Jądrowy wychwyt elektronu w jonach wodoro-, helo- i lito-podobnych

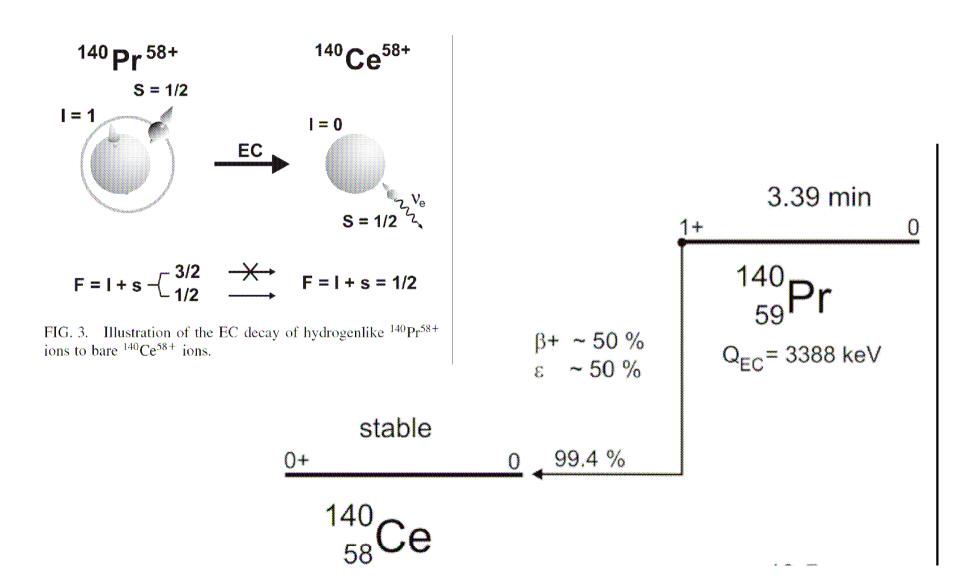
Zygmunt Patyk

Zakład Fizyki Teoretycznej NCBJ

Plan wykładu

- 1. Elementy teorii jądrowego wychwytu elektronu.
- 2. Wychwyt elektronu w jonach lito-podobnych.
- 3. Niezachowanie parzystości w stanach wzbudzonych jonów helopodobnych
- 4. Przyczyny różnic czasów życia jonów wodoro- i helo-podobnych
- 5. Pomiary czasów życia w jonach wodoro-, helo-podobnych na wychwyt elektronu .

Electron Capture in ¹⁴⁰Pr



Electron capture

$$(Z, A) + e^{-} \rightarrow (Z - 1, A) + \nu_{e}$$

The EC decay probability per time unit is given by Fermi's golden rule [19]:

$$W = \frac{2\pi}{\hbar} |\langle f | \hat{O} | i \rangle|^2 \rho_f, \tag{2}$$

where ρ_f is the density of the neutrino final states per energy unit, which is proportional to the square of the decay energy $Q_{\rm EC}$.

The weak interaction operator \hat{O} , responsible for the EC decay [20], satisfies the following general conditions:

- (a) it acts only on nuclear and leptonic variables involved in the process (emitted neutrino and captured electron),
- (b) it has nonzero matrix elements only between states with identical total momentum and its projection, and
- (c) the square of its expectation value does not depend on the projection of the total momentum.

Study of nuclear electron capture in Li-like ions

Katarzyna Siegień-Iwaniuk¹ and Zygmunt Patyk¹
¹Soltan Institute for Nuclear Studies, Hoża 69, 00-681 Warsaw, Poland

The nuclear electron capture (EC) process in lithium-like ions is studied. We find that the daughter helium-like ions are created mostly in the 2^1S_0 and 2^3S_1 excited states with probabilities denoted as P_0 and P_1 , respectively. The ratio of probabilities depends on spin I of a mother nucleus and the type of the EC transition. For allowed EC transitions $I \to I \pm 1$ the ratio has a simple form $\frac{P_0}{P_1} = \frac{2I+1}{2(I\pm 1)+1}$. Additionally, we found the simple relation between probabilities of EC decay for lithium- and hydrogen-like ions $P_{Li} = P_H(\frac{2(I\pm 1/2)+1}{(2I+1)} + \frac{\rho^{2s}(Z)}{2\rho^{1s}(Z)})(1-\frac{q}{Z})^3$, where q=0.464. We also discuss applications of excited states formed in helium-like ions, especially the parity nonconservation effect.

Wave function for an H-like ion

The initial (final) wave function of the hydrogen-like ion contains the nuclear part of the mother (daughter) nucleus with spin I ($I \pm 1$), coupled with an orbital electron (neutrino) to spin $I \pm \frac{1}{2}$:

$$|I \pm \frac{1}{2}, M\rangle_{X} = \sum_{k=-\frac{1}{2}}^{\frac{1}{2}} (\frac{1}{2}, k, I \pm 1, M - k|I \pm \frac{1}{2}, M)$$
$$\times |I \pm 1, M - k\rangle_{N} |\frac{1}{2}, k\rangle_{l}, \tag{7}$$

where two indexes (X) and (l) denote a H-like system (H) with one electron (e) in the initial state or a bare nucleus (B) with emitted neutrino ν in the final state.

Wave functions for He-like ions

helium-like wave functions are defined as following:

$$|1^{1}S_{0},0\rangle_{1,2} = \frac{\mathcal{A}[\chi_{1s}^{+}(1)\chi_{1s}^{-}(2)]}{\sqrt{2}},$$

$$|2^{1}S_{0},0\rangle_{1,2} = \frac{\mathcal{A}[\chi_{1s}^{+}(1)\chi_{2s}^{-}(2) - \chi_{1s}^{-}(1)\chi_{2s}^{+}(2)]}{2},$$

$$|2^{3}S_{1},0\rangle_{1,2} = \frac{\mathcal{A}[\chi_{1s}^{+}(1)\chi_{2s}^{-}(2) + \chi_{1s}^{-}(1)\chi_{2s}^{+}(2)]}{2},$$
$$|2^{3}S_{1},\pm 1\rangle_{1,2} = \frac{\mathcal{A}[\chi_{1s}^{\pm}(1)\chi_{2s}^{\pm}(2)]}{\sqrt{2}}.$$

The wave function for a Li-like ion

The electronic ground state wave function of lithium-like ion with spin 1/2 could be expressed as the anti-symmetrized product of 1s and 2s normalized relativistic Dirac spinors $\chi_{ns}^{i}(k)$ [21–23], each with spin 1/2,

$$|1/2, 1/2\rangle_{3e} = \frac{\mathcal{A}[\chi_{1s}^{+}(1)\chi_{1s}^{-}(2)\chi_{2s}^{+}(3)]}{\sqrt{6}}.$$
 (1)

Wave functions for ions (electrons +nucleus)

$$|I \pm 1/2, M\rangle_{Li} = \sum_{i=-1/2}^{1/2} (1/2, i, I, M - i|I \pm 1/2, M)$$

$$\times |1/2, i\rangle_{3e} |I, M - i\rangle_{N}, \qquad (4)$$

where the expression $(1/2, i, I, M - i | I \pm 1/2, M)$ denotes the Clebsch-Gordan coefficient.

Parity nonconservation in high-Z heliumlike ions

R. W. Dunford

Physics Division, Argonne National Laboratory, Argonne, Illinois 60439
(Received 11 August 1995; revised manuscript received 26 February 1996)

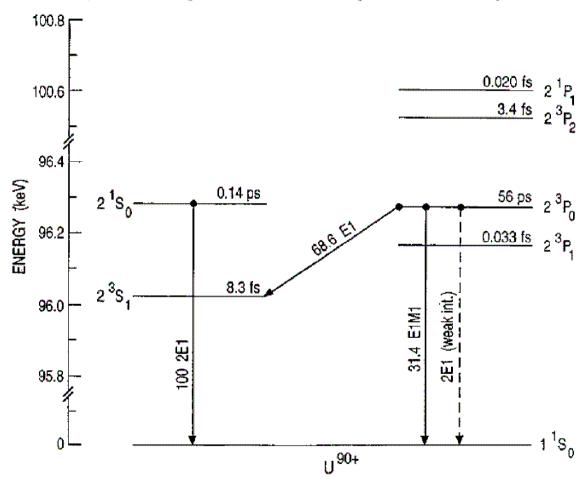


FIG. 1. Low-lying energy levels of He-like U^{90+} including life-times. The decay modes and branching ratios (in percent) are given for the $2\,{}^3P_0$ and $2\,{}^1S_0$ states.

$$|I \pm 1/2, M, 2^{1}S_{0}\rangle_{He} =$$

$$= \sum_{n=-1/2}^{1/2} (1/2, n, I \pm 1, M - n|I \pm 1/2, M)$$

$$\times |2^{1}S_{0}, 0\rangle_{1,2} |1/2, n\rangle_{\nu} |I \pm 1, M - n\rangle_{N}.$$
 (5)

However, the excited state 2^3S_1 could be coupled together with neutrino to the spin 1/2 or 3/2 (denoted later as S) and the final wave function is then given by the formula

$$|I \pm 1/2, M, 2^{3}S_{1}, S\rangle_{He} =$$

$$= \sum_{n=-1/2}^{1/2} \sum_{i=-S}^{S} (S, i, I \pm 1, M - i | I \pm 1/2, M)$$

$$\times (1, i - n, 1/2, n | S, i)$$

$$\times |2^{3}S_{1}, i - n\rangle_{1,2} |1/2, n\rangle_{\nu} |I \pm 1, M - i\rangle_{N}.$$
 (6)

The probability P_0 that a lithium-like ion in the EC process decays into the 2^1S_0 excited helium-like ion is expressed by the matrix element of hydrogen-like ion and is proportional to

$$P_{0} \propto 3_{He} \langle I \pm 1/2, M, 2^{1} S_{0} | \hat{O} | I \pm 1/2, M \rangle_{Li}^{2}$$

$$= \frac{1}{2} (1 - \frac{q}{Z})^{3} {}_{B} \langle I \pm \frac{1}{2}, M | \hat{O} | I \pm \frac{1}{2}, M \rangle_{H}^{2}, \quad (9)$$

The effective charge we found to be equal to Z-q with q=0.464.

In a similar way can be estimated the probability P_1 to reach the state 2^3S_1 in a helium-like ion

$$P_{1} \propto 3_{He} \langle I \pm 1/2, M, 2^{3}S_{1}, 1/2|\hat{O}|I \pm 1/2, M\rangle_{Li}^{2} + 3_{He} \langle I \pm 1/2, M, {}^{3}S_{1}, 3/2|\hat{O}|I \pm 1/2, M\rangle_{Li}^{2}$$
(10)
$$= \frac{2(I \pm 1) + 1}{2(2I + 1)} (1 - \frac{q}{Z})^{3}_{B} \langle I \pm \frac{1}{2}, M|\hat{O}|I \pm \frac{1}{2}, M\rangle_{H}^{2}.$$

Finally, the ratio of probabilities P_0 and P_1 is given by the simple expression

$$\frac{P_0}{P_1} = \frac{2I+1}{2(I\pm 1)+1}. (12)$$

Then the probability P_{Li} equals the sum of probabilities that lithium-like ion decays into the 1^1S_0 ground state or into the 2^1S_0 , 2^3S_1 excited states. The EC probability into the ground state is proportional to $P_0 \frac{\rho^{2s}(Z)}{\rho^{1s}(Z)}$. Adding three probabilities P_0 and P_1 , given by equations 9, 11, and the probability to reach the ground state we obtain the simple relation

$$P_{Li} = \left(\frac{2(I \pm 1/2) + 1}{(2I+1)} + \frac{\rho^{2s}(Z)}{2\rho^{1s}(Z)}\right) (1 - \frac{q}{Z})^3 P_H.(14)$$

where we put $P_H \propto {}_H \langle I \pm \frac{1}{2}, M | \hat{O} | I \pm \frac{1}{2}, M \rangle_H^2$.

The states are mixed by the weak interaction and the resulting state $2^1S'_0$ has the form

$$|2^{1}S_{0}'\rangle = |2^{1}S_{0}\rangle + \delta_{w}|2^{3}P_{0}\rangle,$$
 (16)

where the coefficient δ_w calculated in the perturbation theory [26, 32] has the form

$$\delta_w = \frac{\langle 2^1 S_0 | \frac{G_F}{2\sqrt{2}} (1 - 4\sin^2 \theta_W - N/Z) \rho \gamma_5 | 2^3 P_0 \rangle}{E_{2^1 S_0} - E_{2^3 P_0}}. \quad (17)$$

In the latter equation G_F denotes Fermi's constant, N the neutron number, Z the proton number, ρ the nuclear density normalized to Z and ϑ_W the Weinberg angle. The separation energy $E_{2^1S_0} - E_{2^3P_0}$ between two states has minimal value in the vicinity of nuclei with Z = 62 or Z = 90 [26–29]. The mixing parameter δ_w is of order 10^{-6} [30, 31]. The nuclear electron capture process responsible for formation of the excited state $2^1S'_0$ occurs around Z = 62 in the light isotopes of Pr, Nd, Pm, Sm, Eu or Gd.

The simplest experimental method to measure the quantity δ_w would be to determine the ϵ_2 polarization and the k_2 wave vector of one of the two photons and averaging over all directions and polarizations of the second one. The probability of the two-photon decay of the $2^1S_0'$ excited state is given by the formula [32, 33]

$$\frac{dw_{2\gamma}}{dk_1} = A + \delta_w B \hat{\mathbf{k}}_1 \cdot \hat{\varepsilon}_1^* \times \hat{\varepsilon}_1, \tag{18}$$

where the average over all directions of the wave vector k_2 and the polarization ϵ_2 of the second photon has been found. A detailed discussion of the coefficients A and B can be found in the paper [32].

H-like ion wave function

The states IN and OUT

$$\left| I \pm \frac{1}{2}, M \right\rangle_{H} = \sum_{k=-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{1}{2}, k, I', M - k \middle| I \pm \frac{1}{2}, M \right)$$

$$\times \left| I', M - k \right\rangle_{N} \left| \frac{1}{2}, k \right\rangle_{l}, \tag{7}$$

where the lepton index l denotes the electron in the initial state or the neutrino in the final state. In Eq. (7) the parent nucleus has spin I' = I and the daughter nucleus has spin I' = I or $I' = I \pm 1$.

By reversing Eq. (7) we obtain

$$|I, M\rangle_N |\pm\rangle_I = \sqrt{\frac{I \pm M + 1}{2I + 1}} \left| I + \frac{1}{2}, M \pm \frac{1}{2} \right\rangle_H$$

$$\mp \sqrt{\frac{I \mp M}{2I + 1}} \left| I - \frac{1}{2}, M \pm \frac{1}{2} \right\rangle_H. \tag{8}$$

He-like ion wave function

The state IN

$$|i, I, M\rangle_{\text{He}} = |I, M\rangle_N \frac{1}{\sqrt{2}} (|+\rangle_{1e}|-\rangle_{2e} - |-\rangle_{1e}|+\rangle_{2e}), \quad (3)$$

where $|+\rangle$ and $|-\rangle$ represent relativistic K-electron states with spin 1/2 and projections +1/2 and -1/2, respectively [21,22].

The state OUT

$$|1, 1\rangle_{v,1e} = |+\rangle_{v}|+\rangle_{1e},$$

$$|1, 0\rangle_{v,1e} = \frac{1}{\sqrt{2}}(|+\rangle_{v}|-\rangle_{1e} + |-\rangle_{v}|+\rangle_{1e}),$$

$$|1, -1\rangle_{v,1e} = |-\rangle_{v}|-\rangle_{1e}.$$

$$|f, I, M, 1\rangle_{He} = \sum_{k=-1}^{1} (1, k, I', M - k|I, M)$$

 $\times |I', M - k\rangle_{N} |1, k\rangle_{v, 1e},$

0.5 [{-
$$_{V}$$
<-| { $_{N}$ <0,0|}| O {|1,0> $_{N}$ } |-> $_{e}$ + $_{V}$ <+| { $_{N}$ <0,0|}| O {|1,0> $_{N}$ } |+> $_{e}$ }]

However,

$$|1,0>|+>=\sqrt{2/3}|3/2,+1/2>_{H}-\sqrt{1/3}|1/2,+1/2>_{H}$$

 $|1,0>|->=\sqrt{2/3}|3/2,-1/2>_{H}+\sqrt{1/3}|1/2,-1/2>_{H}$

$$0.5\sqrt{1/3}$$
 [- H<1/2, -1/2| O |1/2, -1/2>_H - H<1/2, 1/2| O |1/2, 1/2>_H]=

$$-\sqrt{1/3}_{\rm H}$$
 <1/2, -1/2 | O | 1/2, -1/2 > $_{\rm H}$

Summing up over two electron indexes of He-like ion we have

$$W_{He} = 2/3 W_H$$

The EC decay probabilities W_{He} and W_{H} for He- and H-like ions, respectively, are proportional to the square of the matrix elements of the operator \hat{O} . In Eqs. (5) and (6) we assigned the index 1 to the remaining (not captured) electron. Because this electron can be captured as well, we must sum over these two possibilities. While neglecting the small difference of the neutrino energies for the EC decay in He- and H-like ions, we arrive at a simple relation:

$$W_{\text{He}} = \frac{2(I \pm \frac{1}{2}) + 1}{2I + 1} W_H, \tag{11}$$

where the plus and minus signs correspond to the nuclear transitions $I \to I + 1$ and $I \to I - 1$, respectively. We illustrate Eq. (11) by three examples:

- (i) For a nuclear transition $0 \to 1$, we get $W_{\text{He}} = 2W_H$.
- (ii) For a nuclear transition $1 \to 0$, we get $W_{\text{He}} = \frac{2}{3}W_H$.
- (iii) For large spins I, we get $W_{\text{He}} \to W_H$.

Screening corrections

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Orbital electron capture of hydrogen- and helium-like ions

Katarzyna Siegień-Iwaniuk, ¹ Nicolas Winckler, ^{2,3} Fritz Bosch, ³ Hans Geissel, ³ Yuri A. Litvinov, ^{2,3} and Zygmunt Patyk ¹ Soltan Institute for Nuclear Studies, Hoza 69, PL-00-681 Warsaw, Poland ² Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany ³ GSI Helmhotzzentrum für Schwerionenforschung, Planckstrasse 1, D-64291 Darmstadt, Germany (Received 22 March 2011; revised manuscript received 17 May 2011; published 6 July 2011)

Corrections to the ratio of electron capture (EC) rates in hydrogen- and helium-like ions are calculated. We find that the most significant contribution is the electron screening effect. The correction has a simple form $(1-5/16Z)^3(1-\delta_3)$ which ranges from almost 50% in helium to 1% in heavier nuclei. We discuss also EC in helium-like ions accompanied by an emission of the remaining electron into the continuum, a new decay channel, for which we calculate the decay probability. It is a very exotic type of Auger electron emission.

By combining Eqs. (3), (7), and (12), and by expressing the electron density at the nucleus in terms of the relativistic spinors with charge Z [as in Eq. (3) for H-like ions] we finally get

$$P_{\text{He}} = \frac{[2(I \pm 1/2) + 1]}{2I + 1} P_H \left(1 - \delta_1 + \frac{\delta_2}{Q_{EC}} \right) \times \left(1 - \frac{5}{16Z} \right)^3 (1 + \delta_3), \tag{13}$$

where $(1 - \frac{5}{16Z})^3$ is the ratio of nonrelativistic electron densities in He-like and H-like ions [34]. The relativistic correction δ_3 can be expressed as [33,34]

$$\delta_3 = \frac{Z^3 \tilde{\rho}_e(Z - q)}{(Z - 5/16)^3 \tilde{\rho}_e(Z)} - 1. \tag{14}$$

TABLE I. Calculated corrections δ_1 , δ_2 (in keV) and δ_3 listed for a few selected nuclei. Z denotes the atomic number. q is the relativistic screening correction.

Z	q	$(1-5/16Z)^3$	$\log_{10}(\delta_1)$	$\delta_2 (\text{keV})$	$10^2\delta_3$
2	0.312	0.601	-0.88	0.0	-0.05
12	0.312	0.924	-2.87	-0.2	-0.25
22	0.309	0.958	-3.43	-0.4	-0.40
32	0.306	0.971	-3.74	-0.6	-0.53
42	0.301	0.978	-3.96	-0.8	-0.64
52	0.294	0.982	-4.11	-1.1	-0.75
62	0.286	0.985	-4.21	-1.3	-0.84
72	0.275	0.987	-4.28	-1.5	-0.92
82	0.263	0.989	-4.31	-1.8	-1.00
92	0.247	0.990	-4.31	-2.1	-1.06

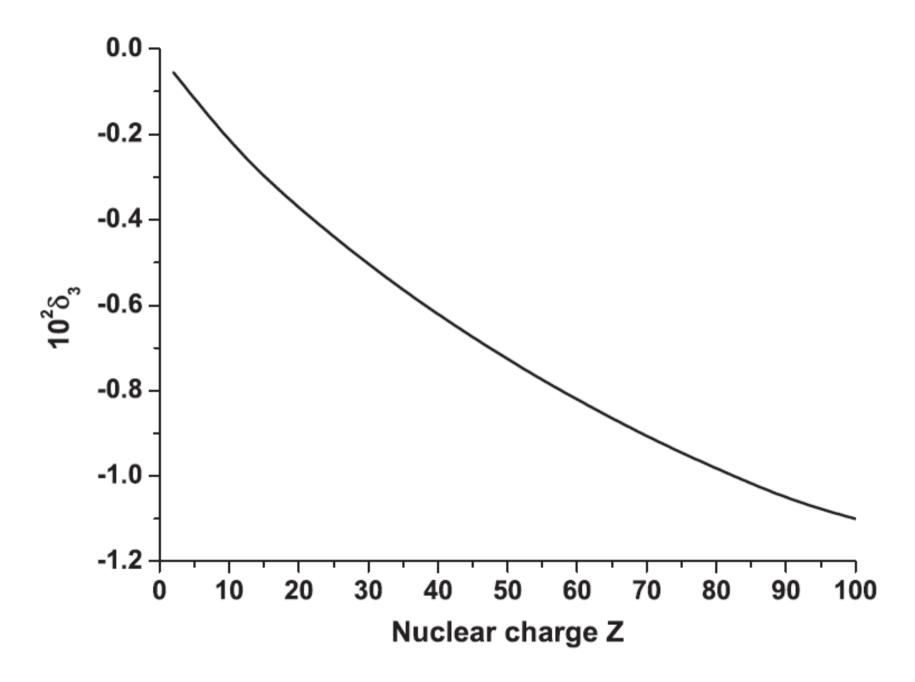


FIG. 4. The relativistic correction δ_3 calculated for uniformly distributed charge as a function of the nuclear charge Z.

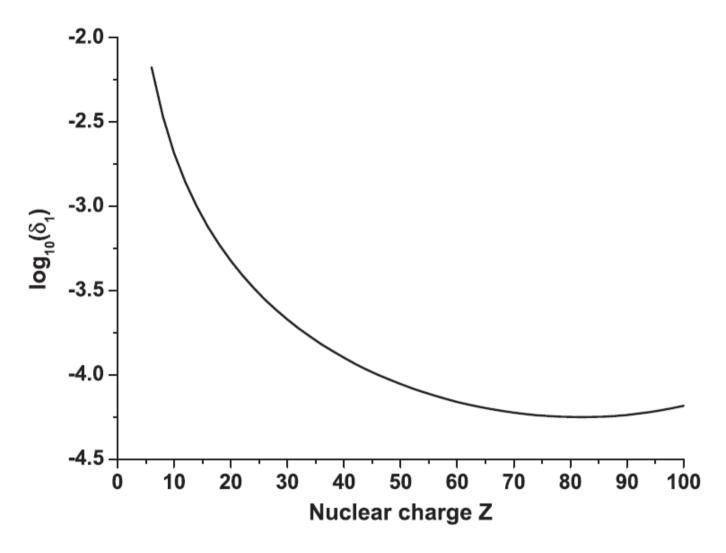
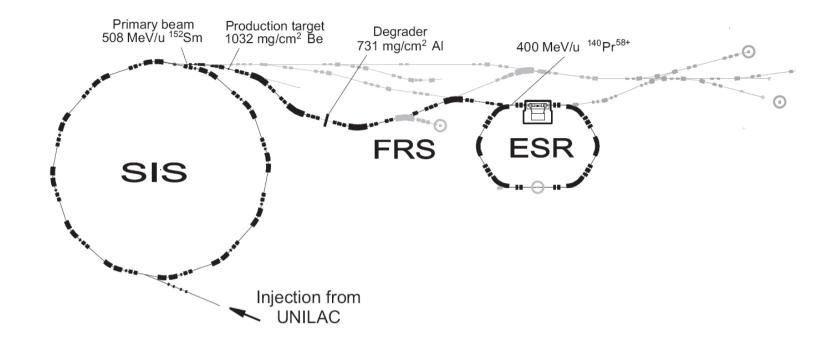
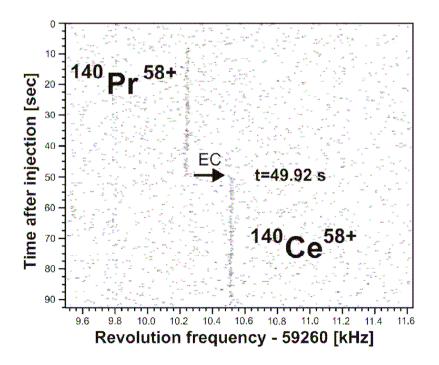


FIG. 2. The calculated probability δ_1 (logarithmic scale) that the He-like ion decays via EC to an atomic nucleus with the remaining electron excited into the continuum. It is about 0.1 for light nuclei and monotonically decreases down to 5.6×10^{-5} for Pb isotopes.





The decay constant can be measured

a) counting decay times for single ions

or

b) assuming the peak area is proportional to the number of circulating ions in the ring and observing the area in time.

Single ion

a) For a single ion decay probability λ per time unit is constant in time.

An ensamble of ions

b) The number of atoms dN decaying in a time interval dt is proportional to the number of atoms N(t) and decay constant λ ,

$$dN = -\lambda N(t) dt$$

$$N(t) = N_0 \exp(-\lambda t)$$

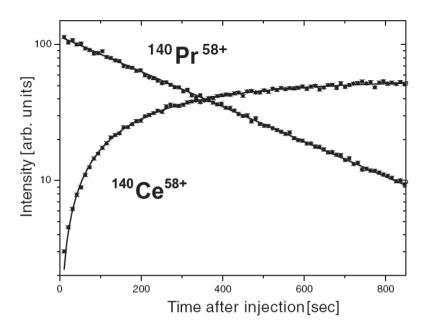


FIG. 2. Decay and growth curves of $^{140}\text{Pr}^{58+}$ and $^{140}\text{Ce}^{58+}$ ions as a function of time. The data points are shown in the laboratory frame and can be converted to the rest frame of the ions using the Lorentz factor $\gamma = 1.43$. The lines represent the fits according to Eqs. (1) and (2).

$$\lambda_{He}/\lambda_{H} = 147/219 = 0.67$$

$$\lambda_{\text{He}} = 2/3 \lambda_{\text{H}}$$

$$N_{\rm Pr}(t) = N_{\rm Pr}(0)e^{-\lambda t},\tag{1}$$

$$N_{\text{Ce}}(t) = N_{\text{Pr}}(0) \frac{\lambda_{\text{EC}}}{\lambda - \lambda_{\text{loss}}} \left[e^{-\lambda_{\text{loss}}t} - e^{-\lambda t} \right] + N_{\text{Ce}}(0) e^{-\lambda_{\text{loss}}t}.$$
(2)

TABLE I. Measured β^+ and EC decay constants obtained for fully ionized, hydrogenlike, and heliumlike ¹⁴⁰Pr ions. The values are given in the rest frame of the ions.

Ion	$\lambda_{\beta^+} (\text{sec}^{-1})$	$\lambda_{\rm EC}~({\rm sec}^{-1})$
¹⁴⁰ Pr ⁵⁹⁺	0.001 58(8)	• • •
¹⁴⁰ Pr ⁵⁸⁺	0.00161(10)	0.002 19(6)
¹⁴⁰ Pr ⁵⁷⁺	0.001 54(11)	0.001 47(7)



Measurement of the β^+ and Orbital Electron-Capture Decay Rates in Fully Ionized, Hydrogenlike, and Heliumlike ¹⁴⁰Pr Ions

Yu. A. Litvinov, ^{1,2} F. Bosch, ¹ H. Geissel, ^{1,2} J. Kurcewicz, ³ Z. Patyk, ⁴ N. Winckler, ^{1,2} L. Batist, ⁵ K. Beckert, ¹ D. Boutin, ² C. Brandau, ¹ L. Chen, ² C. Dimopoulou, ¹ B. Fabian, ² T. Faestermann, ⁶ A. Fragner, ⁷ L. Grigorenko, ⁸ E. Haettner, ² S. Hess, ¹ P. Kienle, ^{6,7} R. Knöbel, ^{1,2} C. Kozhuharov, ¹ S. A. Litvinov, ^{1,2} L. Maier, ⁶ M. Mazzocco, ¹ F. Montes, ^{1,9} G. Münzenberg, ^{1,10} A. Musumarra, ^{11,12} C. Nociforo, ¹ F. Nolden, ¹ M. Pfützner, ³ W. R. Plaß, ² A. Prochazka, ¹ R. Reda, ⁷ R. Reuschl, ¹ C. Scheidenberger, ^{1,2} M. Steck, ¹ T. Stöhlker, ¹ S. Torilov, ¹³ M. Trassinelli, ¹ B. Sun, ^{1,14} H. Weick, ¹ and M. Winkler

We report on the first measurement of the β^+ and orbital electron-capture decay rates of 140 Pr nuclei with the simplest electron configurations: bare nuclei, hydrogenlike, and heliumlike ions. The measured electron-capture decay constant of hydrogenlike 140 Pr $^{58+}$ ions is about 50% larger than that of heliumlike 140 Pr $^{57+}$ ions. Moreover, 140 Pr ions with one bound electron decay faster than neutral 140 Pr $^{0+}$ atoms with 59 electrons. To explain this peculiar observation one has to take into account the conservation of the total angular momentum, since only particular spin orientations of the nucleus and of the captured electron can contribute to the allowed decay.

Table 2 The averaged β^+ and EC decay constants (in the rest frame of the ions) determined for fully-ionized, hydrogen-like, and helium-like ¹⁴²Pm ions. The data for the neutral atom are taken from Ref. [31].

Ion	λ_{β^+} [s ⁻¹]	$\lambda_{EC} [s^{-1}]$	$EC/(\beta^+ + EC)$
¹⁴² Pm ⁶¹⁺	0.0123(7)	_	
¹⁴² Pm ⁶⁰⁺	0.0126(3)	0.0051(1)	$(29.0\pm1.3)\%$
¹⁴² Pm ⁵⁹⁺	0.0139(6)	0.0036(1)	$(20.2 \pm 1.0)\%$
¹⁴² Pm ⁰⁺	0.0132(5)	0.0039(5)	$(22.9\pm2.7)\%$

electron screening modifies the β^+ decay rate by less than 3% in fully-ionized nuclei relative to neutral atoms [1,36].

The ratio of the EC decay constants of H-like and He-like ions is

$$\lambda_{EC}^{H}/\lambda_{EC}^{He} = 1.44(6)$$
.

This ratio is in excellent agreement with the results obtained in the EC-decay measurements of H-like and He-like ¹⁴⁰Pr ions [14], where the corresponding ratio of the decay constants is 1.49(8).

Streszczenie

- 1. W GSI-Darmstadt w ostatnich latach zmierzono stałą rozpadu na wychwyt elektronu dla jonów wodoro i helopodobnych dla przejść 1→0. Wyniki doświadczalne są zgodne z teorią wychwytu elektronu.
- 2. Efekty ekranowania największe są dla lekkich jąder i sięgają 40% dla helu.
- 3. Wychwyt elektronu w jonach lito-podobnych umożliwi wytwarzanie określonych stanów wzbudzonych w jonach helo-podobnych. Badanie tych stanów pozwoli badać niezachowanie parzystości w układach atomowych.

$$H_{EC} = \frac{G_F}{\sqrt{2}} \left[\overline{\psi}_n \, \gamma_\mu \left(1 - \lambda \, \gamma^5 \right) \psi_p \right] \left[\overline{\psi}_\nu \, \gamma^\mu \left(1 - \gamma^5 \right) \psi_e \right]$$

 $G_F = 1.166\,39(1)\,10^{-5}\,\text{GeV}^{-2}$ and $\lambda = 1.26992(69)$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \ \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}, \ \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Properties of H-like ions

$$W_{F^{\pm}\to F^{\mp}} = \frac{4\alpha}{3} \delta E^3 \frac{1}{\hbar m^2 c^4} \frac{2F^{\mp} + 1}{2I + 1}.$$
 (20)

The hyperfine splitting and the relaxation times $\tau_{1/2}$ are presented in Table I for several selected H-like ions. Note that the half-life $\tau_{1/2}$ depends on the atomic number Z of the parent nucleus as Z^{-9} .

In contrast to H-like ions, the He-like ions are typically in the 1^1S_0 ground state. An excited 2^3P_0 state in He-like ions, can be quenched by means of hyperfine interaction yielding a strong reduction of its half-life, which has been intensively explored both theoretically as well as experimentally [29,30].

TABLE I. The hyperfine splittings δE and the half-lives $\tau_{1/2}$ for the transitions between hyperfine levels for several H-like ions. Experimental magnetic moments, μ , are taken from Ref. [28]. In the last column, the total half-lives $T_{1/2}$ of neutral atoms in the nuclear ground state are given for comparison.

H-like ion	$I_i^{\pi_i} o I_f^{\pi_f}$	μ/μ_N	$\delta E \text{ (eV)}$	$ au_{1/2}$	$T_{1/2}$
¹⁹ Ne	$\frac{1}{2}^+ \rightarrow \frac{1}{2}^+$	-1.89	0.004	3 d	17.3 s
³⁷ Ar	$\frac{3}{2}^+ \rightarrow \frac{3}{2}^+$	+1.15	0.01	10 h	35 d
⁶⁴ Cu	$1^+ \rightarrow 0^+$	-0.22	0.009	7 h	12.7 h
⁶⁸ Ga	$1^+ \rightarrow 0^+$	+0.01	0.001	5 yr	67.7 min
⁷¹ Ge	$\frac{1}{2}^- \rightarrow \frac{3}{2}^-$	+0.55	0.041	12 min	11.2 d
108 Ag	$1^+ \rightarrow 0^+$	+2.69	0.53	0.24 s	2.37 min
¹³¹ Cs	$\frac{5}{2}^+ \rightarrow \frac{3}{2}^+$	+3.54	0.98	31 ms	9.69 d
¹⁴¹ Nd	$\frac{3}{2}^+ \rightarrow \frac{5}{2}^+$	+1.01	0.43	0.4 s	2.5 h
¹⁷⁸ Ta	$1^+ \rightarrow 0^+$	+2.74	2.87	1.6 ms	9.31 min

He-like ion matrix elements in the basis of H-like ion

$$\langle I, M, 1, f | \hat{O} | i, I, M \rangle_{\text{He}} \sqrt{2(2I+1)}$$

$$= \mp \frac{I + \frac{1}{2} \pm \frac{1}{2} + M}{\sqrt{2(I \pm \frac{1}{2}) + 1}} \left\langle I \pm \frac{1}{2}, M - \frac{1}{2} \middle| \hat{O} \middle| I \pm \frac{1}{2}, M - \frac{1}{2} \middle\rangle_{H}$$

$$\mp \frac{I + \frac{1}{2} \pm \frac{1}{2} - M}{\sqrt{2(I \pm \frac{1}{2}) + 1}} \left\langle I \pm \frac{1}{2}, M + \frac{1}{2} \middle| \hat{O} \middle| I \pm \frac{1}{2}, M + \frac{1}{2} \middle\rangle_{H}.$$
(9)

The matrix elements of the operator \hat{O} do not depend—for both H- and He-like systems—on the spin projection M due to the property (c). This condition is only satisfied when the two amplitudes with the same momentum but different projections are equal:

$$\left\langle I \pm \frac{1}{2}, M + \frac{1}{2} \middle| \hat{O} \middle| I \pm \frac{1}{2}, M + \frac{1}{2} \middle\rangle_{H} = \left\langle I \pm \frac{1}{2}, M - \frac{1}{2} \middle| \hat{O} \middle| I \pm \frac{1}{2}, M - \frac{1}{2} \middle\rangle_{H}.$$
 (10)