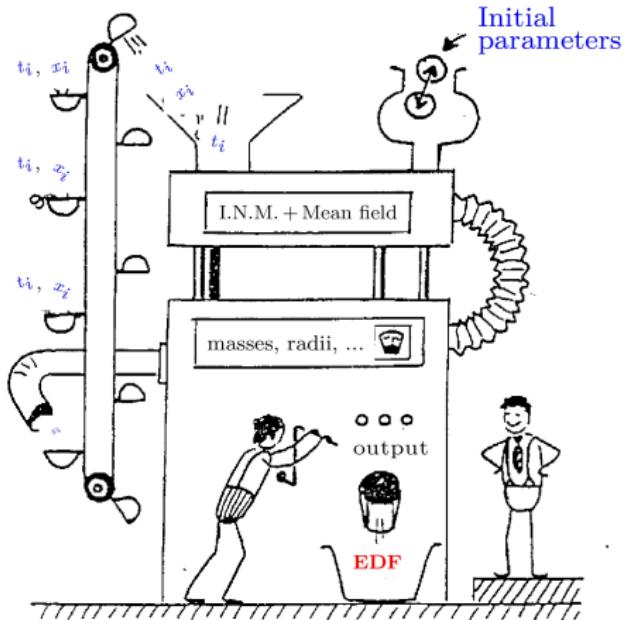
$$\begin{aligned} \mathcal{H}^{\text{tensor}} &= B_t^T \left( \mathbf{s}_t \mathbf{T}_t - \sum_{\mu, \nu=x}^z J_{t,\mu\nu} J_{t,\mu\nu} \right) + B_t^{\nabla s} (\nabla \cdot \mathbf{s}_t)^2 + B_t^{\Delta s} \mathbf{s}_t \Delta \mathbf{s}_t \\ &+ B_t^F \left[ \mathbf{s}_t \mathbf{F}_t - \frac{1}{2} \left( \sum_{\mu=x}^z J_{t\mu\mu} \right)^2 - \frac{1}{2} \sum_{\mu, \nu=x}^z J_{t,\mu\nu} J_{t,\nu\mu} \right] \end{aligned}$$

[E . Perlinska et al. Phys. Rev C 69, 014316 (2004)) ]

[D . Davesne et. al Phys. Rev C (2009) ]

# Fitting the parameters of the functional

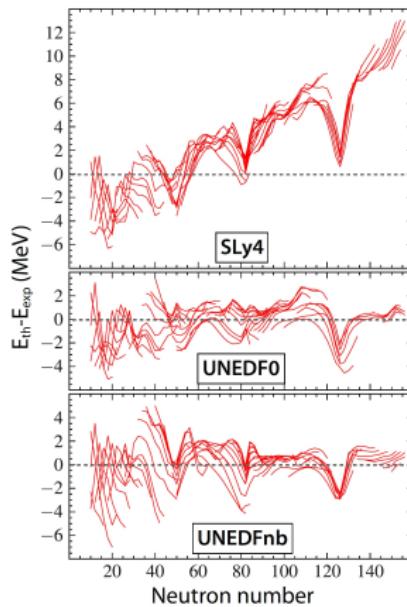
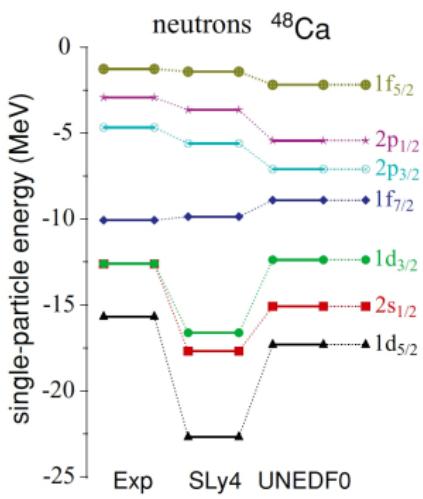
EDF fitting...



**Figure:** Picture by J. Dechargé, from “Approches de champ moyen et au-delà”, J.-F. Berger, École Joliot-Curie: “Les noyaux en pleine forme”, 1991.

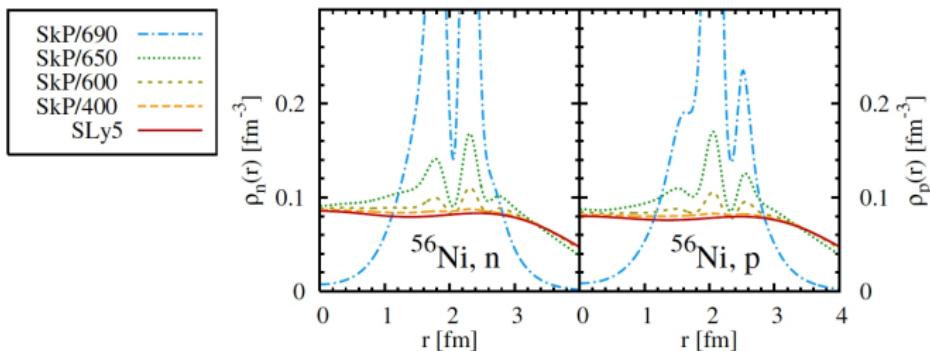
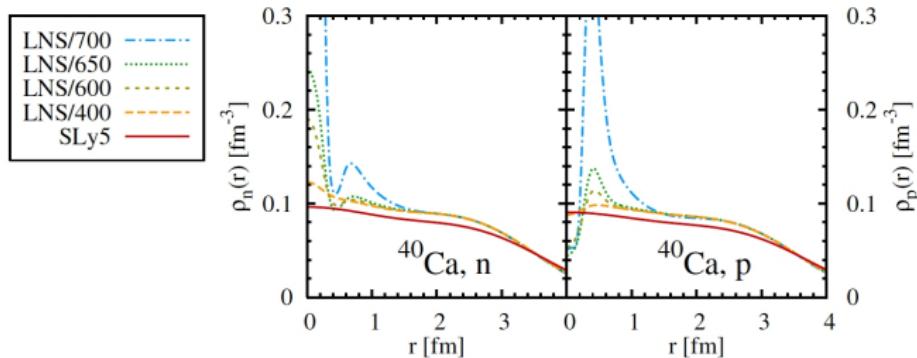
## Fitting the parameters of the functional

Obtaining a good set of (stable) parameters is not an easy task



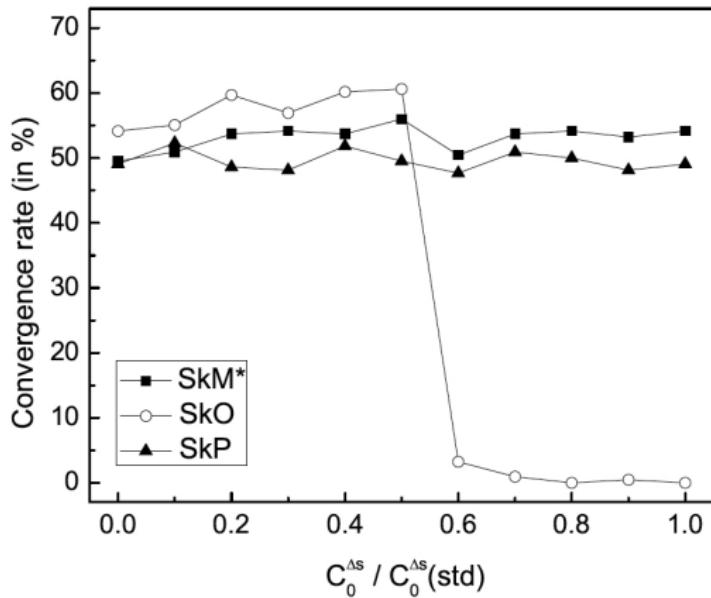
# Instabilities

Instabilities can occur with the Skyrme functional, but not easy to detect!



# Instabilities

... especially in odd systems



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# Landau parameters to detect instabilities

As a first approximation we can derive the Landau parameters from a Skyrme force [L.G. Cao et al. Phys. Rev C 81, 044302 (2010)]

$$V_{ph}^{\text{Landau}}(\mathbf{q}, \mathbf{q}_1, \mathbf{q}_2) = \lim_{\mathbf{q} \rightarrow 0, \mathbf{q}_{1,2} \rightarrow k_F} V_{ph}(\mathbf{q}, \mathbf{q}_1, \mathbf{q}_2)$$

According to Liu [K.-F. Liu et al. Nucl. Phys. A534 (1991) 1-24]

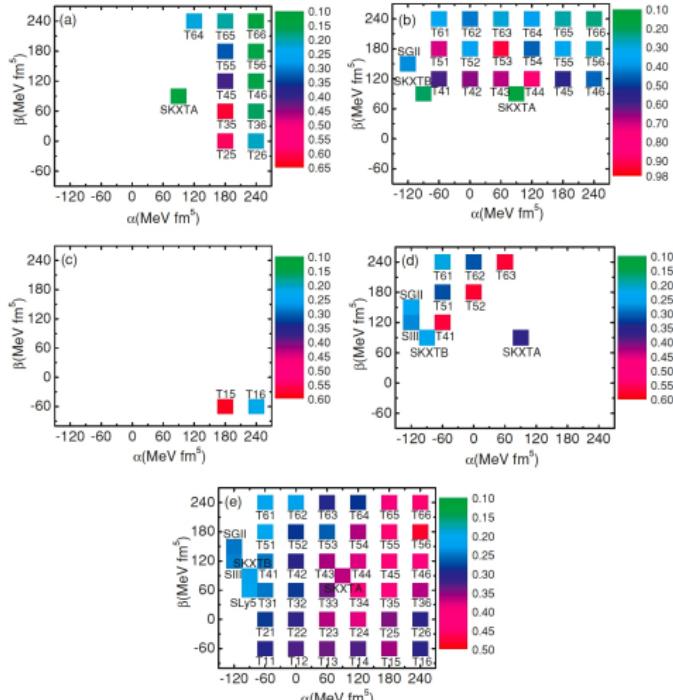
$$\begin{aligned} V_{ph} &= \delta_{\mathbf{r}_2, \mathbf{r}_1} N_0^{-1} \sum_{l=0}^1 \left[ F_l + F'_l \hat{\tau}_a \circ \hat{\tau}_b + G_l \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b + G'_l \hat{\tau}_a \circ \hat{\tau}_b \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \right. \\ &\quad \left. + \frac{k_{12}^2}{k_F^2} H_l S_{ab} + \frac{k_{12}^2}{k_F^2} H_l S_{ab} \hat{\tau}_a \circ \hat{\tau}_b \right] P_l(\cos \theta) \end{aligned}$$

For a Skyrme force we have

$$\begin{aligned} H_0 &= N_0 \frac{k_F^2}{24} (T + 3U) = N_0 \frac{k_F^2}{5} (\alpha_T + \beta_T) \\ H'_0 &= N_0 \frac{k_F^2}{24} (-T + U) = N_0 \frac{k_F^2}{5} (\alpha_T + \beta_T) \end{aligned}$$

[T. Lesinski et al. Phys. Rev. C 76, 014312 (2007)]

# Landau approximation: stability condition



$$a) J = 0^- : 1 + \frac{1}{3} G_1 - \frac{10}{3} H_0 > 0$$

$$b) \left(1 + \frac{G_0}{2}\right) \pm \frac{1}{2} \sqrt{G_0^2 + 8H_0^2} > 0$$

$$c) J = 0^- : 1 + \frac{1}{3} G'_1 - \frac{10}{3} H'_0 > 0$$

$$d) J = 1^- : 1 + \frac{1}{3} G'_1 + \frac{5}{3} H'_0 > 0$$

$$e) \left(1 + \frac{G'_0}{2}\right) \pm \frac{1}{2} \sqrt{G'^2_0 + 8H'^2_0} > 0$$

[L.G. Cao et al. Phys. Rev C 81, 044302 (2010) ]

[T. Lesinski et al. Phys. Rev C 76, 014312 (2007) ]

# Landau parameters from a generic functional

$$\left\{ \begin{array}{l} N_0^{-1} F_0 = 2C_0^{\rho 0} + (2 + \gamma)(1 + \gamma)C_0^{\rho\gamma}\rho_0^\gamma + 2k_F^2 C_0^\tau \\ N_0^{-1} F_1 = -2k_F^2 C_0^\tau \\ N_0^{-1} F'_0 = 2C_1^{\rho 0} + 2C_1^{\rho,\gamma}\rho_0^\gamma + 2k_F^2 C_1^\tau \\ N_0^{-1} F'_1 = -2k_F^2 C_1^\tau \\ N_0^{-1} G_0 = 2C_0^{s,0} + 2C_0^{s\gamma}\rho_0^\gamma + 2k_F^2 \textcolor{red}{C}_0^T + \frac{2k_F^2}{3} \textcolor{red}{C}_0^F \\ N_0^{-1} G'_0 = 2C_1^{s,0} + 2C_1^{s\gamma}\rho_0^\gamma + 2k_F^2 \textcolor{red}{C}_1^T + \frac{2k_F^2}{3} \textcolor{red}{C}_1^F \\ N_0^{-1} G_1 = -2k_F^2 \textcolor{red}{C}_0^T - \frac{2k_F^2}{3} \textcolor{red}{C}_0^F \\ N_0^{-1} G'_1 = -2k_F^2 \textcolor{red}{C}_1^T - \frac{2k_F^2}{3} \textcolor{red}{C}_1^F \\ N_0^{-1} H_0 = \frac{k_F^2}{3} \textcolor{red}{C}_0^F \\ N_0^{-1} H'_0 = \frac{k_F^2}{3} \textcolor{red}{C}_1^F \end{array} \right.$$

Only in the case of a Skyrme force the tensor coupling constant enter only in  $H_0, H'_0$

# Landau Sum Rules

Which Sum Rule you have to use with the tensor?

An equivalent form of the sum rules,

$$S_1 = \sum_l \left[ \frac{F_l}{1 + F_l/(2l+1)} + \sum_J \frac{2J+1}{2l+1} \mathcal{A}_{ll}^{JT=1} \right] = 0 ,$$
$$S_2 = \sum_l \left[ \frac{2}{3} \frac{F_l}{1 + F_l/(2l+1)} + \frac{F'_l}{1 + F'_l/(2l+1)} \right. \\ \left. + \frac{1}{3} \sum_J \frac{2J+1}{2l+1} \mathcal{A}_{ll}^{JT=0} \right] = 0 , \quad (16)$$

is obtained by eliminating first  $C_l$  and then  $C'_l$  in eq. (15). Here we have also used eqs. (5) and (12). If the tensor part of the quasiparticle interaction is neglected, eq. (16) reduces to

$$S'_1 = \sum_l \left[ \frac{F_l}{1 + F_l/(2l+1)} + 3 \frac{G'_l}{1 + G'_l/(2l+1)} \right] = 0 ,$$
$$S'_2 = \sum_l \left[ \frac{2}{3} \frac{F_l}{1 + F_l/(2l+1)} \right. \\ \left. + \frac{F'_l}{1 + F'_l/(2l+1)} + \frac{G_l}{1 + G_l/(2l+1)} \right] = 0 , \quad (17)$$

which when all Landau parameters are small further reduces to

$$\sum_l (F_l + 3G'_l) = 0 , \quad \sum_l (\frac{2}{3}F_l + F'_l + G_l) = 0 . \quad (18)$$

These relations have been written down, in truncated form, by Bauer et al. [7] and Backman et al. [8, eq. (5)]. However, these “sum rules” do not apply to nuclear matter since the Landau parameters are not small. Therefore, the full sum rules,  $S_1$  and  $S_2$ , must be satisfied.

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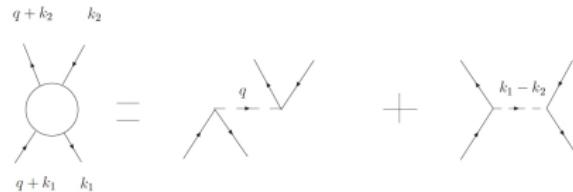
4 RPA:  $V_{ph} \neq 0$

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# RPA formalism

We consider an infinite medium in the Hartree-Fock formalism ( $T=0$ ).  
We act with an external field

$$\sum_j \exp^{i\mathbf{qr}} \Theta_\alpha^j \quad \Theta_\alpha^j = 1, \sigma^j, \tau^j, \sigma^j \tau^j$$



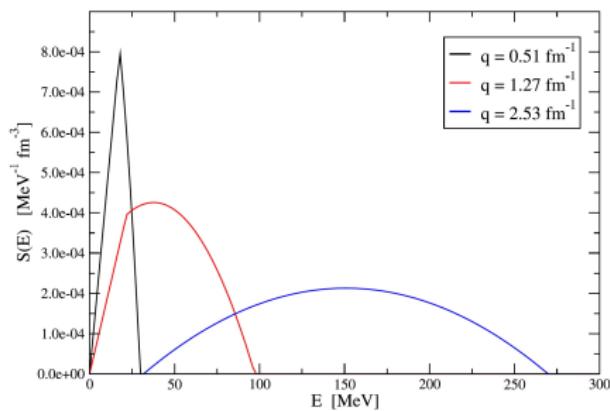
Within the Green function formalism we have for non interacting system

$$G_{HF}(q, \omega, \mathbf{k}_1) = \frac{\theta(k_F - k_1) - \theta(k_F - |\mathbf{k}_1 + \mathbf{q}|)}{\omega + \varepsilon(\mathbf{k}_1) - \varepsilon(|\mathbf{k}_1 + \mathbf{q}|) + i\eta\omega}$$

# Response function: non-interacting system

$$\begin{aligned}\chi_0(q, \omega) &= g \int \frac{d^3 k_1}{(2\pi)^3} G_{HF}^\alpha(q, \omega, \mathbf{k}_1) \\ S^\alpha(q, \omega) &= -\frac{1}{\pi} \text{Im} \chi_{RPA}^\alpha(q, \omega)\end{aligned}$$

The Lindhardt function can be written explicitly [C . Garcia-Recio , Ann. Phys. 214, 293-340, 1992 ]



$$\begin{aligned}\chi_0 &= -\frac{m^* k_F}{4\pi^2} [1 + A_+(\nu, k) + A_-(\nu, k)] \\ A_\pm(\nu, k) &= \frac{1}{4k} [1 - (k \pm \nu)^2] \\ &\times \left( \ln \left( \frac{k \pm \nu + 1}{k \pm \nu - 1} \right) \mp \text{sign}(\nu) \theta(1 - (k \pm \nu)^2) \right)\end{aligned}$$

[F . Raimondi, Ms. Thesis, 2007 ]

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# Residual interaction

Second functional derivative of the Skyrme functional [E . Perlinska et al. Phys. Rev C 69, 014316 (2004)) ]

$$\begin{aligned}
 V_{ph} &= \frac{1}{4} W_1^{00} + \frac{1}{4} W_1^{01} \hat{\tau}_a \circ \hat{\tau}_b + \frac{1}{4} W_1^{10} \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b + \frac{1}{4} W_1^{11} \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \hat{\tau}_a \circ \hat{\tau}_b \\
 &+ \frac{1}{4} (W_2^{00} + W_2^{01} \hat{\tau}_a \circ \hat{\tau}_b + W_2^{10} \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b + W_2^{11} \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \hat{\tau}_a \circ \hat{\tau}_b) \\
 &\times \left[ q_1^2 + q_2^2 - \frac{8\pi}{3} q_1 q_2 \sum_{\mu=-1,0,1} Y_\mu^{(1)*}(\hat{q}_1) Y_\mu^{(1)}(\hat{q}_2) \right] \\
 &+ \left[ +2\bar{\rho} C_1^{\rho,\gamma} \gamma \rho_0^{\gamma-1} \circ (\hat{\tau}_a + \hat{\tau}_b) + 2\gamma C_0^{s\gamma} \rho_0^{\gamma-1} \mathbf{s}_0 \cdot (\boldsymbol{\sigma}_a + \boldsymbol{\sigma}_b) + 2\gamma C_1^{s\gamma} \rho_0^{\gamma-1} \vec{\mathbf{s}} \cdot (\boldsymbol{\sigma}_a \circ \hat{\tau}_a + \boldsymbol{\sigma}_b \circ \hat{\tau}_b) \right] \\
 &+ 2(C_0^{\nabla s} + C_1^{\nabla s} \hat{\tau}_a \circ \hat{\tau}_b) \mathbf{q} \cdot \boldsymbol{\sigma}_a \mathbf{q} \cdot \boldsymbol{\sigma}_b + (C_0^F + C_1^F \hat{\tau}_a \circ \hat{\tau}_b) \left\{ \mathbf{k}_{12} \cdot \boldsymbol{\sigma}_a \mathbf{k}_{12} \cdot \boldsymbol{\sigma}_b - \frac{1}{2} \mathbf{q} \cdot \boldsymbol{\sigma}_a \mathbf{q} \cdot \boldsymbol{\sigma}_b \right\} \\
 &- i(C_0^{\nabla J} + C_1^{\nabla J} \hat{\tau}_a \circ \hat{\tau}_b) (\boldsymbol{\sigma}_a + \boldsymbol{\sigma}_b) \cdot [\mathbf{q} \times \mathbf{q}_1 - \mathbf{q} \times \mathbf{q}_2]
 \end{aligned}$$

$$\frac{1}{4} W_1^{00} = 2C_0^{\rho 0} + (2 + \gamma)(1 + \gamma)C_0^{\rho\gamma} \rho_0^\gamma + \gamma(\gamma - 1)C_1^{\rho,\gamma} \rho_0^{\gamma-2} \bar{\rho}^2 + \gamma(\gamma - 1)C_0^{s\gamma} \rho_0^{\gamma-2} \mathbf{s}_0^2$$

$$+ \gamma(\gamma - 1)C_1^{s\gamma} \rho_0^{\gamma-2} \vec{\mathbf{s}}^2 - \left[ 2C_0^{\Delta\rho} + \frac{1}{2} C_0^\tau \right] q^2$$

$$\frac{1}{4} W_1^{01} = 2C_1^{\rho 0} + 2C_1^{\rho,\gamma} \rho_0^\gamma - \left[ 2C_1^{\Delta\rho} + \frac{1}{2} C_1^\tau \right] q^2 \quad \frac{1}{4} W_1^{10} = 2C_0^{s,0} + 2C_0^{s\gamma} \rho_0^\gamma - \left[ 2C_0^{\Delta s} + \frac{1}{2} C_0^T \right] q^2$$

$$\frac{1}{4} W_1^{11} = 2C_1^{s,0} + 2C_1^{s\gamma} \rho_0^\gamma - \left[ 2C_1^{\Delta s} + \frac{1}{2} C_1^T \right] q^2$$

$$\frac{1}{4} W_2^{00} = C_0^\tau \quad \frac{1}{4} W_2^{01} = C_1^\tau \quad \frac{1}{4} W_2^{10} = C_0^T \quad \frac{1}{4} W_2^{11} = C_1^T$$

# Matrix elements of the tensor part [S=S'=1]

The residual interaction is

$$V_{ph}^{\alpha\alpha'}(q, \mathbf{k}_1, \mathbf{k}_2) = \langle \mathbf{q} + \mathbf{k}_1, \mathbf{k}_1^{-1}, (\alpha) | V | \mathbf{q} + \mathbf{k}_2, \mathbf{k}_2^{-1}, (\alpha') \rangle$$

$M$	$M' = 1$	$M' = 0$	$M' = -1$
1	$(-2B_t^T - 8B_t^{\Delta s})q^2$ $4B_t^T(k_{12})_0(k_{12})_0$ $(-8B_t^T - 4B_t^F)(k_{12})_1(k_{12})_{-1}$	$-4B_t^F(k_{12})_{-1}(k_{12})_0$	$-4B_t^F(k_{12})_{-1}(k_{12})_{-1}$
0	$4B_t^F(k_{12})_0(k_{12})_1$	$(-2B_t^T + 8B_t^{\nabla s} - 8B_t^{\Delta s} - 2B_t^F)q^2$ $(4B_t^T + 4B_t^F)(k_{12})_0(k_{12})_0$ $-8B_t^T(k_{12})_1(k_{12})_{-1}$	$4B_t^F(k_{12})_{-1}(k_{12})_0$
-1	$-4B_t^F(k_{12})_1(k_{12})_1$	$-4B_t^F(k_{12})_1(k_{12})_0$	$(-2B_t^T - 8B_t^{\Delta s})q^2$ $4B_t^T(k_{12})_0(k_{12})_0$ $(-8B_t^T - 4B_t^F)(k_{12})_1(k_{12})_0$

[D. Davesne et al. Phys. Rev. C ]

# Response function: interacting system

The RPA correlated Green function is the solution of Bethe-Salpeter equation

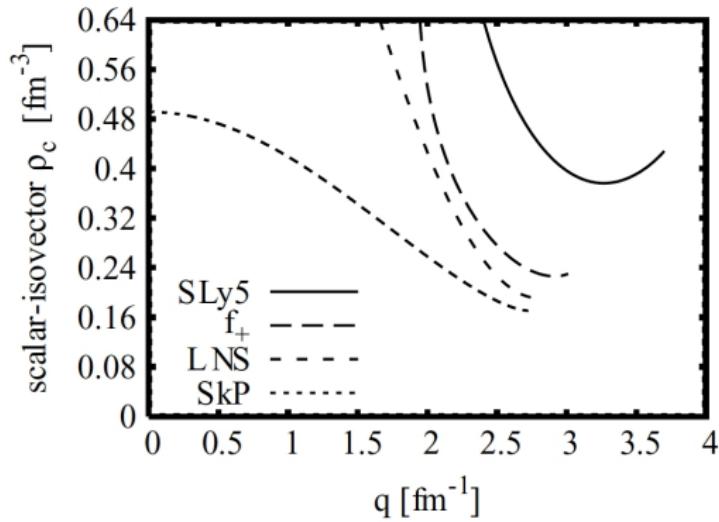
$$\begin{aligned} G_{RPA}^{\alpha}(q, \omega, \mathbf{k}_1) &= G_{HF}(q, \omega, \mathbf{k}_1) \\ &+ G_{HF}(q, \omega, \mathbf{k}_1) \sum_{\alpha'} \int \frac{d^3 k_2}{(2\pi)^3} V_{ph}^{\alpha\alpha'}(q, \mathbf{k}_1, \mathbf{k}_2) G_{RPA}^{\alpha'}(q, \omega, \mathbf{k}_2) \end{aligned}$$

The response function is now defined as

$$\begin{aligned} \chi_{RPA}^{\alpha}(q, \omega) &= g \int \frac{d^3 k_1}{(2\pi)^3} G_{RPA}^{\alpha}(q, \omega, \mathbf{k}_1) \\ S^{\alpha}(q, \omega) &= -\frac{1}{\pi} \text{Im} \chi_{RPA}^{\alpha}(q, \omega) \end{aligned}$$

# Instabilities

We look for the poles of  $\chi(\omega = 0, q)$ , zero energy modes.



# Symmetric nuclear matter (SNM)

The response function is a combination of generalized Lindhardt functions

$$\chi_{2i}(\mathbf{q}) = \int \frac{d_3^q}{(2\pi)^4} G_0(q_3, q) \frac{1}{2} \left[ \left( \frac{q_3^2}{k_F^2} \right)^i \left( \frac{(\mathbf{q} + \mathbf{q}_3)^2}{k_F^2} \right)^i \right]$$

[C . Garcia-Recio , Ann. Phys. 214, 293-340, 1992 ]

$$\begin{aligned} \frac{\chi_{HF}}{\chi_{RPA}^{(1I0)}} &= \left[ 1 + 2B_I^F \frac{m^* k_F^3}{3\pi^2} \right]^2 - \tilde{W}_1^{(1I0)} \chi_0 \\ &+ W_2^{(1I)} \left\{ \frac{q^2}{2} \chi_0 \left[ 1 + 4B_I^F \frac{m^* k_F^3}{3\pi^2} \right] - 2k_F^2 \chi_2 + 4B_I^F \frac{m^* k_F^5}{3\pi^2} (\chi_0 - \chi_2) \right\} \\ &+ [W_2^{(1,I)}]^2 k_F^4 \left[ \chi_2^2 - \chi_0 \chi_4 + \left( \frac{m^* \omega}{k_F^2} \right)^2 \chi_0^2 - \frac{m^*}{6\pi^2 k_F} q^2 \chi_0 \right] \\ &+ 2\chi_0 \left( \frac{m^* \omega}{q} \right)^2 \frac{W_2^{(1,I)} [1 + \frac{m^* k_F^3}{3\pi^2} X^{(1I0)}]}{1 - \frac{m^* k_F^3}{3\pi^2} [W_2^{(1,I)} - X^{(1I0)}]} \end{aligned}$$

# Symmetric nuclear matter (SNM)

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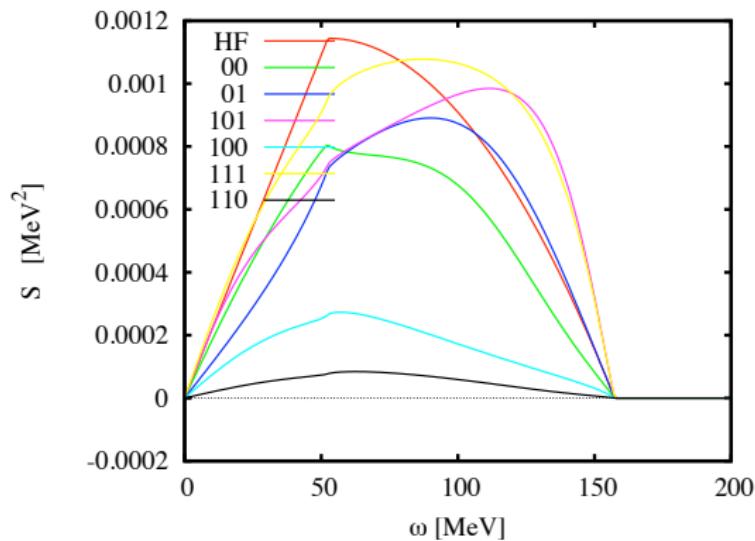
$$\chi_{2i}(\mathbf{q}) = \int \frac{d_3^q}{(2\pi)^4} G_0(q_3, q) \frac{1}{2} \left[ \left( \frac{q_3^2}{k_F^2} \right)^i \left( \frac{(\mathbf{q} + \mathbf{q}_3)^2}{k_F^2} \right)^i \right]$$

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$$\begin{aligned} \frac{\chi_{HF}}{\chi_{RPA}^{(1I0)}} &= \left[ 1 + 2B_I^F \frac{m^* k_F^3}{3\pi^2} \right]^2 - \tilde{W}_1^{(1I0)} \chi_0 \\ &+ W_2^{(1I)} \left\{ \frac{q^2}{2} \chi_0 \left[ 1 + 4B_I^F \frac{m^* k_F^3}{3\pi^2} \right] - 2k_F^2 \chi_2 + 4B_I^F \frac{m^* k_F^5}{3\pi^2} (\chi_0 - \chi_2) \right\} \\ &+ [W_2^{(1,I)}]^2 k_F^4 \left[ \chi_2^2 - \chi_0 \chi_4 + \left( \frac{m^* \omega}{k_F^2} \right)^2 \chi_0^2 - \frac{m^*}{6\pi^2 k_F} q^2 \chi_0 \right] \\ &+ 2\chi_0 \left( \frac{m^* \omega}{q} \right)^2 \frac{W_2^{(1,I)} \left[ 1 + \frac{m^* k_F^3}{3\pi^2} \textcolor{red}{X}^{(1I0)} \right]}{1 - \frac{m^* k_F^3}{3\pi^2} \left[ W_2^{(1,I)} - \textcolor{red}{X}^{(1I0)} \right]} \end{aligned}$$

# Linear response for T22 [Lesinski et al.]

For  $k = k_F$  at  $\rho_0$  we compare different response functions in different channels S,I,M



# Sum Rules

To calculate the results we check the odd-power sum rules

$$M_p^{(\alpha)}(\mathbf{q})/N = -\frac{1}{\pi\rho} \int_0^{+\infty} d\omega \omega^p Im \chi^\alpha(\omega, \mathbf{q})$$

$$\chi^{(\alpha)}(\omega, \mathbf{q}) \approx 2\rho \sum_{p=0}^{+\infty} (\omega)^{-(2p+2)} M_{2p+1}(\mathbf{q})/N$$

$$\chi^{(\alpha)}(\omega, \mathbf{q}) \approx -2\rho \sum_{p=0}^{+\infty} (\omega)^{2p} M_{-(2p+1)}(\mathbf{q})/N$$

$$M_1 = \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle$$

$$M_3 = \frac{1}{2} \langle 0 | [F, H], [H, [H, F]] | 0 \rangle$$

## Sum Rules: $M_1$

The analytical expression of  $M_1$  reads

$$M_1/N = \frac{q^2}{2m^*} \left( 1 - W_2^{(SI)} \frac{m^* \rho}{2} \right)$$

Where

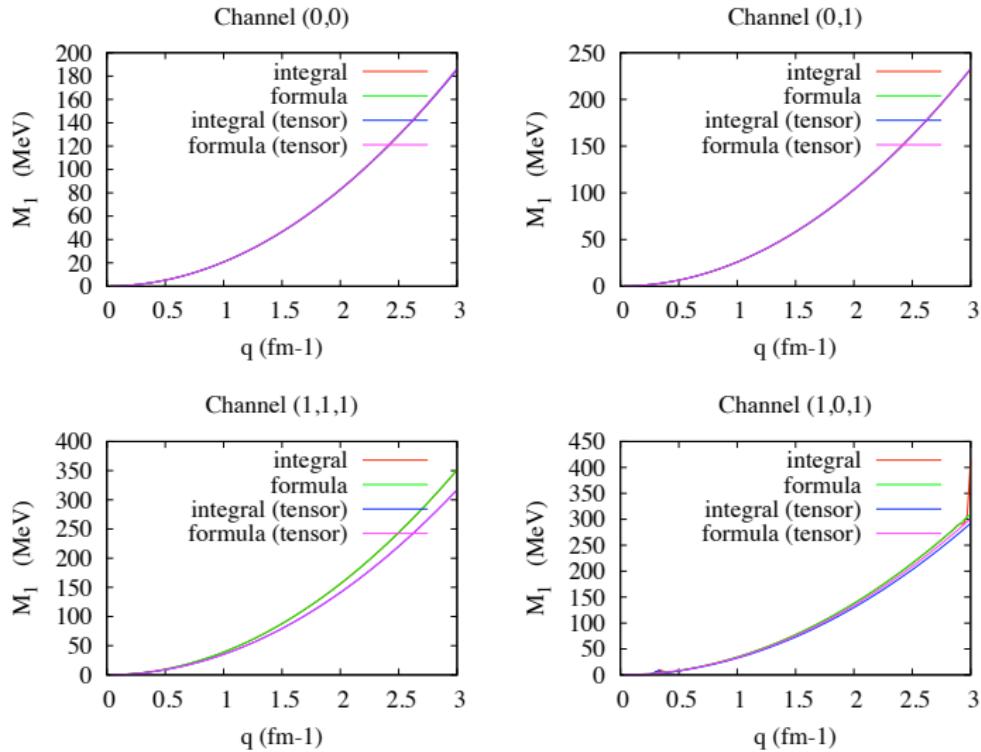
$$\frac{1}{4}W_2^{00} = C_0^\tau \quad \frac{1}{4}W_2^{01} = C_1^\tau$$

$$\frac{1}{4}W_2^{10} = C_0^T \quad \frac{1}{4}W_2^{11} = C_1^T$$

The tensor affects the coupling constant

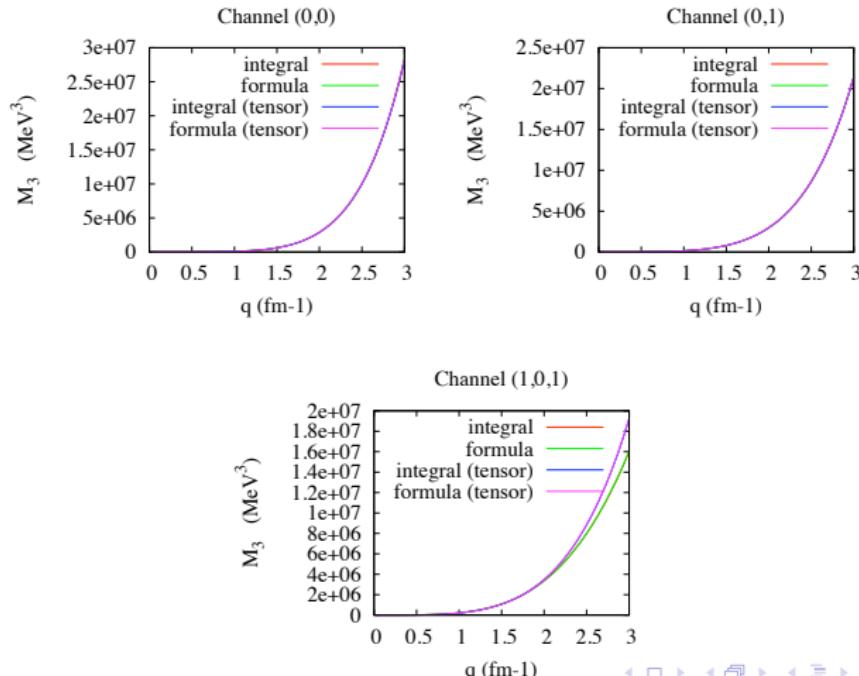
$$C_I^T = A_I^T + \textcolor{blue}{B}_I^T$$

# Sum Rules: $M_1$



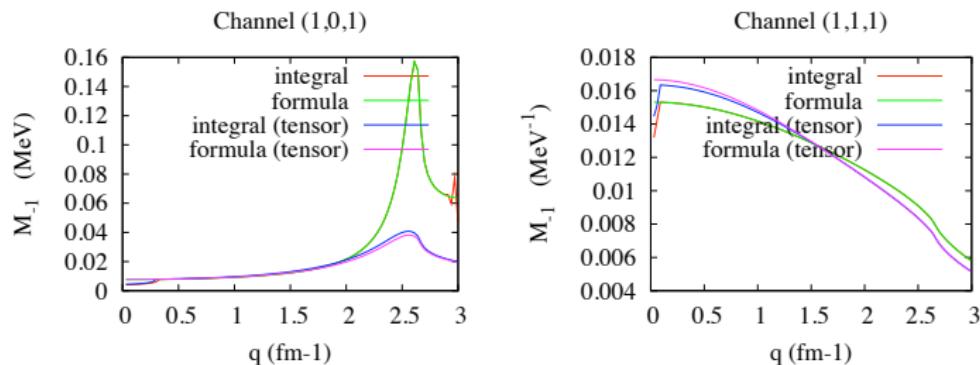
# Sum Rule: $M_3$

The sum rules  $M_3$  involve a more complicated combination of coupling constants. It mainly depends on  $W_2^{(ST)}$  and  $B_I^F$



# Sum Rule: $M_{-1}$

Used to test the low energy part of the response.



It is a combination of  $W_2^{(ST)}$  and  $B_I^F$  parameters.

# Sum Rules: applications

The sum rules are not simply a numerical test for the accuracy of the calculation. We can define

$$E_3 = \sqrt{M_3/M_1} \quad E_1 = \sqrt{M_1/M_{-1}}$$

A general theorem shows

$$E_1 \leq E_{av} = M_1/M_0 \leq E_3$$

Possibility to understand the position of the giant resonances?

[E. Lipparini and S. Stringari, Phys. Rep. 175, 103-261, (1989)]

# Outline

1 Introduction

2 Landau

3 RPA:  $V_{ph}=0$

4 RPA:  $V_{ph} \neq 0$

5 Summary

# Summary and Perspective

Status of the work...

- Derivation of the response function for Symmetric Nuclear Matter for a given Skyrme functional
- Derivation of the response function for Pure Neutron Matter for a given Skyrme functional
- Derivation of the Sum Rules  $M_1, M_3, M_{-1}$

... and future development

- Derive the response function for asymmetric nuclear matter
- Inclusion of the temperature
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- Extensive comparison among finite nuclei and infinite matter.

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.... thank you!!!