Kolektywne stany kwadrupolowe w teorii średniego pola

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Wstęp

- Punkt wyjścia: teoria średniego pola z oddziaływaniami Skyrme'a (czasem RMF)
- Deformacje kwadrupolowe
- Stany kolektywne (o parzystości dodatniej), hamiltonian Bohra, ATDHFB
- Model rdzeń-cząstka-dziura dla jąder nieparzysto-nieparzystych
- Zastosowania: jądra z $A \geqslant 70$, raczej nie magiczne

Zmienne kwadrupolowe

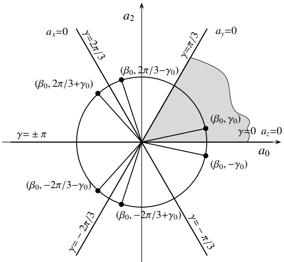
1. Tensor kwadrupolowy rozkładu masy $Q_{2\mu} = \langle \Psi | \sum_i r_i^2 Y_{2\mu}(i) \Psi \rangle$
2. Rozwinięcie powierzchni jądra w Y_{lm}, $r(\boldsymbol{\alpha}) = r_0(1 + \sum_\mu \alpha_\mu^* Y_{2\mu})$
3. Elipsoida (np. powierzchni jądra lub potencjału jednocząstkowego)

Układ osi głównych (wewnętrzny)

$$\{\alpha_\mu\} \xrightarrow{R(\Omega)} \{\tilde{\alpha}_0, \tilde{\alpha}_1 = \tilde{\alpha}_{-1} = 0, \tilde{\alpha}_2 = \tilde{\alpha}_{-2}\}$$

Zmienne β, γ opisujące deformację

$$\beta \cos\gamma = \tilde{\alpha}_0$$
$$\beta \sin\gamma / \sqrt{2} = \tilde{\alpha}_2 = \tilde{\alpha}_{-2}$$



Zmienne kwadrupolowe w teorii średniego pola

Zmienne β, γ

$$\beta\cos\gamma = cq_0 = c\langle\Psi|Q_0|\Psi\rangle = \langle\Psi|\sum_{i=1}^{A}(3z_i^2 - r_i^2)|\Psi\rangle$$
$$\beta\sin\gamma = cq_2 = c\langle\Psi|Q_2|\Psi\rangle = \langle\Psi|\sum_{i=1}^{A}\sqrt{3}(x_i^2 - y_i^2)|\Psi\rangle$$

$$c = \sqrt{\pi/5}/A\overline{r^2} \qquad \overline{r^2} = 3/5r_0^2A^{2/3}, \quad r_0 = 1.2 \text{ fm}$$

Obliczenia typu HFB z więzami $\longrightarrow \Psi(\beta,\gamma)$

$$\delta\langle\Psi|H_{\mathrm{micr}} - \lambda_0 Q_0 - \lambda_2 Q_2|\Psi\rangle = 0$$
$$\langle\Psi|Q_0|\Psi\rangle = q_0, \quad \langle\Psi|Q_2|\Psi\rangle = q_2$$

Szczegóły obliczeń

- Dobrze znane wersje oddziaływań Skyrme — SIII i SLy4, SkM*
- Oddziaływanie pairing: seniority (stałe G), $G \sum_\nu a^+_\nu a^+_{\bar\nu} a_\nu a_{\bar\nu}$
oddziaływanie δ: $V_0 \delta(\boldsymbol{r} - \boldsymbol{r}')$, lub $V_0(\rho(\boldsymbol{r}))\delta(\boldsymbol{r} - \boldsymbol{r}')$, e.g. $V_0(\rho) = 1 - \rho(\boldsymbol{r})/\rho_0$
- Natężenie oddziaływania pairing z różnic mas
- Po ustaleniu średniego pola — tylko Z i N

Adiabatic Time Dependent HFB

ATDHFB plus kilka przybliżeń

Hamiltonian kolektywny $(\boldsymbol{q})$

Energia kinetyczna, tensor masowy B

$$T(\boldsymbol{q}) = -\frac{\hbar^2}{2}\frac{1}{\sqrt{\det B}}\sum_{k,j}\frac{\partial}{\partial q_k}\sqrt{\det B}\,(B^{-1})_{kj}\frac{\partial}{\partial q_j}$$

Energia potencjalna

$$V(\boldsymbol{q}) = \langle\Psi(\boldsymbol{q})|H_{\text{micr}}|\Psi(\boldsymbol{q})\rangle$$

ATHFB w układzie wewnętrznym

Wibracyjne parametry masowe

$$B_{q_i q_j} = \hbar^2 (S_{(1)}^{-1} S_{(3)} S_{(1)}^{-1})_{ij}, \quad i, j = 0, 2$$

$$(S_{(n)})_{ij} = \sum_{\mu,\nu} \frac{\langle \mu | Q_i | \bar{\nu} \rangle \langle \bar{\nu} | Q_j | \mu \rangle}{(E_\mu + E_\nu)^n} (u_\mu v_\nu + u_\nu v_\mu)^2$$

Momenty bezwładności

$$J_k = \hbar^2 \sum_{\mu,\nu} \frac{|\langle \nu | j_k | \bar{\mu} \rangle|^2 (u_\mu v_\nu - u_\nu v_\mu)^2}{(E_\mu + E_\nu)}$$

Czynnik 1.3

Hamiltonian kwantowy w układzie wewnętrznym

Ogólny hamiltonian Bohra

$$H_{\mathrm{Bohr}} = T_{\mathrm{vib}} + T_{\mathrm{rot}} + V$$

$$T_{\mathrm{vib}} = -\frac{1}{2\sqrt{wr}} \left\{ \frac{1}{\beta^4} \left[\partial_\beta \left(\beta^4 \sqrt{\frac{r}{w}} B_{\gamma\gamma} \right) \partial_\beta - \partial_\beta \left(\beta^3 \sqrt{\frac{r}{w}} B_{\beta\gamma} \right) \partial_\gamma \right] + \right.$$

$$\left. + \frac{1}{\beta \sin 3\gamma} \left[-\partial_\gamma \left(\sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\gamma} \right) \partial_\beta + \frac{1}{\beta} \partial_\gamma \left(\sqrt{\frac{r}{w}} \sin 3\gamma B_{\beta\beta} \right) \partial_\gamma \right] \right\}$$

$$T_{\mathrm{rot}} = \frac{1}{2} \sum_{k=1}^{3} I_k^2(\Omega)/J_k; \qquad J_k = 4B_k(\beta,\gamma)\beta^2 \sin^2(\gamma - 2\pi k/3)$$

$$w = B_{\beta\beta}B_{\gamma\gamma} - B_{\beta\gamma}^2; \quad r = B_x B_y B_z$$

Energie, B(E2), momenty kwadrupolowe

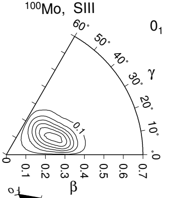
Hamiltonian kwantowy cd.

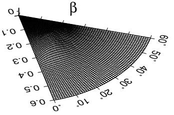
Funkcje falowe

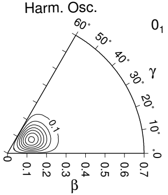
$$\Psi^{\text{(coll)}}_{IM\xi}(\beta,\gamma,\Omega) = \sum_{K=0(2),\text{even}}^{I \text{ or } I-1} F_{IK\xi}(\beta,\gamma)\phi^{I}_{MK}(\Omega)$$

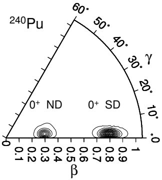
Rozkłady prawdopodobieństwa

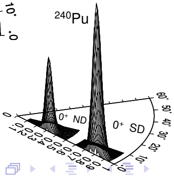
$$p_{I\xi}(\beta,\gamma) = \sum_{K} |F_{IK\xi}(\beta,\gamma)|^2 \sqrt{wr}\beta^4 |\sin 3\gamma|$$









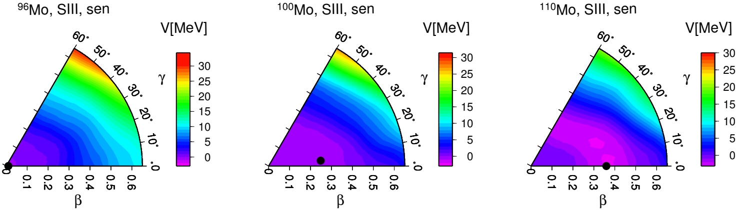


Izotopy $^{84-110}$Mo (p-p)

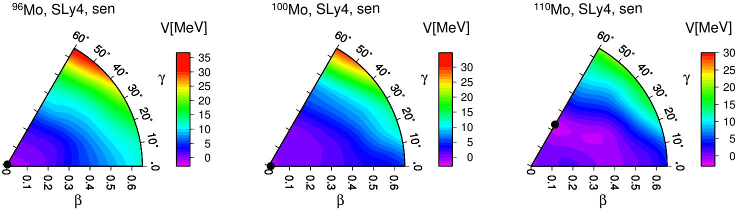
- Izotopy $^{84-110}$Mo (prawie wszystkie spośród znanych), nieco dokładniej:
- $^{96-100}$Mo, M. Zielińska, K. Wrzosek-Lipska
- ^{110}Mo

Energia potencjalna (wybrane izotopy)

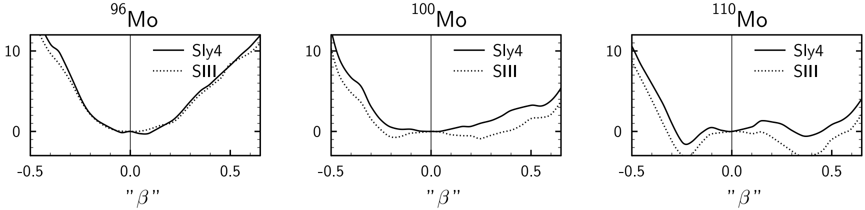
Wersja SIII



Wersja SLy4

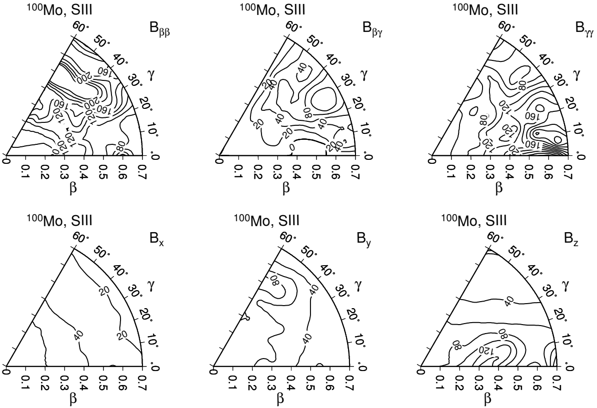


Energia potencjalna cd.



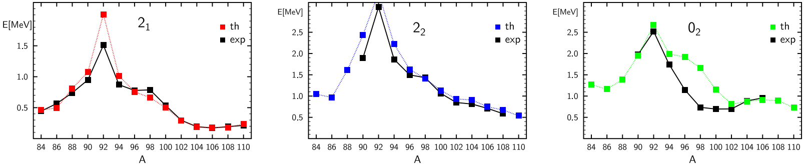
Izotop ^{100}Mo. Parametry masowe

Wersja Skyrme SIII

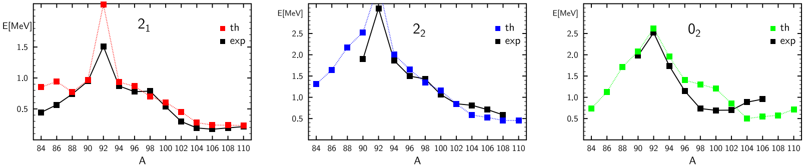


Wyniki. Poziomy energetyczne

SIII

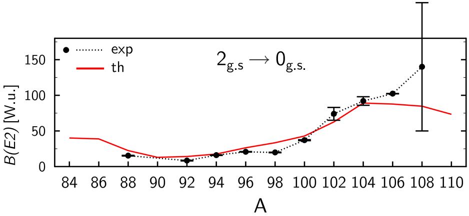


SLy4

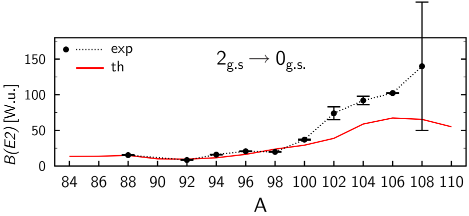


Wyniki. Przejścia E2 $2_1 \rightarrow 0_{\mathrm{g.s.}}$

SIII



SLy4

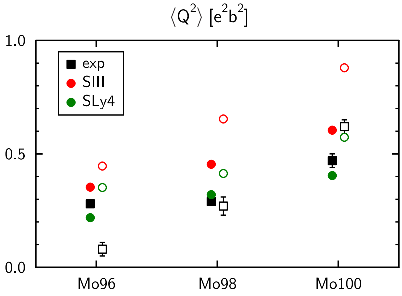


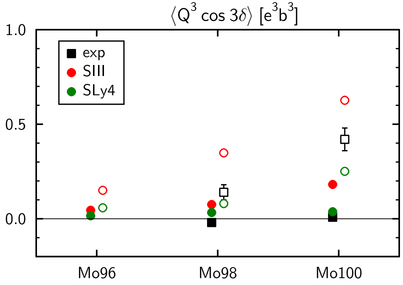
Niezmienniki E2. Izotopy $^{96-100}$Mo

$$[E2 \times E2]^0 \sim Q^2$$
$$[[E2 \times E2]^2 \times E2]^0 \sim Q^3 \cos 3\delta$$

$$\langle i \| Q^2 \| i \rangle \sim \sum_t \langle i \| E2 \| t \rangle \langle t \| E2 \| i \rangle$$

Poziomy $0_{g.s}$ i 0_2^+

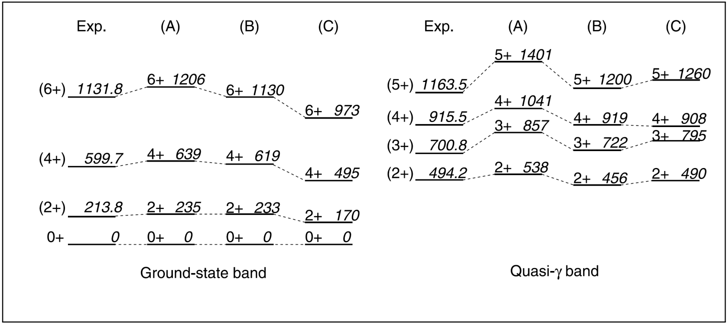




Daleko od β stabilności. ^{110}Mo (i ^{114}Ru)

Czas życia 0.27 s

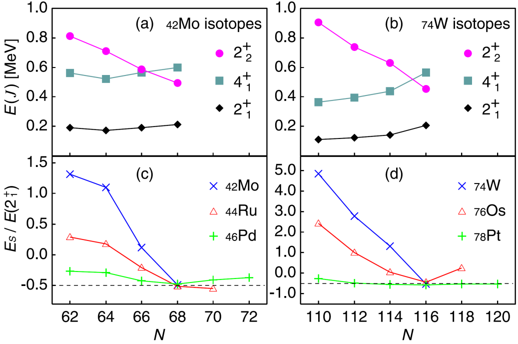
H. Watanabe et al, Phys.Lett. B 704 (2011) 270



(A) SIII, (B) Sly4, (C) QRPA Q+P (Hinohara et al.)

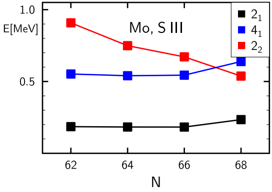
^{110}Mo, cd.

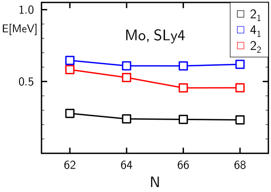
Exp



$$E_S = E_{2_2} - E_{4_1}$$

Th



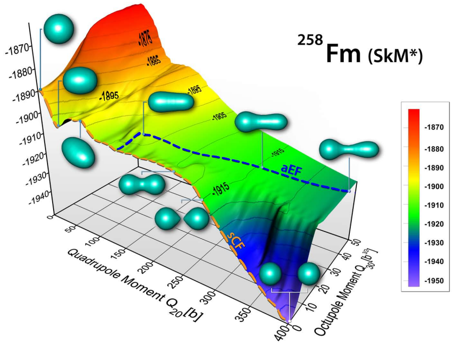


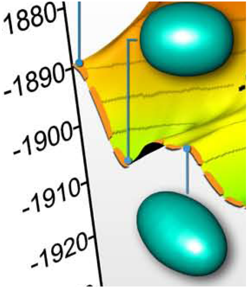
Kilka uwag

- Niezła zgodność z doświadczeniem
- Interesujące problemy:
  zamknięta powłoka $N = 50$,
  nisko leżące stany 0^+ w $^{96-100}$Mo
- Jakie średnie pole wybrać?

Aktynowce ^{238}U, ^{240,242}Pu, ^{246,248}Cm, ^{250,252}Cf

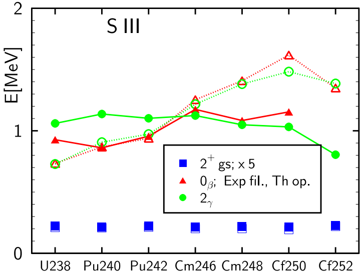
Jądra superciężkie, spontaniczne rozszczepienie.

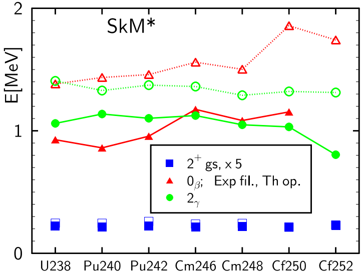




A.Staszczak, A.Baran, J.Dobaczewski, W.Nazarewicz, Phys.Rev. C **80**, 014309 (2009)

Aktynowce ^{238}U, ^{240,242}Pu, ^{246,248}Cm, ^{250,252}Cf
Poziomy 2_1, 0_2, 2_2

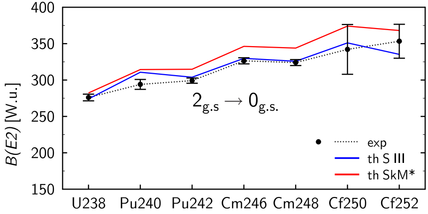


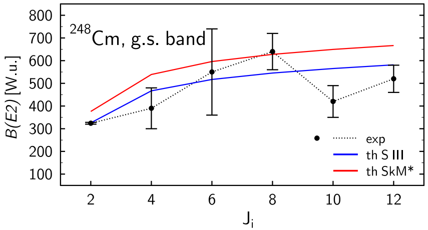


Poprawka wibracyjna dla 2_1^+: 10–15 keV

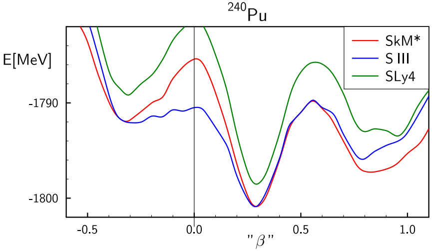
$2(2+1)/2J(\min)$

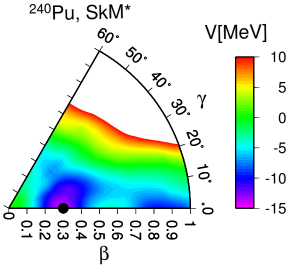
Aktynowce ^{238}U, ^{240,242}Pu, ^{246,248}Cm, ^{250,252}Cf
Przejścia E2

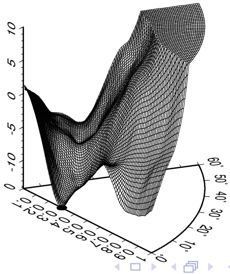




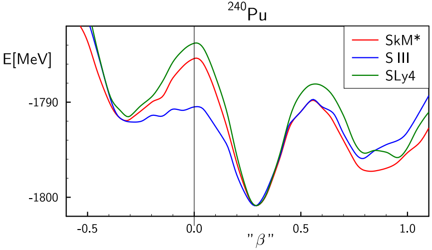
Energia potencjalna ^{240}Pu

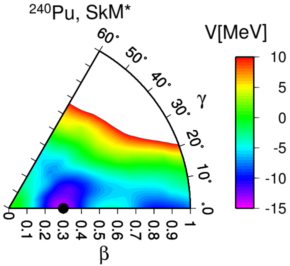


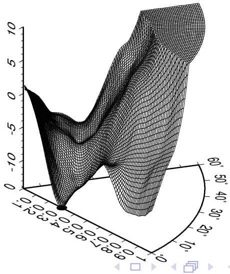




Energia potencjalna ^{240}Pu





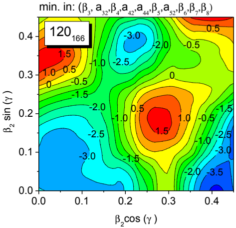


Stany w drugim minimum, ^{240}Pu

	J	#	$\langle\beta\rangle$	$\langle\gamma\rangle$	E	E_{SD}	Exp
ND	0	1	0.299	8.09			
	2	1	0.299	8.07	0.046		0.043
	0	2	0.315	10.54	1.420		0.861
	2	3	0.315	10.50	1.470		0.900
	2	2	0.314	13.07	1.347		1.137
SD	0	8	0.829	2.94	4.321		2.55
	2	13	0.829	2.94	4.342	0.021	0.020
	0	12	0.836	3.52	5.472	1.151	0.770
	2	20	0.835	3.53	5.494	1.173	0.785

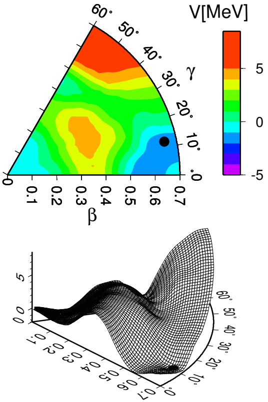
Jądra SDO (superoblate), $Z \sim 118$, $N \sim 164$

P. Jachimowicz, M. Kowal and J. Skalski
Phys. Rev. C 83, 054302 (2011)



$Z = 120$, $N = 166$, SkM*

$^{286}120$



Model Core-Particle-Hole Coupling

$$|\mathrm{odd-odd}, i\rangle = \sum_{j,k,m} U^i_{jkm} |\mathrm{core}, j\rangle |\mathrm{p}, k\rangle |\mathrm{n}, m\rangle$$

$$H_{\mathrm{o-o}} = H_{\mathrm{core}} - \chi Q q_{\mathrm{p}} - \chi Q q_{\mathrm{n}} - \chi' q_{\mathrm{p}} q_{\mathrm{n}} + h_{\mathrm{p}} + h_{\mathrm{n}}$$

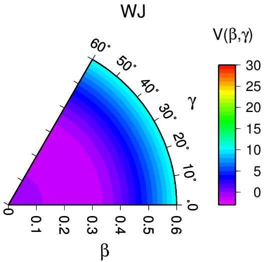
$Q, q_{\mathrm{p,n}}$— operatory kwadrupolowe

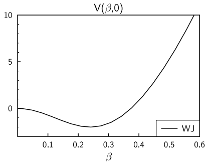
- Rdzenie:
  Hamiltonian Bohra (β, γ, Ω);
  sztywny rotor (Davydov-Filipov); (Ω); [$\tilde{\beta}, \tilde{\gamma}$ — fixed]
- Sektor jednocząstkowy:
  $\mathrm{p}h_{11/2} \otimes \mathrm{n}h^{-1}_{11/2}, \quad A \sim 130$

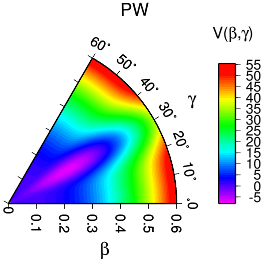
Ch. Droste, S.G. Rohoziński, K. Starosta

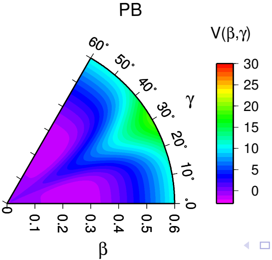
Przykłady S symetrycznych hamiltonianów

Wilets-Jean, studnia potencjału, bariera (w zmiennej γ).
SKE (jeden stały parametr masowy)

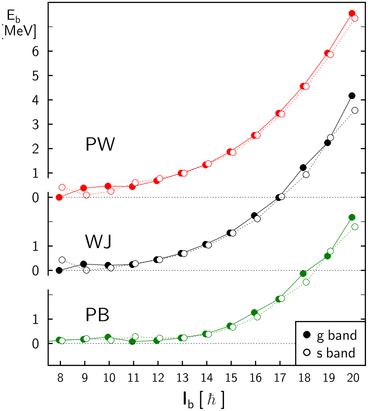




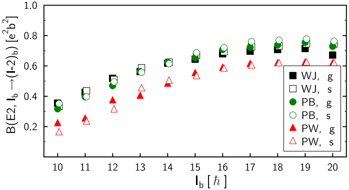




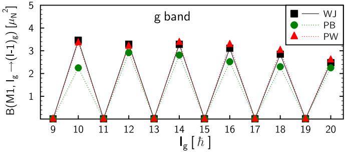
Wyniki dla hamiltonianów S symetrycznych

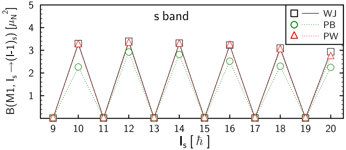


$\Delta I = 2$ E2 transitions



M1 transitions





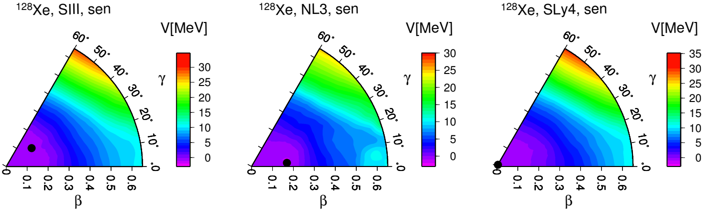
Rdzenie obliczone mikroskopowo

$$^{128}\text{Xe} + p \otimes n^{-1} \longrightarrow {}^{128}\text{Cs}$$
$$^{128}\text{Ba} + p \otimes n^{-1} \longrightarrow {}^{128}\text{La}$$

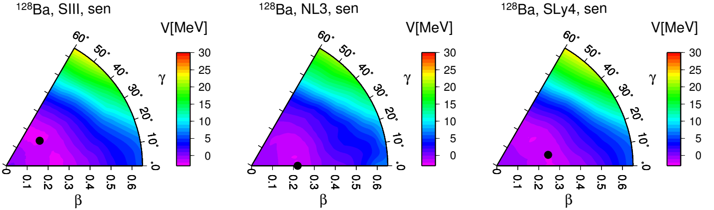
- Pole średnie: Skyrme SIII i SLy4, RMF NL3

Rdzenie obliczone mikroskopowo, energia potencjalna

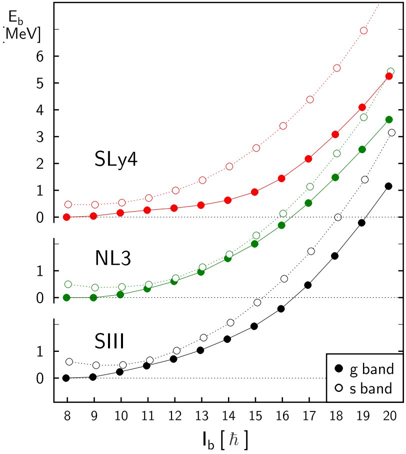
^{128}Xe



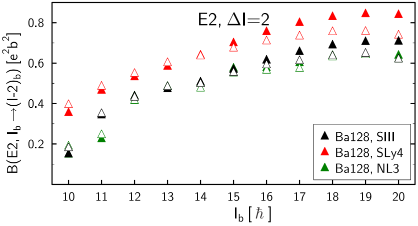
^{128}Ba



Jądro odd-odd z rdzeniem ^{128}Ba



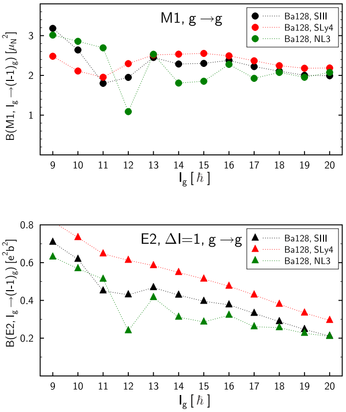
E2 in-band transitions



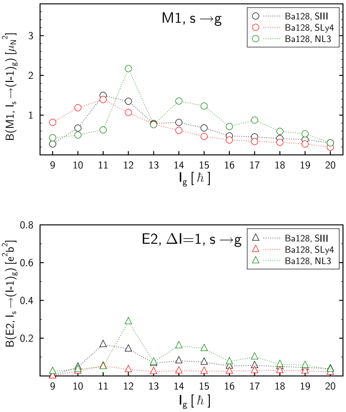
Jądro odd-odd z rdzeniem ^{128}Ba, cd.

$\Delta I = 1$ M1 and E2 transitions

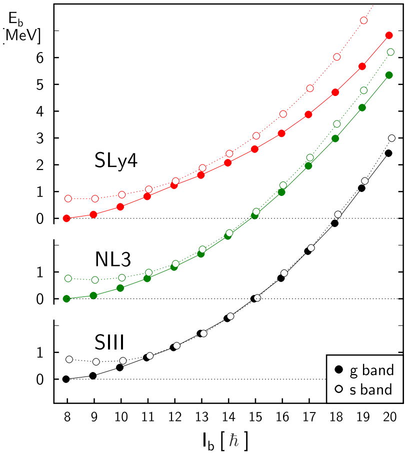
In-band



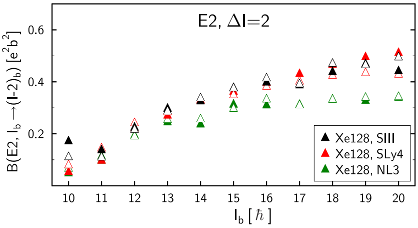
Inter-band



Jądro odd-odd z rdzeniem ^{128}Xe



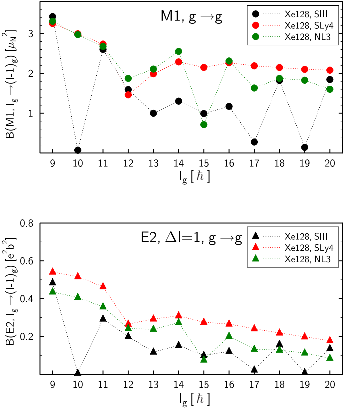
E2 in-band transitions



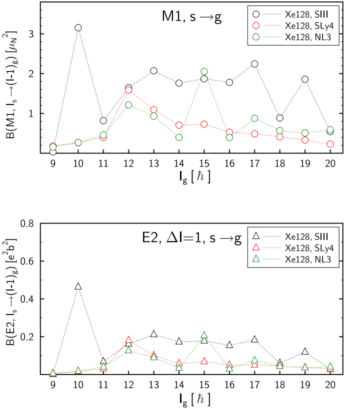
Jądro odd-odd z rdzeniem ^{128}Xe, cd.

Przejścia $\Delta I = 1$ M1 i E2

In-band



Inter-band



Kilka uwag

- Dublety pasm nie wymagają istnienia sztywnej deformacji
- Symetria S — pożyteczna w obliczeniach modelowych (tylko?)
- Dublety typu chiralnego można otrzymać dla rdzeni „mikroskopowych”